Renormalization of overlap quark bilinear operators in RI/MOM and RI/SMOM schemes

Yujiang Bi
on behalf of χQCD Collaboration

Wuhan University & IHEP

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Motivation

- Renormalization constants are necessary to convert lattice results to the continuous $\overline{\text{MS}}$ scheme.
- Quark mass and chiral quark condensate Σ [C. Wang etc., Chin. Phy. 2017].
- Decay constants such as f_{D^*}, f_V^T.
- Quark momenta fraction $\langle x \rangle$ in the nucleon [M. Sun, 1502.05482v3, M. Deka, PRD91(2015)]
- RI-MOM scheme is a common method of calculating RCs [G. Martinelli et al., NPB445(1995)].
- RI-SMOM scheme is a modified scheme which avoids the exceptional momenta [C. Strum et al., PRD80(2009).]
- Interesting to compare RI-MOM and RI-SMOM schemes numerically.

Renormalization procedure

- Green function:

$$G_{\mathcal{O}}(p_1, p_2) = \sum_{x,y} e^{-i(p_1\cdot x - p_2\cdot y)} \langle \psi(x)\mathcal{O}(0)\overline{\psi}(y)\rangle, p_1 = p_2.$$

- Quark Propagator:

$$S(p) = \sum_{x} e^{-ip\cdot x} \langle \psi(x)\overline{\psi}(0)\rangle.$$

- Vertex Function:

$$\Lambda_{\mathcal{O}}(p_1, p_2) = S^{-1}(p_1) G_{\mathcal{O}}(p_1, p_2) S^{-1}(p_2)$$

- RCs convention:

$$\psi_R = Z_q^{1/2}\psi_B, \mathcal{O}_R = Z_{\mathcal{O}}\mathcal{O}_B, \Lambda_{\mathcal{O},R} = (Z_{\mathcal{O}}/Z_q)\Lambda_{\mathcal{O},B}$$

- $\Lambda_{\mathcal{O}}^{\text{tree}} = I, \gamma_5, \gamma_\mu, \gamma_5\gamma_\mu, \sigma_{\mu\nu}$, denoted by S, P, V, A & T.

RI-MOM vs RI-SMOM

- Exceptional or non-exceptional momenta:

$$\text{RI-MOM: } q = p_1 - p_2 = 0, p_1^2 = p_2^2 = \mu^2$$
$$\text{RI-SMOM: } q = p_1 - p_2 \neq 0, p_1^2 = p_2^2 = q^2 = \mu^2.$$

- Axial vector renormalization condition:

$$\text{RI-MOM:} \quad \lim_{m_R \to 0} \frac{1}{48} Z_q^{-1} Z_A \mathrm{Tr}\left[\Lambda^\mu_{A,B}(p_1,p_2)\gamma_5\gamma_\mu\right]\Big|_{p_1^2=p_2^2=\mu^2} = 1$$
$$\text{RI-SMOM:} \quad \lim_{m_R \to 0} \frac{1}{12q^2} Z_q^{-1} Z_A \mathrm{Tr}\left[q_\mu\Lambda^\mu_{A,B}(p_1,p_2)\gamma_5 \not{q}\right]\Big|_{q^2=\mu^2} = 1$$

  More in C. Strum et al., PRD80, 014501(2009).

- Smaller non-perturbative effects at small momentum scale and conversion ratio $C_{\mathcal{O}}^{\text{RI-SMOM}}$ converges faster.
- $Z_A = Z_V$ is satisfied better at small momentum in RI-SMOM.
- Using Z_A from Ward Identity to derive other RCs in both schemes.

