Towards leading isospin breaking effects in mesonic masses with $O(a)$ improved Wilson fermions

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Motivation

- Isospin symmetry broken by different masses and electric charges of up and down quarks $\Rightarrow$ expect $O(1\%)$ systematic error [1].
- Recent lattice calculations with isosymmetric setting achieve precision of $O(1\%)$ in many observables [2] $\Rightarrow$ investigation of isospin breaking effects becomes necessary.

Finite Volume QED

- Several attempts to define non-compact QED on finite spatial volume with periodic boundary conditions exist.
- Naive QED:
  - IR divergence in electromagnetic correction to pion mass $\Rightarrow$ inconsistent theory [3].
- $\mathrm{QED}_{\mathrm{TL}}$:
  - Remove $p = 0$ mode $\Rightarrow$ IR finite theory, but reflection positivity is violated.
  - Taking infinite time limit in fixed spatial volume $\Rightarrow$ IR divergence reappears. Setting required for mass extraction [4].
- $\mathrm{QED}_{\mathrm{L}}$:
  - Remove $\vec{p} = 0$ modes $\Rightarrow$ IR finite theory with reflection positivity [3,4].
  - Action in Feynman gauge:

$$\widetilde{S}^{(0)}_{\gamma}[\widetilde{A}] = \frac{1}{2}\widetilde{A}_{\mathbf{a}}\widetilde{\Delta}_{\mathbf{ab}}\widetilde{A}_{\mathbf{b}} \qquad \widetilde{\Delta}_{p\mu q\nu} = \hat{p}^2\delta_{\mu\nu}\delta_{-p,q}$$

$$\widetilde{A}_{p\mu} = 0 \text{ for } \vec{p} = 0 \qquad \hat{p}^2 = 4\sum_{\mu}\left(\sin\left(\frac{p_\mu}{2}\right)\right)^2.$$

Reweighting and perturbative treatment of isospin breaking

- Reuse gauge configurations describing isosymmetric QCD ($\mathrm{QCD}_{\mathrm{iso}}$) generated according to

$$Z_{\mathrm{g}} = \int DU \exp(-S^{(0)}_{\mathrm{g}}[U])Z^{(0)}_{\mathrm{q}}[U]$$

  $\Rightarrow$ apply reweighting techniques [5,6,7].
- Express expectation values in $\mathrm{QCD}+\mathrm{QED}$ in terms of expectation values in $\mathrm{QCD}_{\mathrm{iso}}$:

$$\langle O[U,\Psi,\overline{\Psi}]\rangle = \frac{\langle R[U]\langle O[U,\Psi,\overline{\Psi}]\rangle_{\mathrm{q},\gamma}\rangle_{\mathrm{g}}}{\langle R[U]\rangle_{\mathrm{g}}} \qquad R[U] = \exp(-S^{\Delta}_{\mathrm{g}}[U])\frac{Z_{\mathrm{q},\gamma}[U]}{Z^{(0)}_{\mathrm{q}}[U]Z^{(0)}_{\gamma}}.$$

- Reweighting factor $R[U]$ composed of partition function of QED in classical QCD background field

$$Z_{\mathrm{q},\gamma}[U] = \int DAD\Psi D\overline{\Psi}\exp(-S_{\gamma}[A] - \overline{\Psi}_{\mathbf{a}}D[U,A]_{\mathbf{ab}}\Psi_{\mathbf{b}})$$

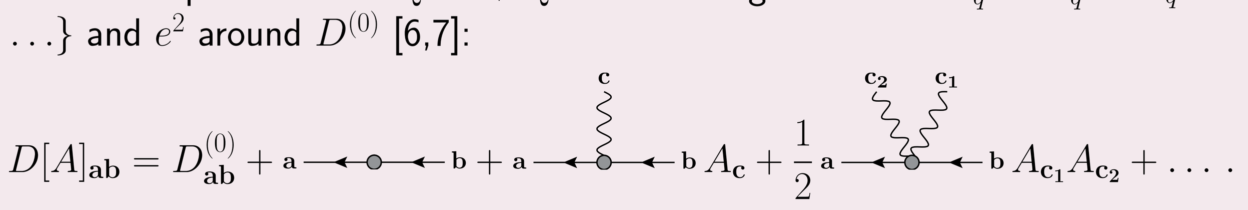
  and partition functions of respective free isosymmetric theories

$$Z^{(0)}_{\gamma} = \int DA\exp(-S^{(0)}_{\gamma}[A]) \qquad Z^{(0)}_{\mathrm{q}}[U] = \int D\Psi D\overline{\Psi}\exp(-\overline{\Psi}_{\mathbf{a}}D^{(0)}[U]_{\mathbf{ab}}\Psi_{\mathbf{b}}).$$

- Isospin breaking now treated in perturbation theory around $\mathrm{QCD}_{\mathrm{iso}}$.
- Only leading order effects considered $\Rightarrow$ e^2 does not renormalise, hence fixed to $e^2 = 4\pi\alpha$.

