QCD Phase Boundary in the Strong Coupling Regime

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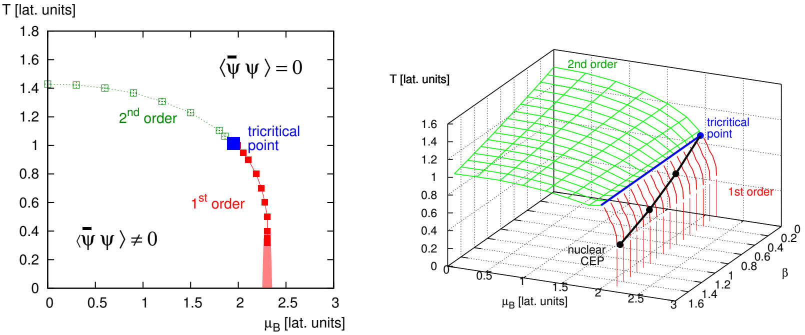
Introduction

- The phase boundary of lattice QCD for staggered fermions in the $\mu_B - T$ has been established via a dual representation in the strong coupling limit.
- Extending this phase boundary towards finite inverse gauge coupling is challenging.



(a) Phase diagram in the strong coupling limit (b) [Langelage, de Forcrand, Philipsen &U.,PRL 113(2014)]

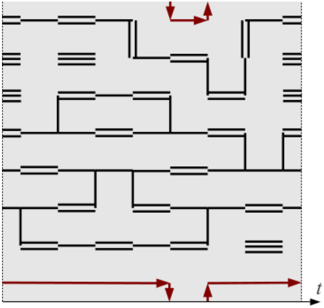
- We present numerical results from the direct simulations away from the strong coupling limit, taking into account the $O(\beta^2)$ corrections via plaquette occupation numbers using Metropolis update in addition to the worm algorithm.
- This allows to study the relation between the nuclear and chiral transition as a function of β.

At the Strong Coupling Limit ($\beta = 0$)

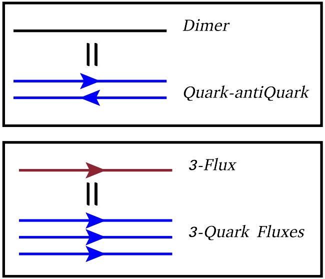
- The partition function after Grassmann integration,

$$Z_F(m_q,\mu,\gamma) = \sum_{k,n,\ell} \underbrace{\prod_{b=(x,\hat\mu)} \frac{(N_c-k_b)!}{N_c!k_b!}\gamma^{2k_b\delta_{\hat 0,\hat\mu}}}_{\text{meson hoppings}} \underbrace{\prod_x \frac{N_c!}{n_x!}(2am_q)^{n_x}}_{\text{chiral condensate}} \underbrace{\prod_\ell w(\ell,\mu)}_{\text{baryon hoppings}}$$

- k_b: bond occupation at bond b
- n_x: monomer numbers at site x
- ℓ: baryon loop



(c) MDP ensemble in the chiral limit



(d) singlet and triplet fluxes

- If we turn on the β, the gauge flux have to be taken into account.
- At finite β, weights are modified.

$$Z(m_q,\mu,\gamma) = \sum_{\{k,n,\ell,n_p\}} \underbrace{\prod_{b=(x,\mu)} \frac{(N_c-k_b)!}{N_c!(k_b-|f_b|)!}\gamma^{2k_b\delta_{\hat{0},\hat{\mu}}}}_{\text{singlet hoppings } M_x M_y} \underbrace{\prod_x \frac{N_c!}{n_x!}(2am_q)^{n_x}}_{\text{chiral condensate } \bar{\psi}\psi}$$

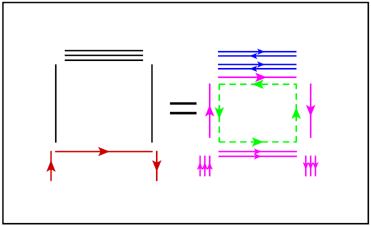
$$\times \underbrace{\prod_{\ell_3} w(\ell_3,\mu)}_{\text{triplet hoppings } \bar{B}_x B_y} \underbrace{\prod_{\ell_f} \tilde{w}(\ell_f,\mu)}_{\text{weight modification}} \underbrace{\prod_P \frac{\left(\frac{\beta}{2N_c}\right)^{n_P+\bar{n}_P}}{n_P!\bar{n}_P!}}_{\text{gluon propagation}}$$