Z_A from Ward Identity

- Using Z_A^{WI} which equals to Z_A^{RI} in RI scheme to derive Z_q in RI-MOM and similar in RI-SMOM,

$$Z_q = Z_A \frac{1}{48} \text{Tr}\left[\Lambda_A(p)\Lambda_A^{\text{tree}}(p)^{-1}\right]\Big|_{p^2=\mu^2}.$$

- From PCAC and $Z_m = Z_P^{-1}$ if with exact chiral symmetry,

$$Z_A^{\text{WI}} \partial_\mu A_\mu = 2 Z_m m_q Z_P P, A_\mu = \overline{\psi}\gamma_\mu\gamma_5\psi, P = \overline{\psi}\gamma_5\psi$$

one gets

$$Z_A \partial_\mu \langle 0|A_\mu|\pi\rangle = 2m_q\langle 0|P|\pi\rangle.$$

- If the pion at rest and from $G_{PP}(\overrightarrow{p} = 0, t) = \sum_{\vec{x}}\langle 0|P(x)P^\dagger(0)|0\rangle$ and $G_{A_4P}(\overrightarrow{p} = 0, t) = \sum_{\vec{x}}\langle 0|A_4(x)P^\dagger(0)|0\rangle$, one gets

$$Z_A = \lim_{m_q\to 0, t\to\infty} \frac{2m_q G_{PP}(\overrightarrow{p} = 0, t)}{m_\pi G_{A_4P}(\overrightarrow{p} = 0, t)}$$

Lattice setup

Calculation for overlap quark bilinears is on $2+1$-flavor domain-wall ensembles from RBC/UKQCD Collaboration [T. Blum et al., PRD93].

- Lattice size: 96×48^3, $N_{\text{conf}}=81$, and $a^{-1}=1.730(4)$ GeV
- Lattice pion mass at the physical point.
- Overlap fermion has good chiral symmetry on the lattice.
- 10 overlap valence quark masses for chiral extrapolation.
- Periodic boundary condition in time-space directions

$$ap=\left(\frac{2k_t\pi}{T},\frac{2k_x\pi}{L},\frac{2k_y\pi}{L},\frac{2k_z\pi}{L}\right)$$

  $k_\mu=-12,-11,\cdots,11,12.$

- Applying momentum cutting off in RI-MOM scheme

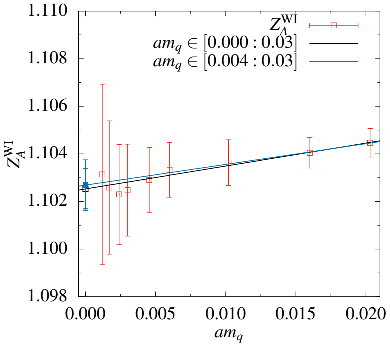
$$(\sum_\mu p_\mu^4)/(\sum_\mu p_\mu^2)^2\leq 0.29$$

Z_A^{WI} from Ward Identity

Table 1: $Z_A^{\mathrm{WI}}(am_q)$ for 10 valence quark masses

am_q	0.001200	0.001700	0.002400	0.003000	0.004550
Z_A	1.1031(38)	1.1026(28)	1.1023(21)	1.1025(19)	1.1029(14)
am_q	0.006000	0.010200	0.016000	0.020300	0.065000
Z_A	1.1033(12)	1.1036(10)	1.10405(64)	1.10447(60)	1.10743(44)

$$Z_A^{\mathrm{WI}} = 1.1025(8),\ \ am_q \in [0.000 : 0.030]$$
$$Z_A^{\mathrm{WI}} = 1.1027(11), am_q \in [0.004 : 0.030]$$



