Thermodynamics near the first order phase transition of SU(3) gauge theory using gradient flow

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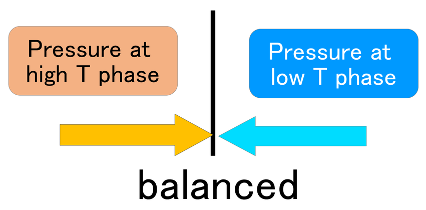
Abstract

We study the gaps between thermodynamic quantities in the hot and cold phases at the first order phase transition point of the SU(3) gauge theory. Performing simulations on lattices with various spatial volume and lattice spacing, we calculate the energy gap (latent heat) by a method using the Yang-Mills gradient flow and compare it with that by the conventional derivative method.

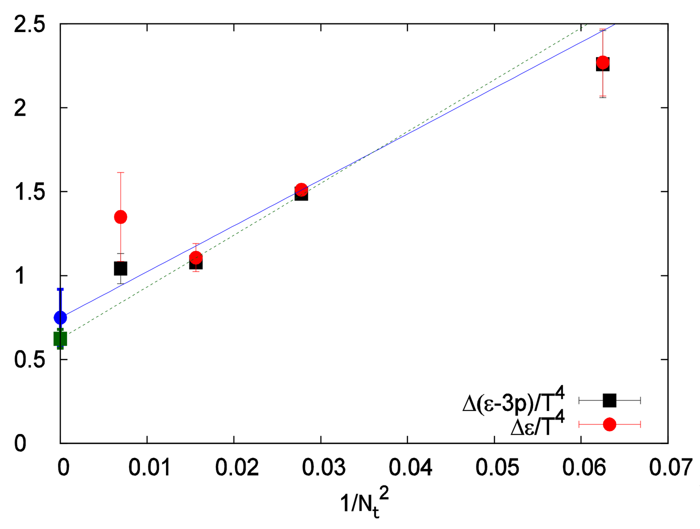
1. Introduction

- First order phase transitions are expected in many interesting systems of lattice field theories. e.g. high density QCD, many flavor etc..
  → important to study first order phase transitions.
- The latent heat (energy gap) the most basic quantity.
- The gap of pressure must vanish.
  → Reliability of the calculation can be confirmed.



- We test the gradient flow method [H. Suzuki, 2013] for the calculation of the equation of state (EoS) at the first order transition of SU(3) gauge theory.
- We compare the results of EoS with those by the derivative method.

Latent heat by the derivative method with the non-perturbative anisotropy coefficients



The continuum limit is

$$\frac{\Delta\epsilon}{T^4} = 0.75 \pm 0.17$$

$$\frac{\Delta(\epsilon-3p)}{T^4} = 0.623 \pm 0.056.$$

Shirogane et al., Phys. Rev. D 96, 014506 (2016)

$1/N_t^2 = a^2 T_c^2$

2. Numerical Simulations

2-1 Simulation parameters

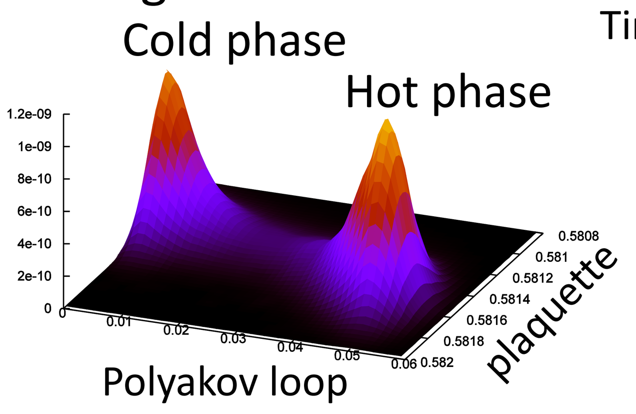
Lattice size $N_s^3\times N_t$	# β	β range	No. of Conf.	interval
$48^3\times 8$	6	6.0560-6.0670	1220000	20
$64^3\times 8$	5	6.0585-6.0650	4535000	20
$48^3\times 12$	3	6.3330-6.3370	4750000	50
$64^3\times 12$	5	6.3320-6.3390	2385000	50
$96^3\times 12$	7	6.3300-6.3390	3547500	50
$96^3\times 16$	3	6.5430-6.5470	720000	50
$128^3\times 16$	3	6.5430-6.5470	332000	50

- Pure SU(3) gauge theory
- Standard plaquette action.
- Pseudo-heat bath algorithm + over-relaxation.
- Simulations are performed at 3-7 β points near the transition point.
- The multi-point reweighting method is used for the measurements.

