CONFORMAL WINDOW IN SU(2) WITH FUNDAMENTAL FERMIONS

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BACKGROUND

SU(N) gauge theories with N_f massless flavors of Dirac fermions in the fundamental representation provide a probe to a variety of gauge theories. At a small $N_f < N_f^{(c)}$ the theory breaks chiral symmetry of the vacuum similar to QCD, while above $N_f = 11N/2$ the theory loses asymptotical freedom. The domain $N_f^{(c)} \leq N_f \leq 11N/2$ forms the conformal window, within which the theory is conformal and the long distance behavior is governed by a nontrivial infrared fixed point (IRFP).

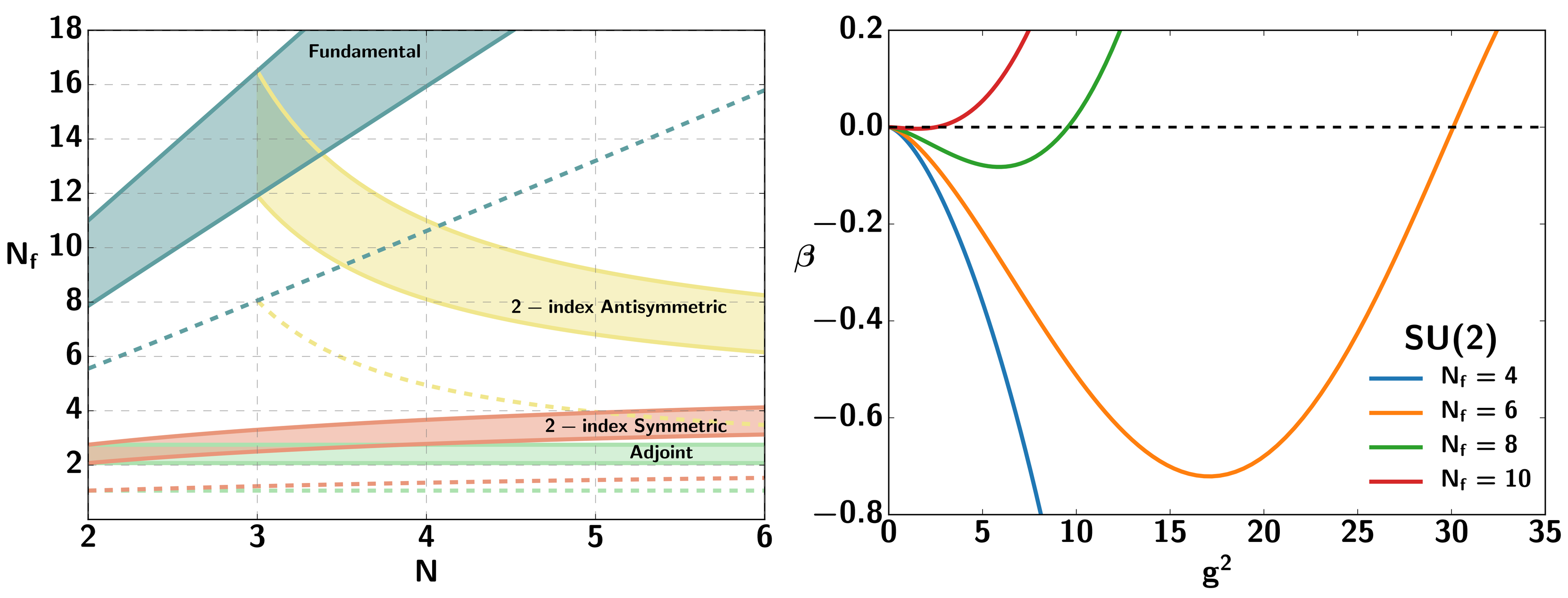

Figure 1: Left: Location of conformal window for different SU(N) [1], N_f and fermion representations. Right: 4-loop $\overline{\mathrm{MS}}$ β-function for different N_f.

