Estimates of Scaling Violations for Pure SU(2) Lattice Gauge Theory

David A. Clarke and Bernd A. Berg

Florida State University

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Outline

- Introduction
- Statistics
- Scaling and asymptotic scaling analysis
- Conclusions

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Action and energy operators

- Studied pure SU(2) LGT in 4D with Wilson action

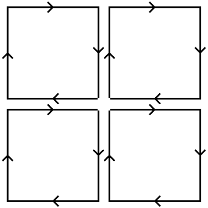
$$S = \beta \sum_{\square} \left(1 - \frac{1}{2} \operatorname{Tr} U_{\square} \right)$$

- Plaquettes with parameterization

$$\langle U_{\square} \rangle = a_0 \mathbf{1} + i \sum_{i=1}^{3} a_i \sigma_i$$

- Examined three discretizations of action:
  - $E_0 \equiv 2\,[1 - a_0]$
  - $E_1 \equiv \sum_{i=1}^{3} a_i^2$
  - $E_4 \equiv \frac{1}{4} \sum_{i=1}^{3} \left(a_i^{(1)} + a_i^{(2)} + a_i^{(3)} + a_i^{(4)} \right)^2$

    suggested by Lüscher[1]



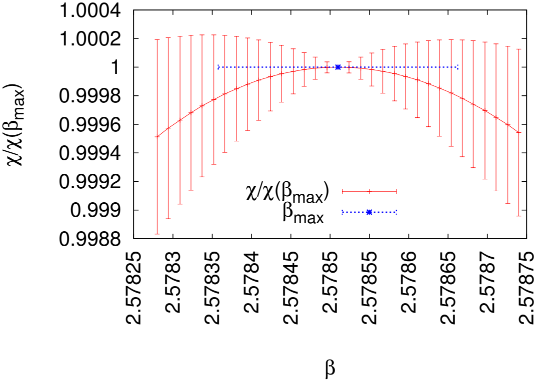
[1] M. Lüscher. In: *JHEP* 2010.8 (2010), pp. 1–18.

Deconfining phase transition temperature

- Determine critical coupling constant $\beta_c(N_\tau)$ from $\beta_c(N_s, N_\tau)$ using three parameter fit

$$\beta_c(N_s, N_\tau) = \beta_c(N_\tau) + A\, N_s^{-B}$$

- Inverting the result gives length $N_\tau(\beta)$ unambiguously
- Example $64^3 \times 10$ lattice



Gradient flow

- Lüscher's gradient flow equation[1]

$$\dot{V}_\mu(x,t) = -g^2 V_\mu(x,t)\partial_{x,\mu} S[V(t)]$$

  - Flow time t and initial condition $V_\mu(x,t)\big|_{t=0} = U_\mu(x)$
  - Gradient flow equation drives action down

- Target value y defines length $s(\beta) = \sqrt{t_y(\beta)}$ implicitly via

$$t^2 \langle E_t \rangle \,|_{t=t_y} = y$$

- Requires no fits or extrapolations
- Found at least 100 times more efficient than $N_\tau(\beta)$
- Some ambiguity in choosing y

QUESTION: Does choosing one target over another result in seriously distinct scaling behavior?

Cooling flow

- Introduced by Berg[2] for $O(3)$ topological charge, has many applications as a smoothing procedure (review[3])
- Proposed by Bonati and D'Elia[4] for scale setting
- Replace link variable with one that locally minimizes action

$$V_\mu(x, n_c) = \frac{V_\mu^\sqcup(x, n_c - 1)}{|V_\mu^\sqcup(x, n_c - 1)|}$$

- In 4D, n_c cooling sweeps corresponds[4] to a gradient flow time

$$t_c = n_c/3$$

- Length $x(\beta) = \sqrt{t_y(\beta)}$ defined implicitly like gradient flow
- ≥ 34 times faster than gradient with Runge-Kutta $\epsilon = 0.01$

QUESTION: Does the cooling flow experience significantly larger scaling violations than the gradient flow?