Feynman Rules

- Expand Wilson-Dirac Operator D of $\mathrm{QCD}+\mathrm{QED}$ to leading order in $\Delta m_q = m_q - m^{(0)}_q$ with $q \in \{\mathrm{u},\mathrm{d},\mathrm{s}\ldots\}$ and e^2 around $D^{(0)}$ [6,7]:



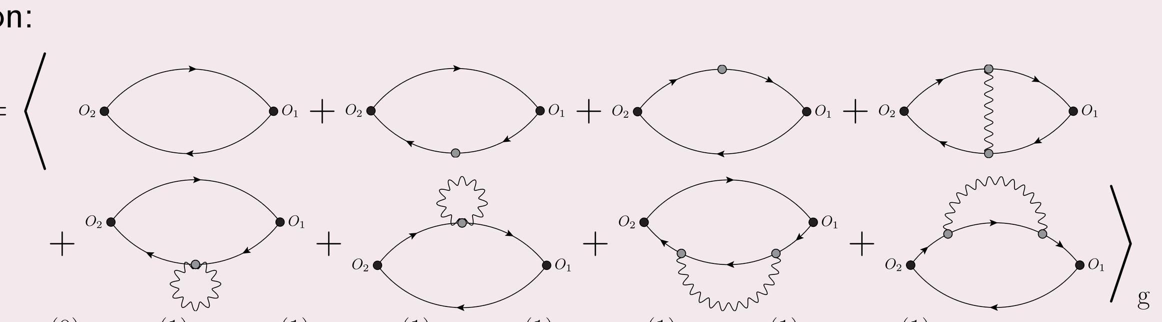
- Propagators defined by

- Investigation of higher order isospin breaking effects $\Rightarrow$ take higher order quark-photon vertices into account, however treatment of multiple photon propagators technically challenging.

Quark connected part of general mesonic 2pt function

- Neglect isospin breaking effects in sea quarks.
- Neglect contributions from disconnected diagrams.
- Consider only insertion of point-like interpolation operators.
- General mesonic 2pt function:

$$\langle\overline{\Psi}_{\mathbf{a}}O_{2,\mathbf{ab}}\Psi_{\mathbf{b}}\overline{\Psi}_{\mathbf{c}}O_{1,\mathbf{cd}}\Psi_{\mathbf{d}}\rangle =$$



$$= C^{(0)}_{\mathrm{iso}} + C^{(1)}_{\mathrm{det1}} + C^{(1)}_{\mathrm{det2}} + C^{(1)}_{\mathrm{exch}} + C^{(1)}_{\mathrm{tad1}} + C^{(1)}_{\mathrm{tad2}} + C^{(1)}_{\mathrm{bow1}} + C^{(1)}_{\mathrm{bow2}}.$$

- Point-split interpolation operators: additional diagrams have to be included due to perturbative expansion of gauge links.

Stochastic treatment of all-to-all photon propagators

- Calculation of all-to-all photon propagators infeasible on larger lattices $\Rightarrow$ stochastic estimation of propagator [7].
- Generate real photon sources J according to $\mathbb{Z}_2$-noise satisfying

$$\langle J_{\mathbf{a}}J_{\mathbf{b}}\rangle_J = \delta_{\mathbf{ab}}.$$

- Define propagated photon source $K[J]$ by

$$K[J]_{\mathbf{b}} = \Delta^{-1}_{\mathbf{bc}}J_{\mathbf{c}}.$$

- Full propagator is now stochastically estimated by

$$\langle K[J]_{\mathbf{b}}J_{\mathbf{a}}\rangle_J = \Delta^{-1}_{\mathbf{bc}}\langle J_{\mathbf{c}}J_{\mathbf{a}}\rangle_J = \Delta^{-1}_{\mathbf{ba}}.$$

- Explicit inversion of differential operator Δ in Fourier representation $\Rightarrow$ propagated source given by

$$K[J]_{x\mu} = \frac{1}{V_\Lambda}\sum_{p\in\tilde{\Lambda},\vec{p}\neq 0}\frac{\exp(ip\cdot(x-y))}{\hat{p}^2}J_{y\mu}.$$