Z_V/Z_A in RI-MOM and RI-SMOM

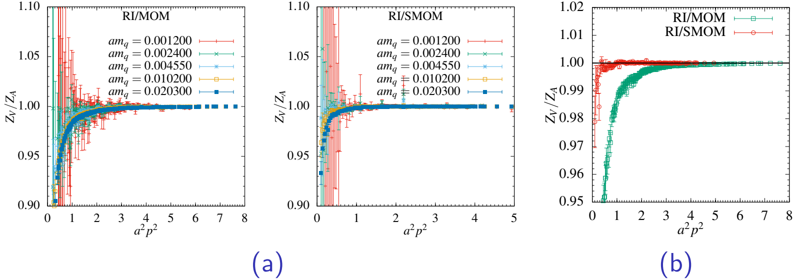
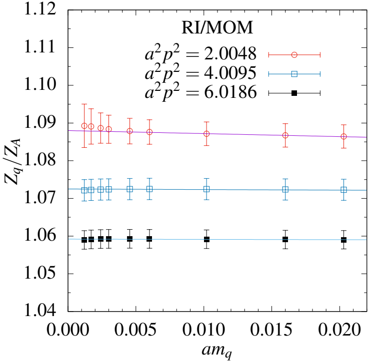
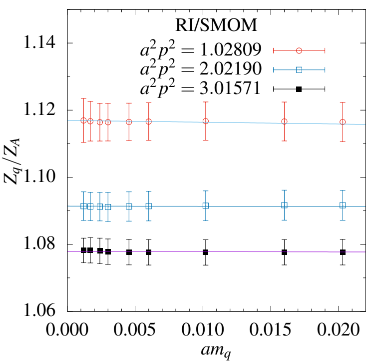


Figure 1: (a): Z_V/Z_A valence quark mass dependence in two schemes; (b) Z_V/Z_A at the chiral limit in two schemes.

Z_V/Z_A equals to 1 at very small a^2p^2 in RI-SMOM scheme

Z_q/Z_A valence quark mass dependence

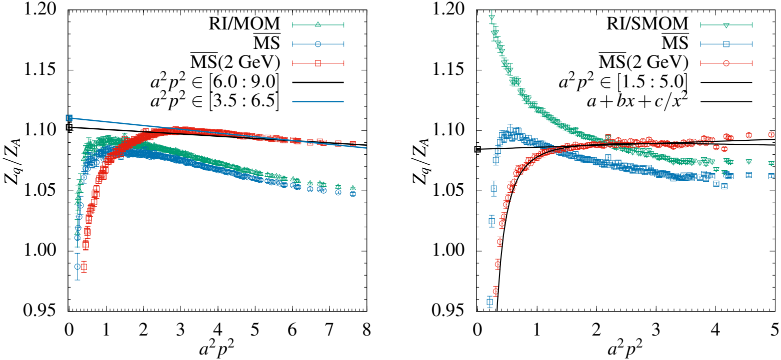




- Small dependence on am_q in both schemes.
- chiral extrapolating linearly

$$Z(am_q) = A + B(am_q)$$

Fitting Z_q/Z_A

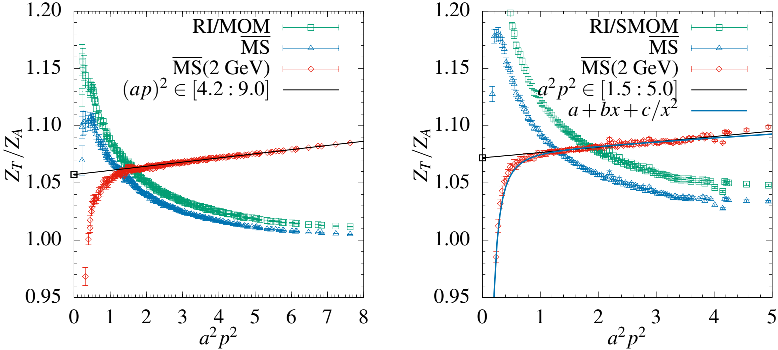


- RI-MOM: $Z_q^{\overline{\text{MS}}}/Z_A(\mu = 2\ \text{GeV}) = 1.1027(43)(73)$, $a^2p^2 \geq 5$.
- RI-SMOM: $Z_q^{\overline{\text{MS}}}/Z_A(\mu = 2\ \text{GeV}) = 1.0832(30)(13)$, $a^2p^2 \geq 1.5$.