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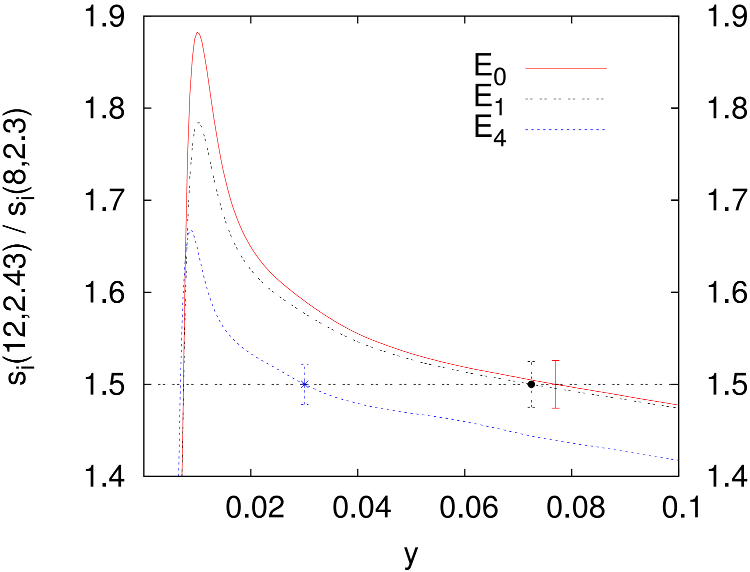
[2] B. A. Berg. In: *Phys. Lett.* 104B (1981), pp. 475–480.

[3] E. Vicari and H. Panagopoulos. In: *Phys. Rep.* 470 (2009), pp. 93–150.

[4] C. Bonati and M. D'Elia. In: *Phys. Rev. D* 89 (2014), p. 105005.

Determination of target value



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Statistics

- Deconfinement: $N_{\text{cnfg}} \geq 32$ with $2^{14} - 2^{18}$ MCOR sweeps
- Gradient and cooling: $N_{\text{cnfg}} = 128$ with $2^{11} - 2^{13}$ MCOR sweeps
- Gradient and cooling flows applied to latter configurations
- Largest pure SU(2) lattices generated (compare[5])

Table: Gradient and cooling lattices

Lattice	β values
16^4	2.300
28^4	2.430, 2.510
40^4	2.574, 2.620, 2.670, 2.710, 2.751
44^4	2.816
52^4	2.875

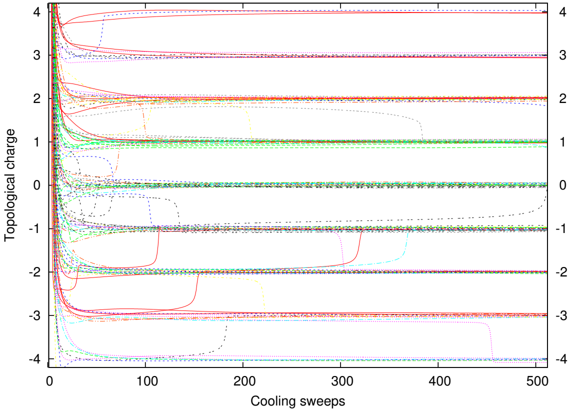
Table: Deconfining lattices

$N_s \times N_\tau$	β_c value
$56^3 \times 4$	2.300
$60^3 \times 6$	2.430
$80^3 \times 8$	2.510
$64^3 \times 10$	2.578
$52^3 \times 12$	2.636

[5] B. Lucini, M. Teper, and U. Wenger. In: *JHEP* 2004.1 (2004).

Statistics

- Estimates of τ_{int} of for time series of 128 measured scale values all statistically compatible with $\tau_{\text{int}} = 1$
- Same thing for topological charge (naive discretization)
- Example $\beta = 2.816$ on 44^4 lattice



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Scaling analysis

- Length ratios exhibit $\mathcal{O}(a^2)$ scaling violations

$$R_{i,j} \equiv \frac{L_i}{L_j} \approx r_{i,j} + k_{i,j} a^2 \Lambda_L^2 = r_{i,j} + c_{i,j} \left(\frac{1}{L_j} \right)^2$$

- Length scales from the project
  - $L_1 - L_6$: gradient length scales (3 operators, 2 targets)
  - $L_7 - L_{12}$: cooling length scales
  - N_τ: deconfining length scale
- In the following we fix $L_j = L_{10}$ and plot

$$\frac{R_{i,10}}{r_{i,10}} = 1 + c'_{i,10} \left(\frac{1}{L_{10}} \right)^2$$

Asymptotic scaling analysis

- Asymptotic scaling relation

$$a\Lambda_L \approx \exp\left(-\frac{1}{2b_0g^2}\right)\left(b_0g^2\right)^{-b_1/2b_0^2}\left(1+q_1g^2\right) \equiv f_{as}^1(\beta)$$

