LATTICE CALCULATION OF HADRONIC TENSOR OF THE NUCLEON

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INTRODUCTION

In recent years, considerable interest has been aroused to calculate the parton distribution functions (PDFs) on the lattice. The nucleon hadronic tensor has been formulated in the Euclidean path-integral formalism [1, 2]. Numerical methods have been proposed to address the inverse Laplace problem in order to convert the the hadronic tensor to the Minkowski space [3]. We should point out that no renomalization is needed for the hadronic tensor which involves vector currents and it is frame-independent so that it can be calculated in any momentum frame of the nucleon. Furthermore, the valence+connected-sea (VCS) partons, the connected sea (CS) anti-partons, and the disconnected-sea (DS) partons and anti-partons are separately calculated in different path-integral diagrams [1, 2] and, thus, $\bar{u}$ and $\bar{d}$ in the CS which are responsible for the Gottfried sum rule violation can be revealed explicitly [4]. The challenging task of converting the Euclidian hadronic tensor calculated on the lattice to the Minkowski space is tackled with different methods in this exploratory work. The calculation is carried out on a $12^3 \times 128$ anisotropic clover lattice generated by the CLQCD collaboration with $M_\pi \sim 640$ MeV, $a_s = 0.18$ fm and $\xi = 5$.

EUCLIDEAN HADRONIC TENSOR

The definition of hadronic tensor in Minkowski space is

$$W_{\mu\nu}(q^2,\nu) = \frac{1}{4\pi}\int d^4z e^{iq\cdot z}\langle p,s|[J_\mu^\dagger(z)J_\nu(0)]|p,s\rangle, \quad (1)$$

where $|p,s\rangle$ is the nucleon state and J_μ is the vector current. The matrix element therein can be extracted on the lattice through the following 4-point function

$$G = \sum_{\vec{x}_f} e^{-i\vec{p}\cdot\vec{x}_f} \sum_{\vec{x}_2\vec{x}_1} e^{-i\vec{q}\cdot(\vec{x}_2-\vec{x}_1)}\langle\chi_N(\vec{x}_f,t_f)J_\mu(\vec{x}_2,t_2)J_\nu(\vec{x}_1,t_1)\bar{\chi}_N(\vec{0},t_0)\rangle, \quad (2)$$

where χ_N is the nucleon operator and the Euclidean hadronic tensor can be obtained by the ratio of this 4-point function to the nucleon 2-point function,

$$\tilde{W}_{\mu\nu}(q^2,\tau) \propto \sum_n \langle p,s|J_\mu|n\rangle\langle n|J_\nu|p,s\rangle e^{-(E_n-E_p)\tau}, \quad (3)$$

where $\tau = t_2 - t_1$, $E_n - E_p = \nu$.

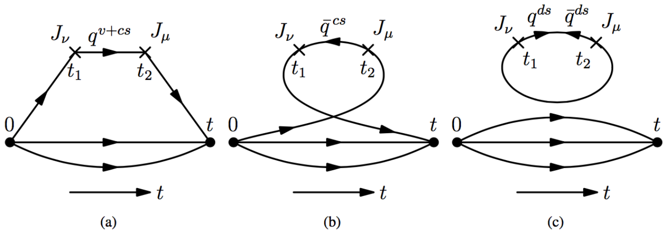

Figure 1: Three topologically distinct diagrams.

There are three topologically distinct diagrams in the Euclidean-path integral formulation of the hadronic tensor (shown in Fig. 1). Diagram (b) reveals the CS anti-parton contribution which is the main topic of this poster. We use two sequential-sources to calculate the 4-point functions. The preliminary numerical results of the CS Euclidean hadronic tensor are presented in Fig. 2 where the nucleon momentum $\vec{p} = 0$ and $\mu = \nu = 1$. For the present case, the errors are all smaller than 1% which help to give non-zero results after carrying out the inverse-Laplace transform.

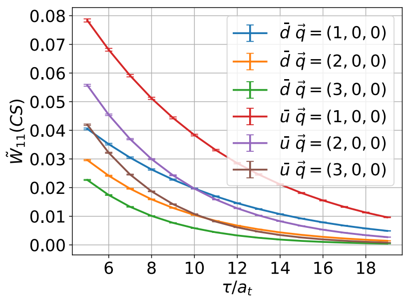

Figure 2: Euclidean hadronic tensors of CS $\bar{u}$ and $\bar{d}$ as a function of τ.

MINKOWSKI HADRONIC TENSOR

$W_{\mu\nu}(q^2,\nu)$ is directly proportional to the spectral density $\rho(\omega)$ with the excitation energy $\omega = \nu$. To solve the inverse-Laplace problem is equivalent to finding the $\rho(\omega)$ in the integral equation

$$D(\tau) = \int d\omega K(\tau,\omega)\rho(\omega) \quad (4)$$

with lattice data $D(\tau) = \tilde{W}_{\mu\nu}(q^2,\tau)$ and kernel function $K(\tau,\omega) = e^{-\omega\tau}$. It can be implemented by several methods, such as the Backus-Gilbert method [5], the improved maximum entropy method [6] and normal χ^2 fitting with model functions. In this poster, the Minkowski hadronic tensor is converted from the Euclidean one by the Backus-Gilbert method and illustrated in Fig. 3.

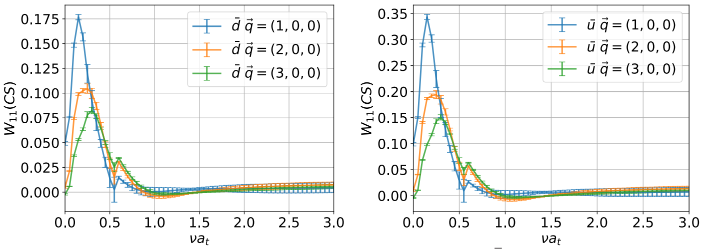

Figure 3: Minkowski hadronic tensors of CS $\bar{u}$ and $\bar{d}$ as a function of ν.

The Bjorken x in the $\vec{p} = 0$ case is defined as $\frac{Q^2}{2M_p\nu}$ with $Q^2 = |\vec{q}|^2 - \nu^2$. For the present study with $|\vec{q}|^2$ (0.3 GeV^2, 1.3 GeV^2 and 3.0 GeV^2) with $\vec{p} = 0$, they are too small to obtain results for physical Q^2 and x. Although in the unphysical region ($\nu a_5 > 1.0$), the results show reasonable behavior. The positions of the largest weight is the expected elastic peak at $x = 1$. In order to calculate the hadronic tensor in the physical region of Q^2 and x, we can let $\vec{p} \sim -\vec{q}$ with $\vec{p}$ and $\vec{q}$ both large. Thus, the elastic peak will appear at $\nu \sim M_p$ and then we can choose a reasonabl larger ν in the deep inelastic region to have $Q^2 \sim 2.5$ GeV and $x < 1$. The large nucleon momentum $\vec{p}$ can be realized by the momentum source [7] with satisfying signal.

SUMMARY

To summarize, we have attempted to calculate the nucleon hadronic tensor to obtain PDF in deep inelastic scattering. Preliminary results are obtained to test the feasibility of our method. Calculation with large momentum $\vec{p}$ and $\vec{q}$ for VCS u, d, and CS $\bar{u}$ and $\bar{d}$ are in progress.

References

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