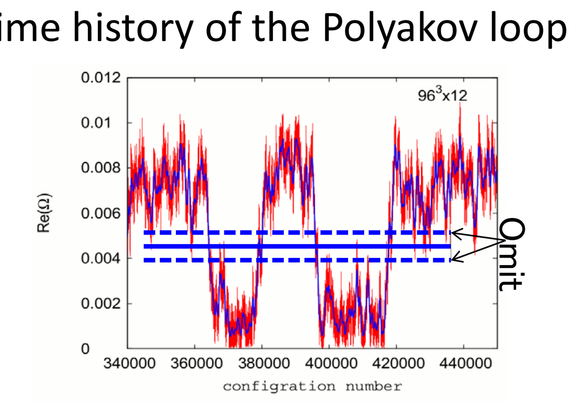
2-2 Separation of the hot and cold phases

- We classify configurations into the hot and cold phases by the Polyakov loop and calculate EoS using configurations in these phases separately.

Histogram

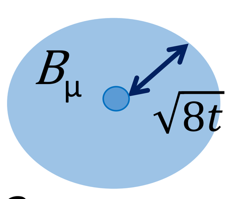


Time history of the Polyakov loop



3. Gradient flow method

3-1 Gradient flow



- Imaginary evolution of the system into a fictitious "time" t.
- We may view the flowed field B_μ as a smeared A_μ over a physical range $\sqrt{8t}$.
- It was shown that operators in terms of B_μ have no UV divergences nor short-distance singularities at finite t.
- GF defines a physical (non-pert.) renormalization scheme, which can be calculated directly on the lattice.

3-2 EoS calculation using Gradient flow

H. Suzuki, Prog. Theor. Exp. Phys. 2014, 083B03 (2014); FlowQCD, Phys. Rev. D90, 011501(2014)

Strategy of the Gradient flow method

(1) Gradient flow

Smeared field strength: $F_{\mu\nu} \xrightarrow{\text{Gradient Flow}} G_{\mu\nu}$

(2) Define Energy-momentum tensor

$$T^R_{\mu\nu} = \lim_{t\to 0}\left\{\frac{1}{\alpha_U(t)}U_{\mu\nu}(t,x) + \frac{\delta_{\mu\nu}}{4\alpha_E(t)}[E(t,x) - \langle E(t,x)\rangle_0]\right\}$$

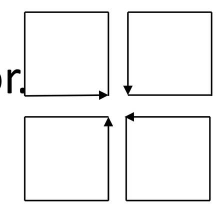
Dim. 4 operators: $E(t,x) = \frac{1}{4}\delta_{\mu\nu}G_{\rho\sigma}(t,x)G_{\rho\sigma}(t,x)$

$$U_{\mu\nu}(t,x) = G_{\mu\rho}(t,x)G_{\nu\rho}(t,x) - \frac{1}{4}\delta_{\mu\nu}G_{\rho\sigma}(t,x)G_{\rho\sigma}(t,x)$$

$$\alpha_U(t) = g^2[1 + 2b_0 s_1 g^2 + \cdots] \qquad \alpha_E(t) = \frac{1}{2b_0}[1 + 2b_0 s_2 g^2 + \cdots]$$

g: running coupling constant with $\overline{\text{MS}}$ scheme based on the 4-loop beta function.

$F_{\mu\nu}$, $G_{\mu\nu}$ are defined by a clover-shaped operator.

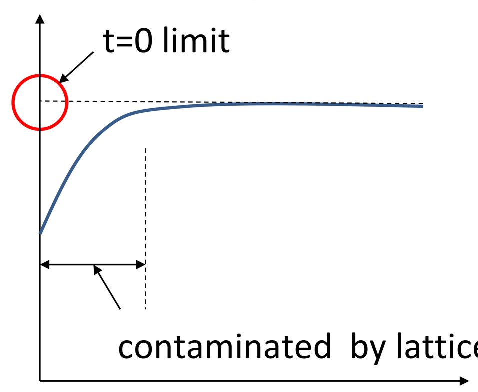


Energy density and pressure

$$\epsilon = \langle T_{00}\rangle \qquad p = \frac{1}{3}\sum_i \langle T_{ii}\rangle$$

(3) Continuum extrapolation $a\to 0$ at fixed flow time t

(4) Remove unwanted contributions at $t > 0$ by taking $t\to 0$.