- Follow Allton's suggestion[6] of including asymptotic scaling corrections. Taylor series expansion of length L_k in powers of a

$$\frac{1}{L_k} = c_k\Lambda_L\left(1+\sum_{j=1}^{\infty}\alpha_{k,j}(a\Lambda_L)^j\right)$$

- Combine equations and truncate power series[7]

$$L_k \approx \frac{c_k}{f_{as}^1(\beta)}\left(1+\sum_{j=1}^{3}\alpha_{k,j}[f_{as}^1(\beta)]^j\right)$$

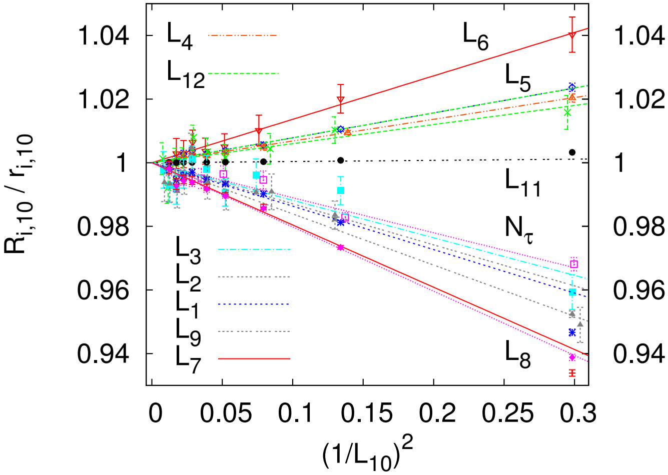
with $q_1 = 0.08324$[8] and enforce $R_{i,j} = \mathcal{O}(a^2)$

[6] C. R. Allton. In: *Nucl. Phys. B (Proc. Suppl.)* 53 (1997), p. 867.

[7] B. A. Berg. In: *Phys. Rev. D* 92 (2015).

[8] B. Allés, A. Feo, and H. Panagopoulos. In: *Nucl. Phys. B* 491 (1997), pp. 498–512.