$$Z_q^{\overline{\text{MS}}}/Z_A = A + B(a^2p^2) + C/(a^2p^2)^2$$

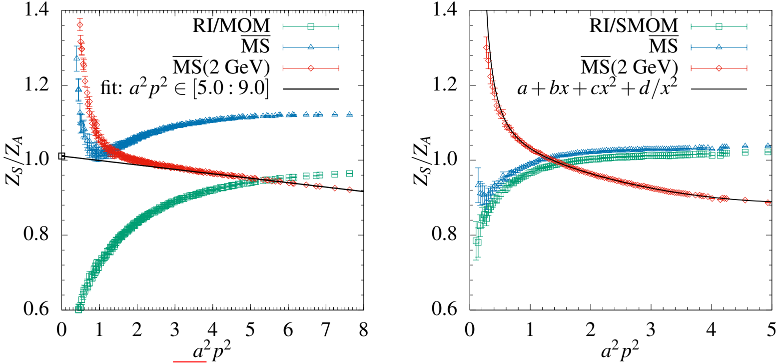
- Only statistical and fitting range or fitting ansatz errors.

Fitting Z_T/Z_A



- MOM: $Z_T^{\overline{\text{MS}}}/Z_A(\mu = 2\ \text{GeV}) = 1.0574(4)$, $a^2p^2 \geq 4.2$.
- SMOM: $Z_T^{\overline{\text{MS}}}/Z_A(\mu = 2\ \text{GeV}) = 1.0719(9)$, $a^2p^2 \geq 1.5$.
- Both fitting linearly.

Scalar density renormalization



- MOM: $Z_S^{\overline{\text{MS}}}/Z_A(\mu = 2\ \text{GeV}) = 1.0109(13)$, $a^2p^2 \geq 5.0$.
- SMOM: $Z_S^{\overline{\text{MS}}}/Z_A(\mu = 2\ \text{GeV}) = 1.0519(92)$, $a^2p^2 \geq 1.5$.

$$Z_S^{\overline{\text{MS}}}/Z_A = A + B(a^2p^2) + C(a^2p^2)^2 + D/(a^2p^2)^2$$

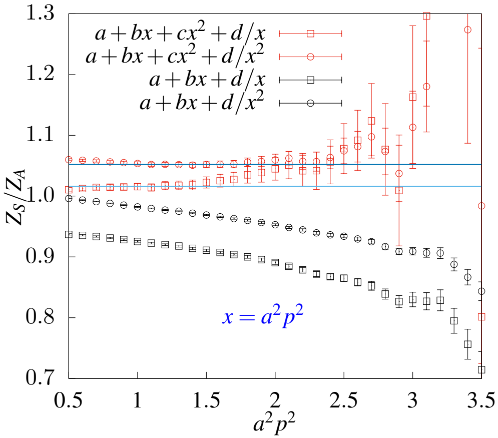
Seems similar to T. Bhattacharya, PRD89, PRD92, PRD94.

Fitting Z_S/Z_A using various models

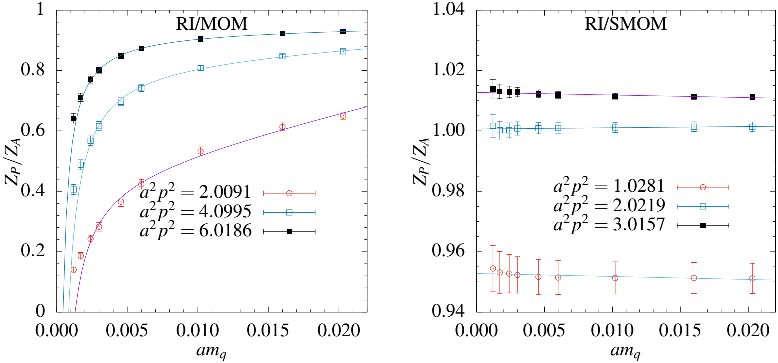
Let $x = a^2p^2$,

$$
\begin{aligned}
\text{(a)}: f(x) &= A + Bx + D/x^2 \\
\text{(b)}: f(x) &= A + Bx + Cx^2 + D/x^2 \\
\text{(c)}: f(x) &= A + Bx + D/x \\
\text{(d)}: f(x) &= A + Bx + Cx^2 + D/x \\
\text{(e)}: f(x) &= A + Bx + Cx^2
\end{aligned}
$$