Lattice artifact in the small t region.

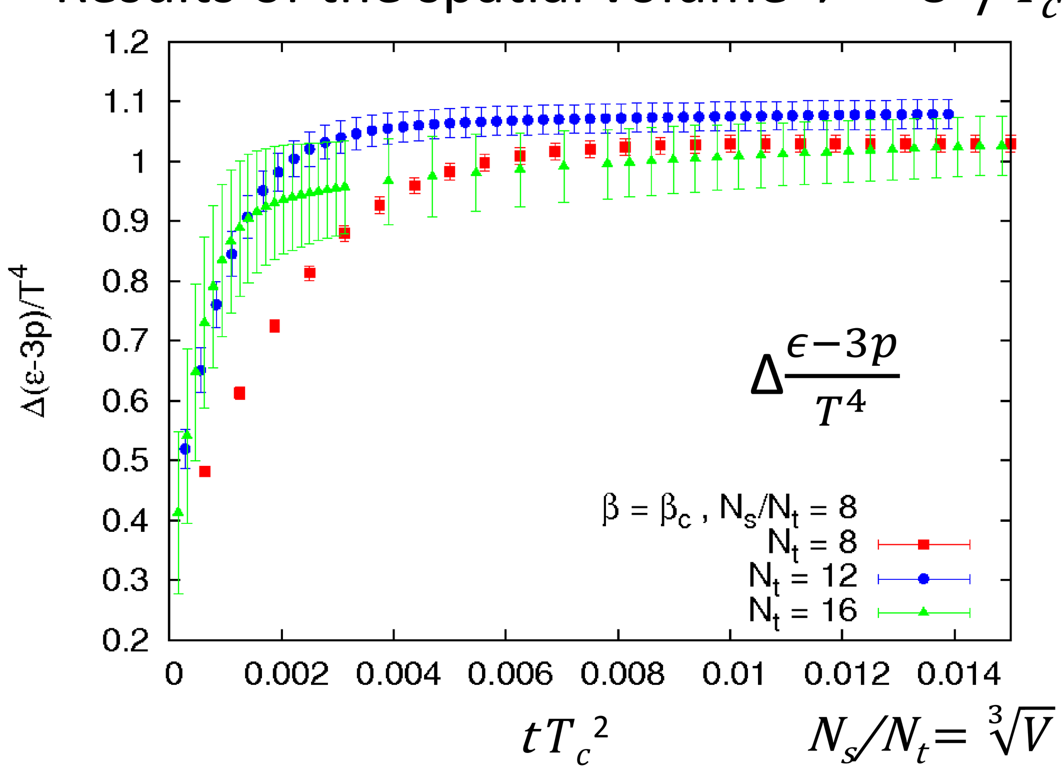
Extrapolation to the flow time zero limit from the large t region.