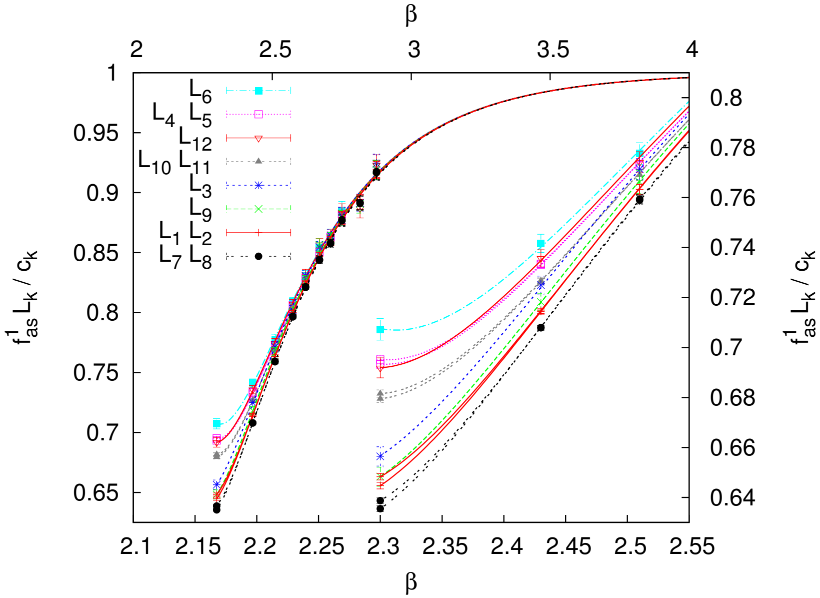
Scaling violations



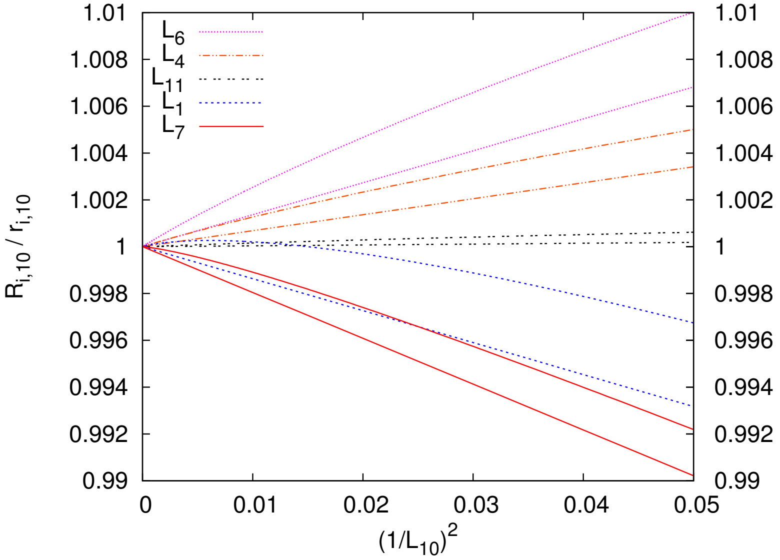
$\sim 10\%$ violation at $\beta = 2.3$,

$\sim 2\%$ violation at $\beta = 2.574$

Asymptotic scaling fits



Comparison scaling and asymptotic scaling for ratios



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Conclusions

- Does choosing different target values for the gradient and cooling flows lead to seriously distinct scaling behavior?
  - Not if using physical input to guide initial scaling values
  - Here we use the deconfinement scale
- Does the cooling length experience significant scaling violations compared to the deconfining and gradient scales?
  - No noticeable loss of accuracy using cooling flow
  - Cooling ≥ 34 times more efficient than gradient with $\epsilon = 0.01$ Runge-Kutta in pure SU(2)
- What is a reasonable estimate for the combined systematic error due to choice of scale and fitting form?
  - For pure SU(2) at around $\beta = 2.6$ it is roughly 2%
  - Therefore pure SU(2) simulations must go rather deep in the scaling region to supress systematic error of this variety to $\sim 1\%$, which can easily outweigh statistical uncertainties
  - Maybe be of interest to continuum limit extrapolations in QCD

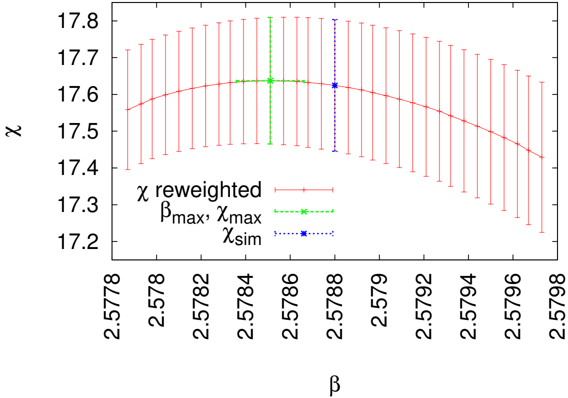
Thank you!

Backup: Defconfinement scale

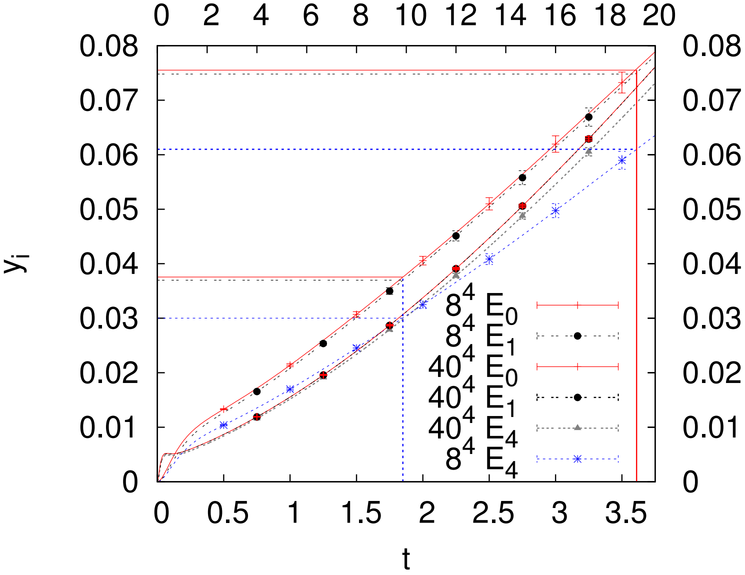
- Deconfining scale error bars

$$\Delta N_\tau = \frac{N_\tau}{L_{10}^3(\beta_c)} \left[L_{10}^3(\beta_c) + L_{10}^3(\beta_c - \Delta\beta_c) \right]$$