SETUP

We study a SU(2) theory with varying number of fermions. We focus on theories with $N_f = 6$ and $N_f = 8$, since the previous studies [2-6] on these models are inconclusive, and it is known that $N_f = 4$ and $N_f = 10$ are outside and inside the conformal window, respectively. We use the following simulation parameters:

- HEX smeared, clover improved Wilson fermions with Schrödinger functional boundary conditions
- β's between $8 \ldots 0.5$ (0.4 for $N_f = 8$)
- Lattice sizes between $L/a = 8 \ldots 30$ (32 for $N_f = 8$)

GRADIENT FLOW COUPLING

We measure the coupling with the τ-improved [7] gradient flow method:

$$g^2_{\mathrm{GF}}(\mu) = \mathcal{N}^{-1} t^2 \langle E(t + \tau_0 a^2)\rangle|_{x_0 = L/2,\, t=1/8\mu^2},$$

The scale is set as: $\mu^{-1} = \sqrt{8t} = c_t L$ and we have chosen $c_t = 0.3$ for $N_f = 6$ and $c_t = 0.4$ for $N_f = 8$.

We then measure β-function from the step scaling function as:

$$\Sigma(u,s,L/a) = g^2_{\mathrm{GF}}(g_0^2, sL/a)\big|_{g^2_{\mathrm{GF}}(g_0^2, L/a)=u}, \quad \sigma(u,s) = \lim_{a/L\to 0} \Sigma(u,s,L/a)$$

$$\beta(g) \approx \frac{g}{2\ln(s)}\left(1 - \frac{\sigma(g^2,s)}{g^2}\right)$$

We also measure the slope of the β-function at IRFP γ_g using a linear fit around the IRFP.

MASS ANOMALOUS DIMENSION γ_M

We measure the pseudoscalar density renormalization constant Z_P using the Schrödinger functional method and measure the γ_m as:

$$\Sigma_P(u,s,L/a) = \frac{Z_P(g_0, sL/a)}{Z_P(g_0, L/a)}\bigg|_{g^2(g_0,L/a)=u}, \quad \sigma_P(u,s) = \lim_{a/L\to 0} \Sigma_P(u,s,L/a)$$

$$\gamma_m(g^2) = -\frac{\log \sigma_P(g^2,s)}{\log s}$$

We also measure the γ_m using the spectral density method where the scaling of massless Dirac operator is governed by the mass anomalous dimension. The mode number in the vicinity of IRFP is approximately a power law [8,9]:

$$\nu(\Lambda) \simeq C\Lambda^{4/(1+\gamma_m)}$$

RESULTS

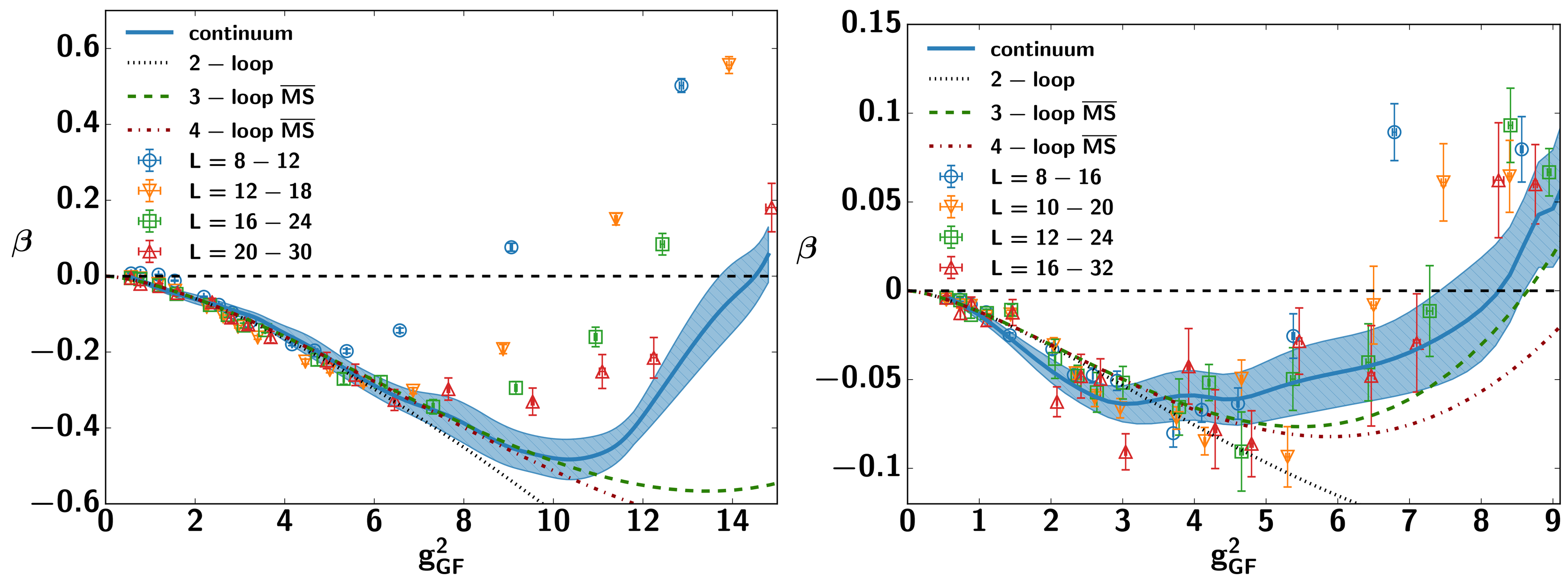

Figure 2: Continuum limit of β-function for $N_f = 6$, $s = 1.5$ (Left) and $N_f = 8$, $s = 2$ (Right) and respective raw measurements

We observe IRFPs for both theories. For $N_f = 6$ at $g_*^2 = 14.5(3)^{+0.41}_{-1.38}$ and for $N_f = 8$ at $g_*^2 = 8.24(59)^{+0.97}_{-1.64}$. For the $N_f = 6$ we have also measured the slope of the β-function and c_t-dependence of the results. These are shown in the table below:

	$c_t = 0.3$	0.35	0.4	0.45
g_*^2	$14.5(3)^{+0.41}_{-1.38}$	$17.3(5)^{+0.77}_{-1.73}$	$22.6(7)^{+1.14}_{-2.89}$	$31(1)^{+1.8}_{-21.1}$
γ_g^*	$0.63(15)^{+0.28}_{-0.27}$	$0.67(11)^{+0.21}_{-0.11}$	$0.69(11)^{+0.11}_{-0.26}$	$0.67(12)^{+0.15}_{-0.55}$