- Uncertainty from models (b) & (d) is about 3%.
- Unclear unexpected non-perturbative effects!!
- Truncation and coupling constant errors etc. to be determined!



Z_P/Z_A valence quark mass dependence

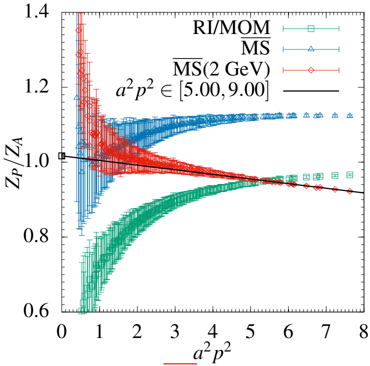


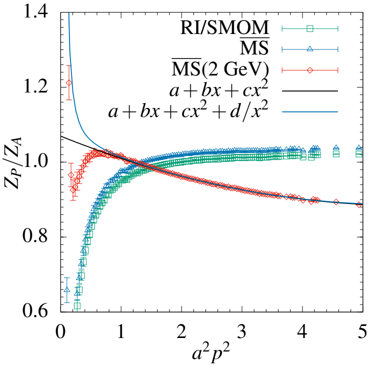
- Removing Goldstone boson contamination in RI-MOM:

$$Z_P^{\text{sub},-1} = A + B(am_q) + C/(am_q)$$

- No apparent sign of Goldstone boson pole effect in RI-SMOM.

Fitting Z_P/Z_A in RI-MOM and RI-SMOM schemes





- MOM: $Z_P^{\overline{\text{MS}}}/Z_A(\mu = 2\text{ GeV}) = 1.0168(55)$, $a^2p^2 \geq 5.0$
- SMOM: $Z_P^{\overline{\text{MS}}}/Z_A(\mu = 2\text{ GeV}) = 1.059(10)$, $a^2p^2 \geq 1.5$, similar to Z_S/Z_A.

Summary

Table 2: Summary of all renormalization constants in $\overline{\mathrm{MS}}$ at $\mu = 2$ GeV.

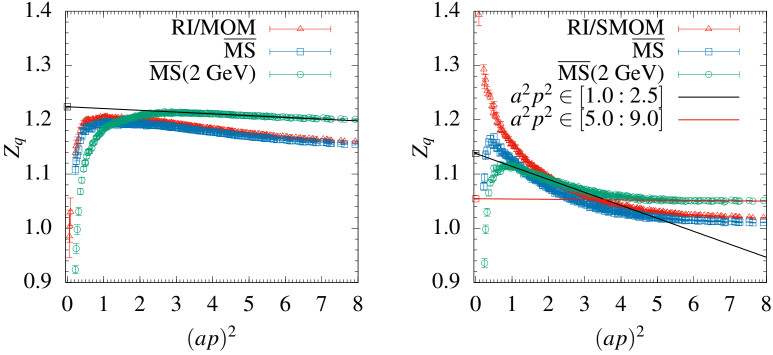
scheme	Z_A	Z_q/Z_A	Z_T/Z_A	Z_S/Z_A	Z_P/Z_A
MOM	1.1028(55)	1.1027(81)	1.0574(4)	1.0109(13)	1.0168(55)
SMOM		1.0844(33)	1.0719(9)	1.0519(92)	1.059(10)
Comment		$< 1.5\%$	$< 1.5\%$	$> 3\%??$	

- Calculating RCs for overlap quark bilinears on domain wall configurations using RI-MOM and RI-SMOM schemes.
- $Z_S = Z_P$ and $Z_A = Z_V$ are satisfied as expected.
- Goldstone boson contamination suppressed for Z_P in RI-SMOM.
- Unexpected non-perturbative effects occur in Z_S & Z_P in RI-SMOM and still unclear!!
- Systematic uncertainty will be determined later.