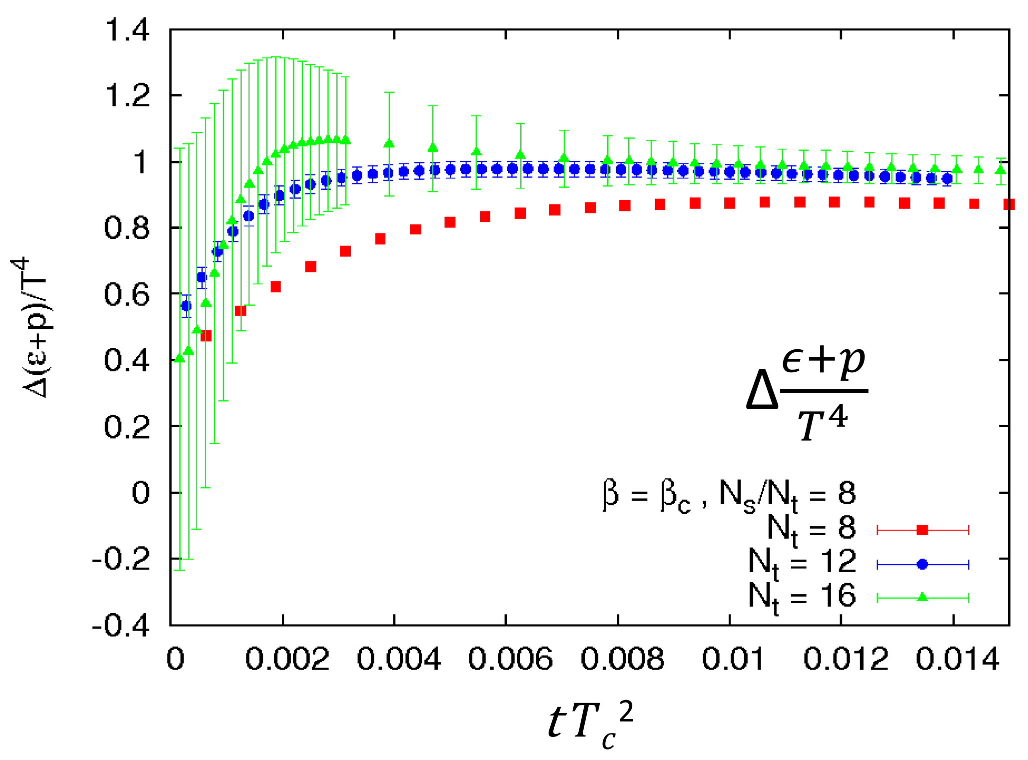
T_c: transition temperature

4. Results of EoS

4-1 Latent heat vs Flow time

Results of the spatial volume $V = 8^3/T_c^3$

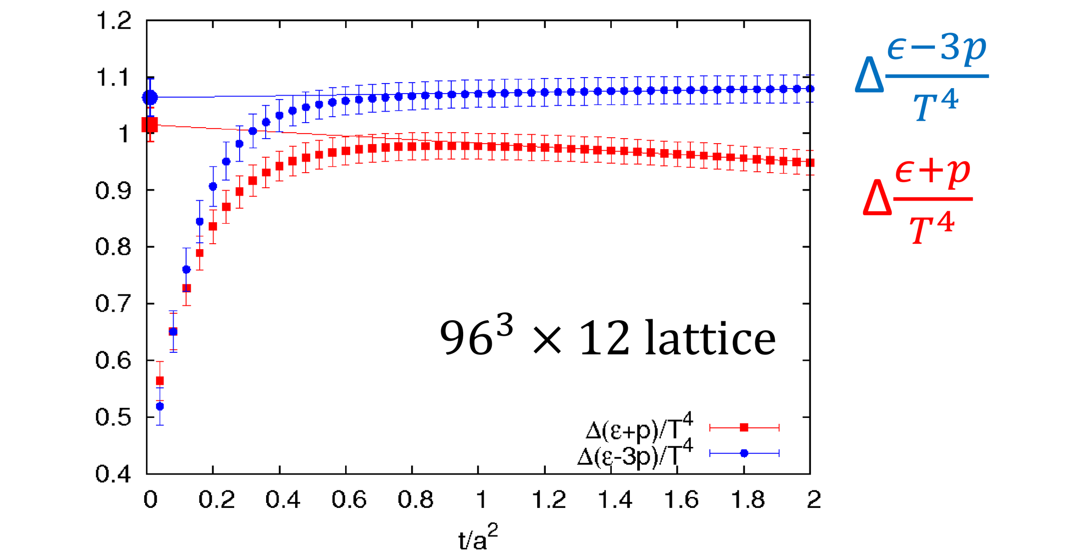




- The results of N_t=12 and 16 are consistent in the range of $tT_c > 0.006$.
- The lattice artifacts are removed at large t.