Lattice ensemble

- Initial exploratory study on CLS 2-flavour lattice E5 [8].
- Two degenerate dynamical light quark flavours, $\mathcal{O}(a)$ improved Wilson fermions, (anti-)periodic boundary conditions.
- Lattice parameters [8,9]:

$\mathrm{T}\times\mathrm{L}^3$	β	κ_{l}	κ_{s}	κ_{c}	a[fm]	m_π[MeV]
64×32^3	5.30	0.13625	0.135802302	0.12724	0.0658	437

Fitting correlation functions

- Investigation of pseudo-scalar meson channel with point-like interpolation operators.
- Isosymmetric contributions fitted with

$$C^{(0)}_{\mathrm{iso}}(t) = a^{(0)}_{\mathrm{iso}}\cosh\left(m^{(0)}_{\mathrm{iso}}\left(\frac{T}{2}-t\right)\right).$$

- For first-order contributions each diagram fitted individually with ratio [6,7]

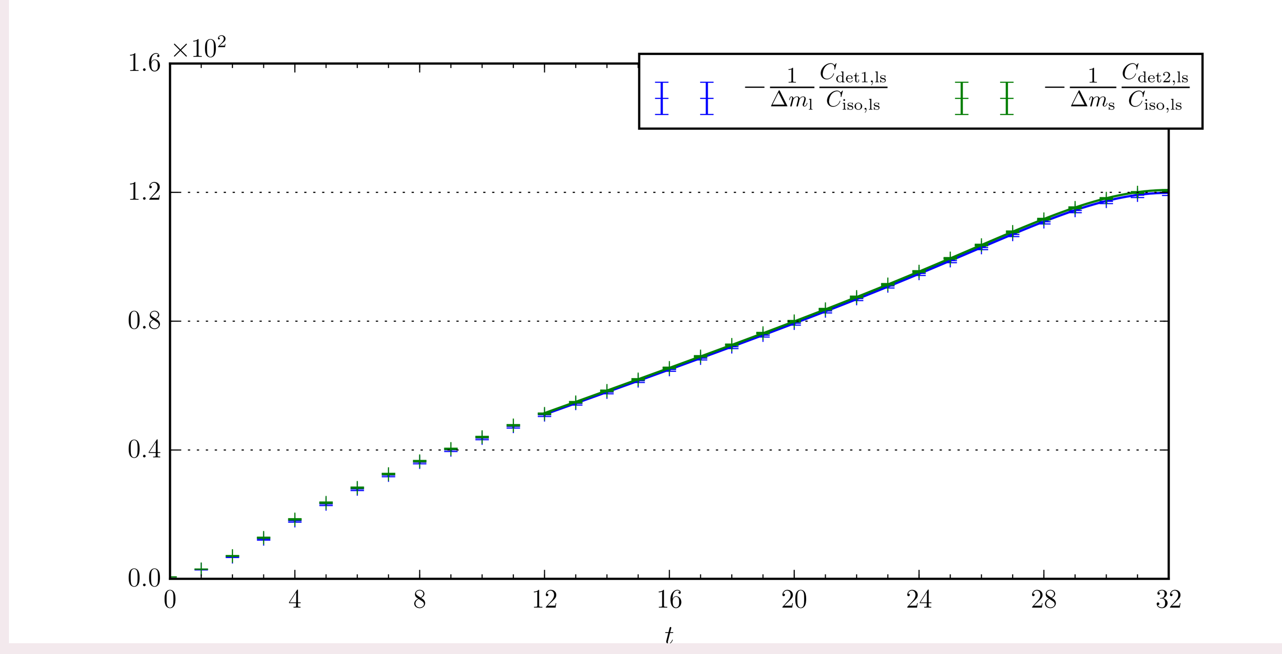
$$\frac{C^{(1)}_{diag}(t)}{C^{(0)}_{\mathrm{iso}}(t)} = \frac{a^{(1)}_{diag}}{a^{(0)}_{\mathrm{iso}}} + m^{(1)}_{diag}\left(\frac{T}{2}-t\right)\tanh\left(m^{(0)}_{\mathrm{iso}}\left(\frac{T}{2}-t\right)\right).$$

- Final mass given by sum of all contributions:

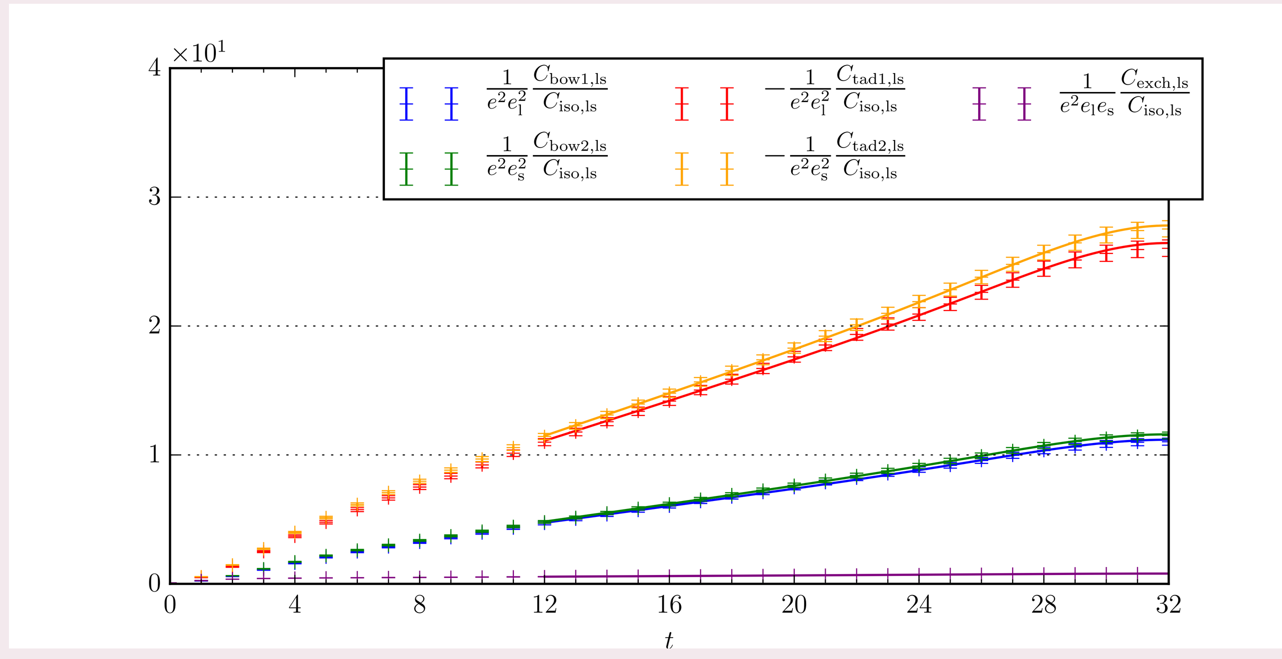
$$m = m^{(0)}_{\mathrm{iso}} + \sum_{diag} m^{(1)}_{diag}.$$

Pseudo-scalar correlation functions

- Statistics: 1000 gauge configurations, 2 point quark sources per gauge configuration, 8 stochastic photon sources per quark source and gauge configuration.
- Pseudo-scalar light-strange channel:
  - Contributions due to explicit quark mass detuning:



  - Contributions due to electromagnetic interaction:



Pseudo-scalar meson masses

- Preliminary results for pseudo-scalar meson masses in lattice units:

$$m_{\pi^+} - m_{\pi^0} = 0.00063(4)(1)$$

$$\frac{1}{2}(m_{\pi^+} + m_{\pi^0}) = 0.1456(5)(70) + (\Delta m_{\mathrm{u}} + \Delta m_{\mathrm{d}})\cdot 4.67(4)(7) + 0.0319(11)(34)$$

$$m_{K^+} - m_{K^0} = (\Delta m_{\mathrm{u}} - \Delta m_{\mathrm{d}})\cdot 3.444(27)(44) + 0.0140(4)(10)$$

$$\frac{1}{2}(m_{K^+} + m_{K^0}) = 0.1934(4)(8) + (\Delta m_{\mathrm{u}} + \Delta m_{\mathrm{d}})\cdot 1.722(14)(22) + \Delta m_{\mathrm{s}}\cdot 3.467(10)(16) + 0.0163(5)(15)$$

- Pion mass splitting is pure first-order electro-magnetic effect $\Rightarrow$ use isosymmetric scale to convert to physical units [7]:

$$m_{\pi^+} - m_{\pi^0} = 1.90(11)(3)\,\mathrm{MeV}$$

- Experimentally determined value: $4.5936(5)\,\mathrm{MeV}$ [10].
- Result still unphysical:
  - Neglected diagrams.
  - No physical pion mass ($437\,\mathrm{MeV}$).
  - No finite volume corrections, believed to be large due to long-range electromagnetic interactions.
  - No continuum limit.

Future work

- Noise reduction by usage of spin-diluted photon sources as well as a symmetric formulation [11].
- Dependence on choice of photon gauges.
- $2+1$ flavour ensembles with open boundary conditions $\Rightarrow$ adaptation of photon propagator required.
- Isospin breaking effects in $g-2$.

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