Thank you!

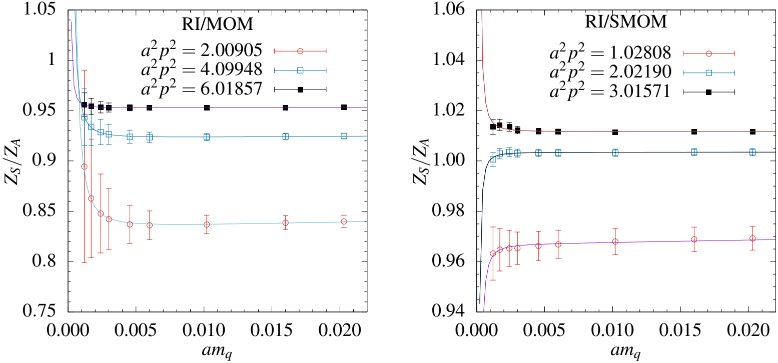
Z_q directly from quark propagator

$$Z_q = \frac{1}{12}\mathrm{Tr}\left[S_B^{-1}(p)S^{\mathrm{tree}}(p)\right]\Big|_{p^2=\mu^2}.$$



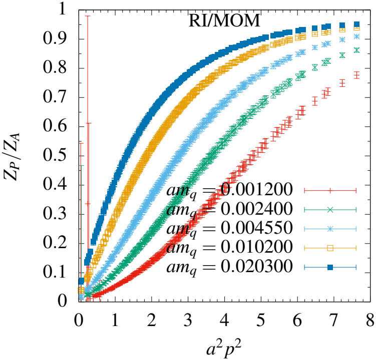
Could not find a reliable fitting range for Z_q!!

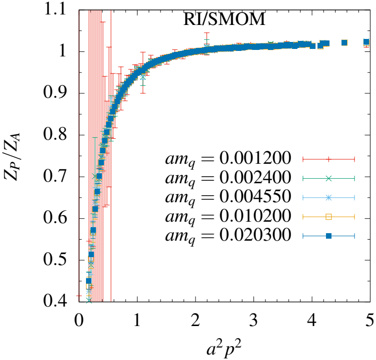
Z_S/Z_A against am_q for different a^2p^2



$$Z_S(am_q) = \mathcal{Z}_S + B(am_q) + \frac{C}{(am_q)^2}$$

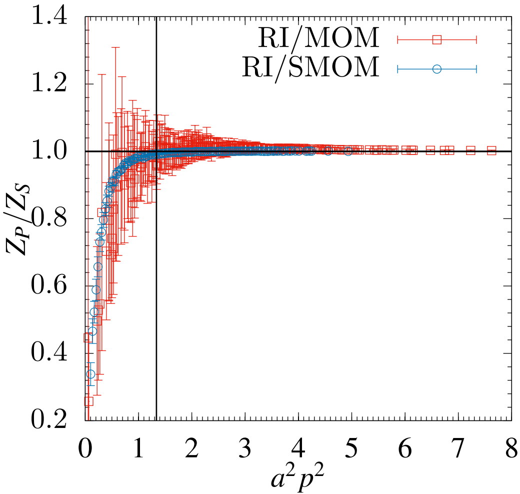
$Z_{\mathcal{P}}/Z_A$ valence quark mass dependence comparison





A much smaller dependence on valence quark mass in RI-SMOM.

Z_P/Z_S in RI-MOM and RI-SMOM



$Z_S = Z_P$ is satisfied in both schemes, and better in RI-SMOM