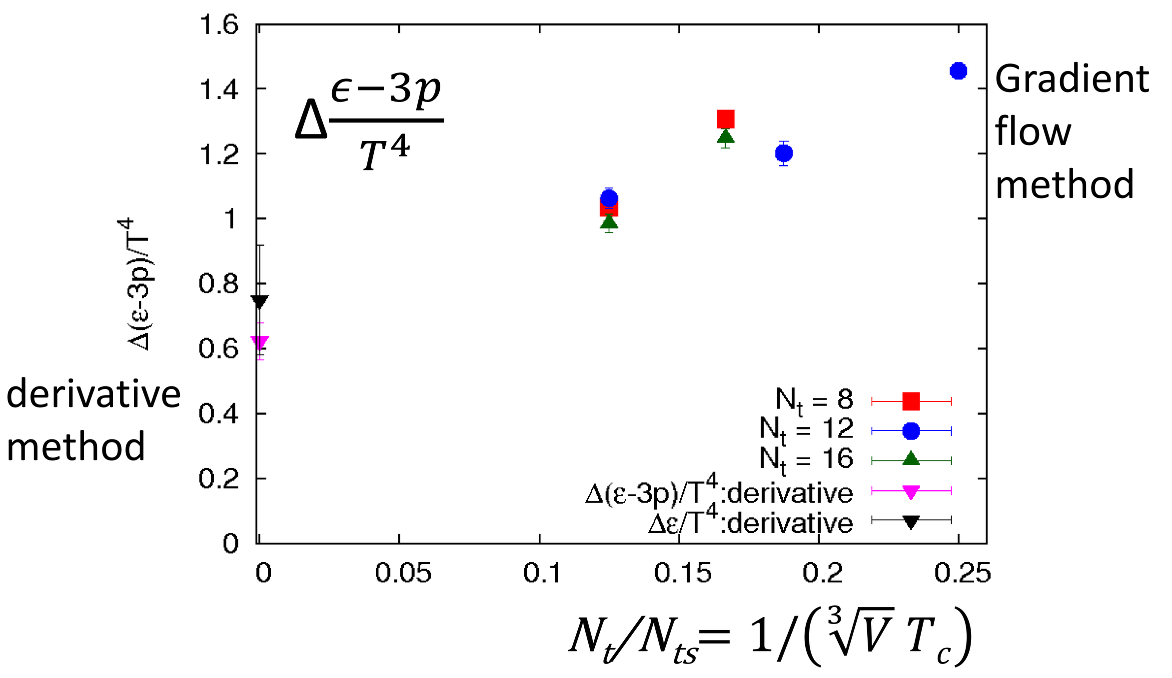
- $t\to 0$ extrapolation is carried out with the fit range: $tT_c > 0.008$ for N_s/N_t=8, $tT_c > 0.01$ for $N_s/N_t \le 6$.