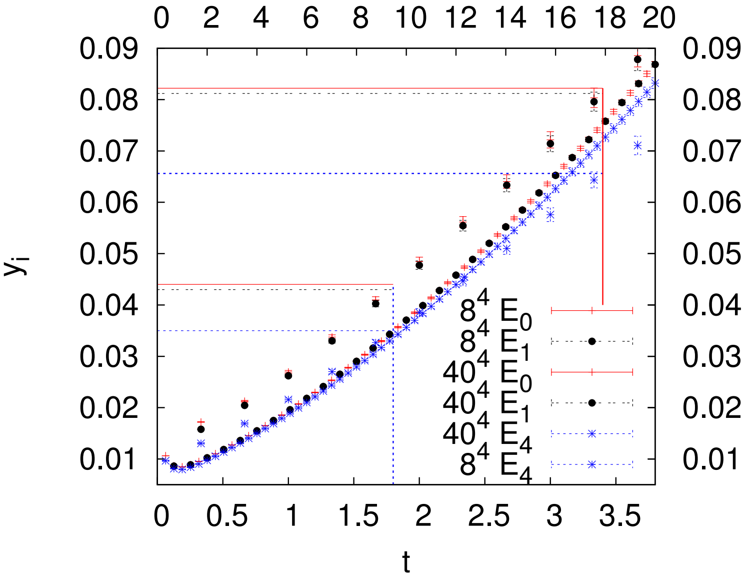
- Example susceptibility on $64^3 \times 10$ lattice



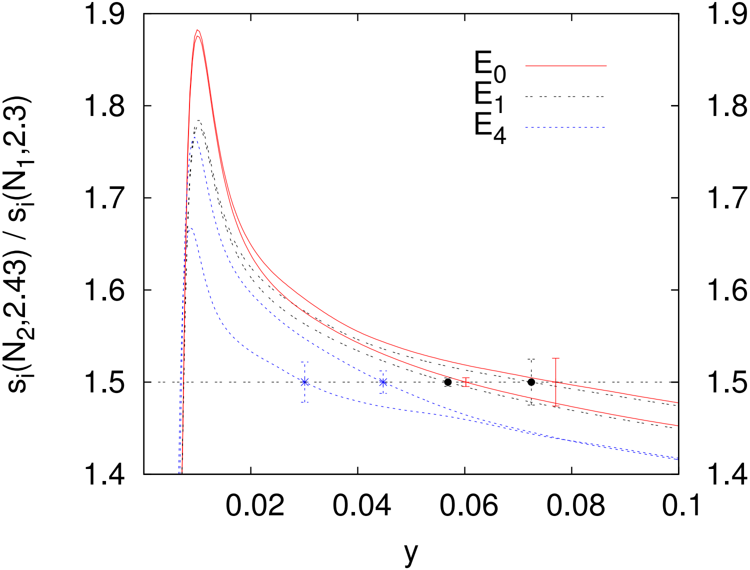
Backup: Determination of gradient scale



Backup: Determination of cooling scale



Backup: Determination of target value



Backup: Topological charge

- Topological charge discretization

$$Q = -\frac{1}{2^9\pi^2}\sum_x \sum_{\mu\nu\rho\sigma=\pm 1}^{\pm 4} \tilde{\epsilon}_{\mu\nu\rho\sigma} \,\mathrm{Tr}\, U_{\mu\nu}(x) U_{\rho\sigma}(x)$$

with

$$\tilde{\epsilon}_{\mu\nu\rho\sigma} = \begin{cases} \epsilon_{\mu\nu\rho\sigma} & \text{if } \mu,\nu,\rho,\sigma > 0 \\ \epsilon_{-(\mu)\nu\rho\sigma} & \text{otherwise.} \end{cases}$$