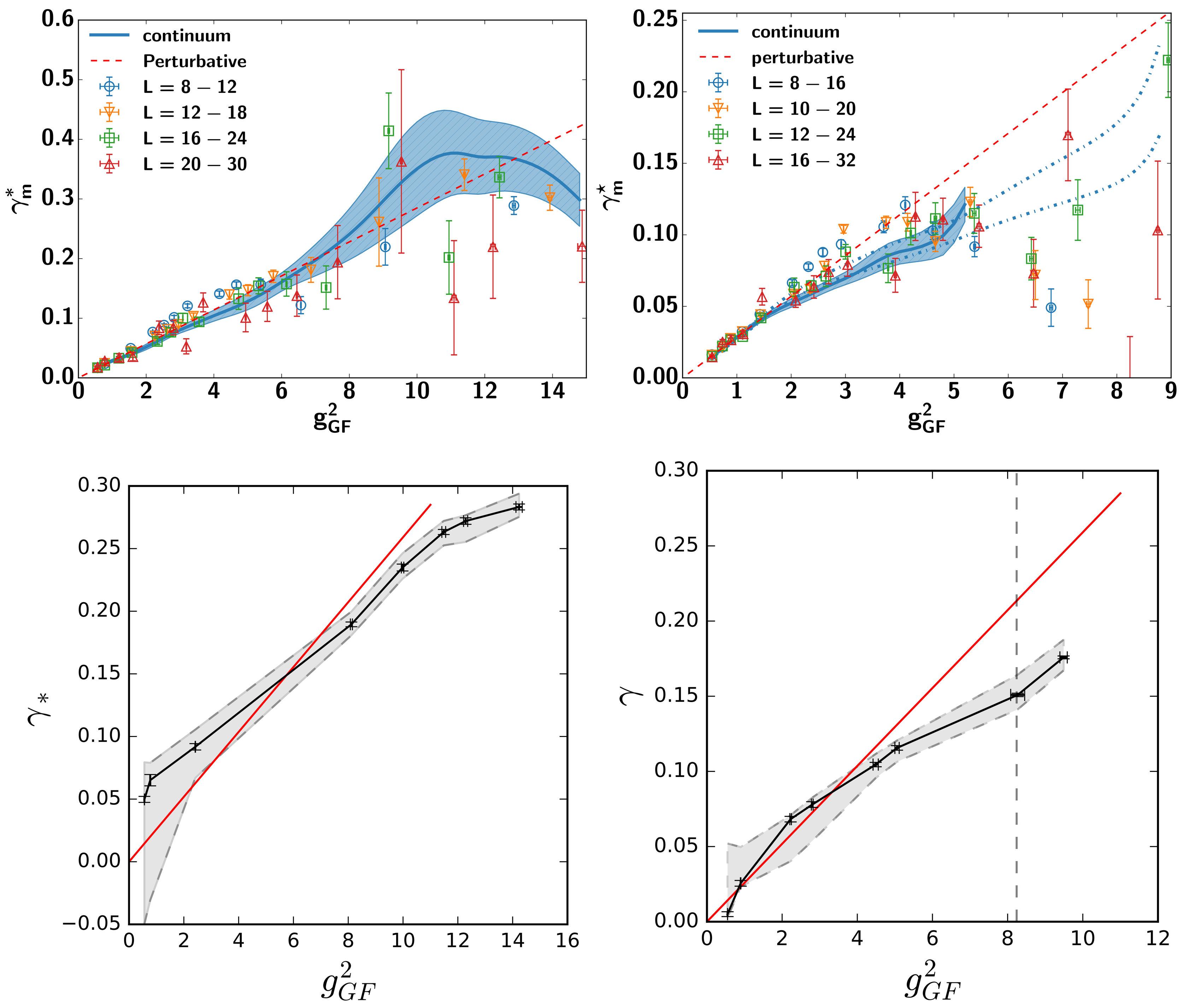

Figure 3: Up: Bare γ_m from Schrödinger functional analysis and its continuum limit. Down: Spectral density γ_m. Left: $N_f = 6$. Right: $N_f = 8$.

As can be seen from the figures, the step scaling method works well on small couplings but becomes unstable at larger couplings. Meanwhile the spectral density method works well on large couplings with bigger errors at small couplings. Using the spectral density method we find $\gamma_m^* \sim 0.275$ for $N_f = 6$ and $\gamma_m^* \sim 0.15$ for $N_f = 8$. We see that even when the step scaling continuum limit results are unreliable, they agree with the spectral density results.

CONCLUSIONS

We observe that both of the theories inspected $N_f = 6$ and $N_f = 8$ develop an IRFP at large couplings. The $N_f = 8$ infrared behavior is close to the perturbative estimates, while the $N_f = 6$ develops IRFP earlier that estimated. Especially the $N_f = 6$ case can be near the lower boundary of conformal window, which makes it an interesting candidate for beyond the standard model theories. However the measured mass anomalous dimensions γ_m^* remain small for most walking technicolor scenarios.

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