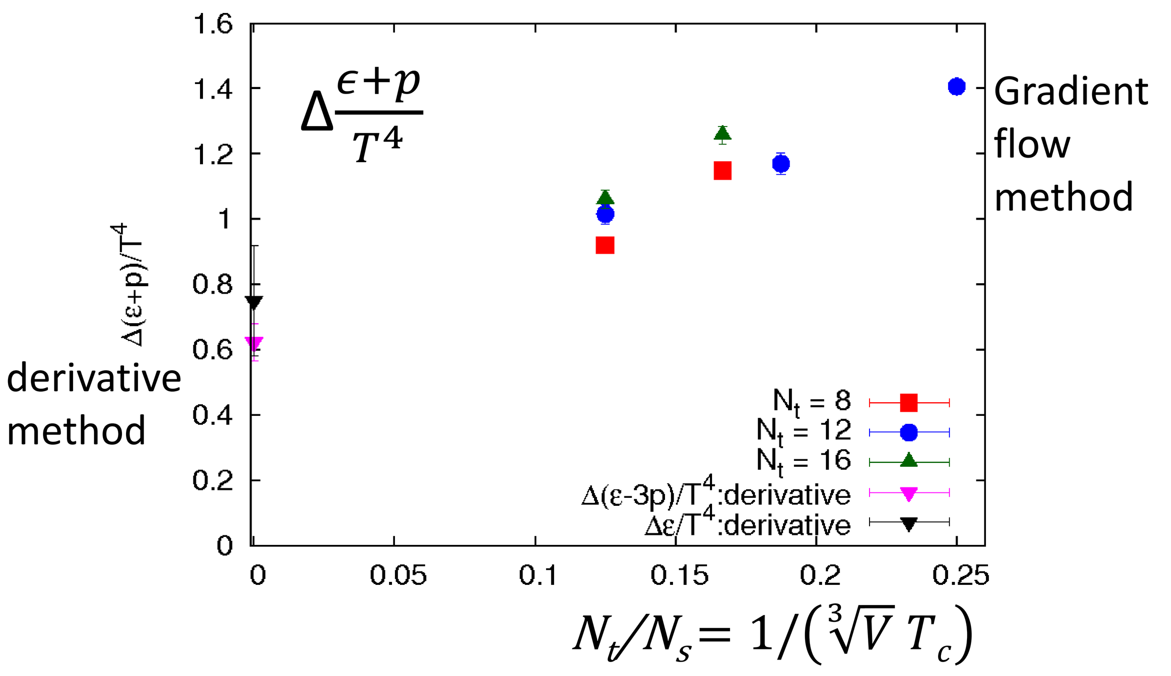
4-2 Vanishing pressure gap at $t = 0$



- $\Delta(\epsilon-3p)/T^4$ and of $\Delta(\epsilon+p)/T^4$ are consistent within the error in the t = 0 limit.
- This means Δp=0.

4-3 Volume dep. of latent heat at $t = 0$

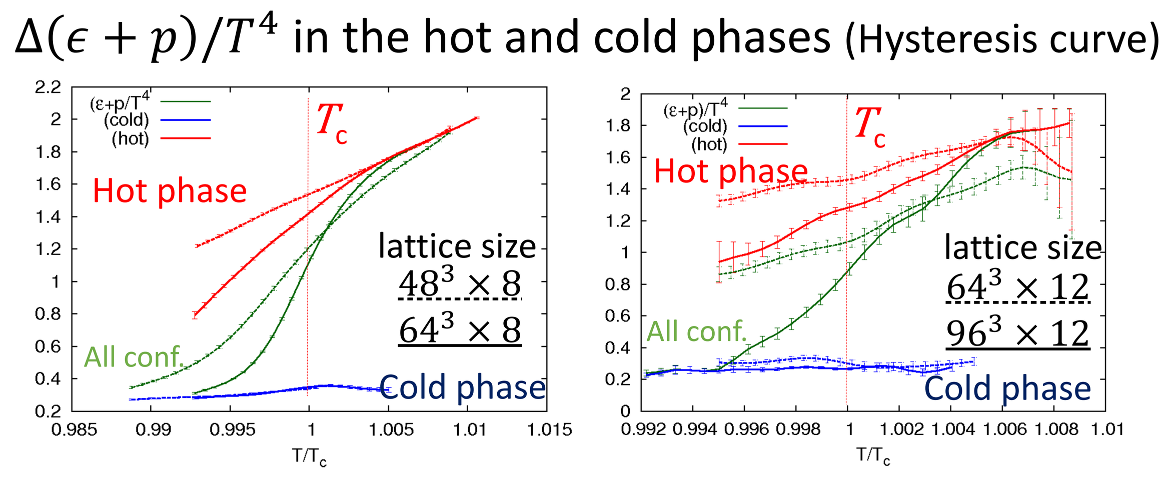


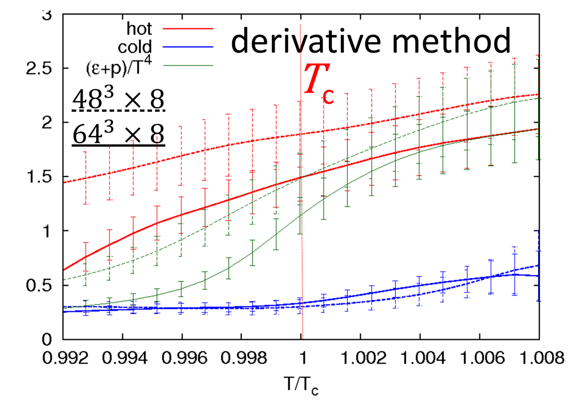


- The spatial volume dependence is observed.
- $\Delta\epsilon/T^4$ approaches the result by the derivative method as the volume increases.

4-4 Gap in the energy density near T_c

$\Delta(\epsilon+p)/T^4$ in the hot and cold phases (Hysteresis curve)





- The volume dependence is visible in the hot phase near T_c and in the meta-stable state.
- No volume dependence in the cold phase.
- A careful treatment may be required only at T_c.

- There may exist the volume dependence in $\Delta\epsilon$ by the derivative method. However, it is smaller than the error from the anisotropy coefficients.

5. Conclusion & Outlook

- Comparing the derivative method and the gradient flow method, we confirmed the reliability of the gradient flow method.
- The statistical error is very small in comparison with the results by the derivative method.
- Then, the volume dependence is visible.
- $t\to 0$ extrapolation is an important issue, which is a source of the systematic error.