- f_b: the number of gauge fluxes at bond b
- n_p: plaquette(counterclockwise) occupation number
- $\bar{n}_p$: plaquette(clockwise) occupation number
- $\tilde{w}(\ell_f)$: weight modification at $O(\beta^2) \rightarrow$ see W.Unger's talk

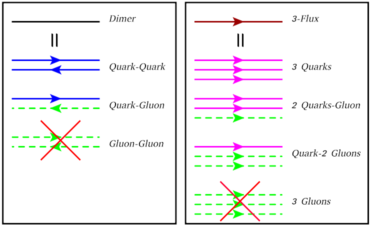
finite β corrections

- Grassmann constraint is modified

$$n_x + \sum_{\hat{\mu}=\pm\hat{0},\ldots\pm\hat{d}} \left(k_{\hat{\mu}}(x) + \frac{N_c}{2}|\ell_{\hat{\mu}}(x)| \right) = N_c + \sum_{\hat{\nu}=\pm\hat{0},\ldots\pm\hat{d}} \frac{1}{2}|f_{\hat{\nu}}(x)|$$



(e) At the corner of a plaquette, the color constraints is $N_C + 1$



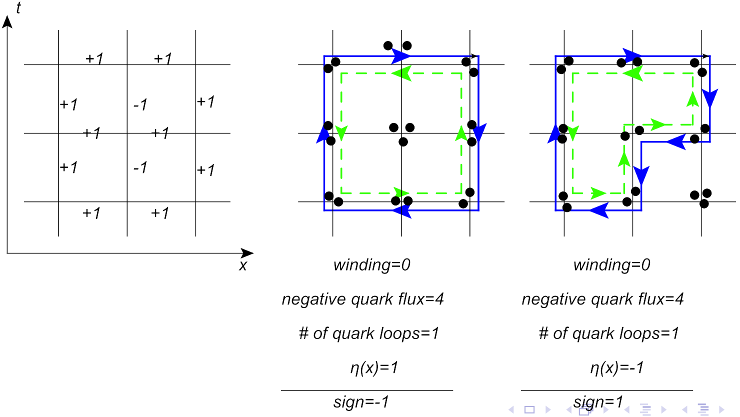
(f) Dimer and 3-Flux can be composed of quark and gluon fluxes

Sign problem of $U(N)$ ensembles at finite β

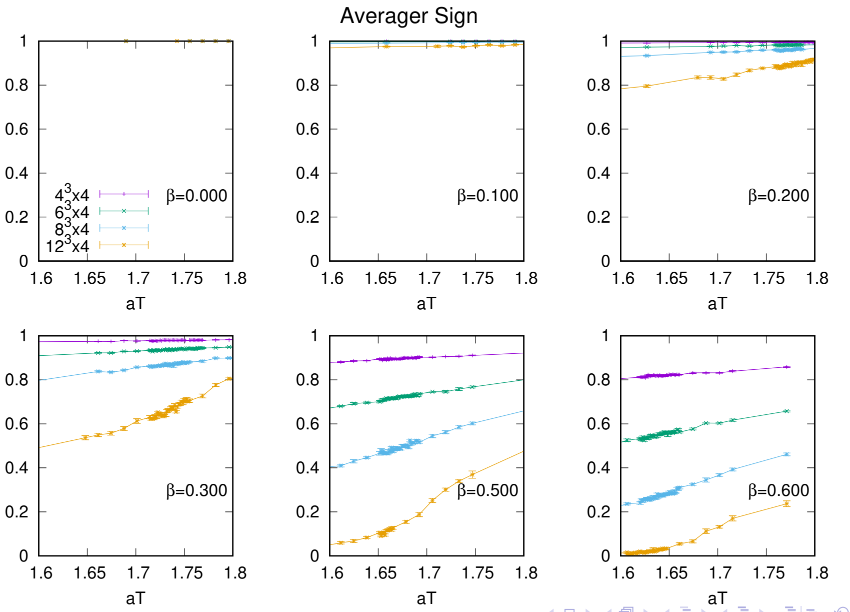
- $U(N)$ ensembles in the strong coupling limit do not have sign problem.
- The sign does not come from the gauge flux but comes only from the quark loop (ℓ_q).

$$\sigma(\ell_q) = (-1)^{w(\ell_q)+N_-(\ell_q)+1} \prod_{b=(x,\hat{\mu})\in\ell_q} \eta_{\hat{\mu}}(x)$$

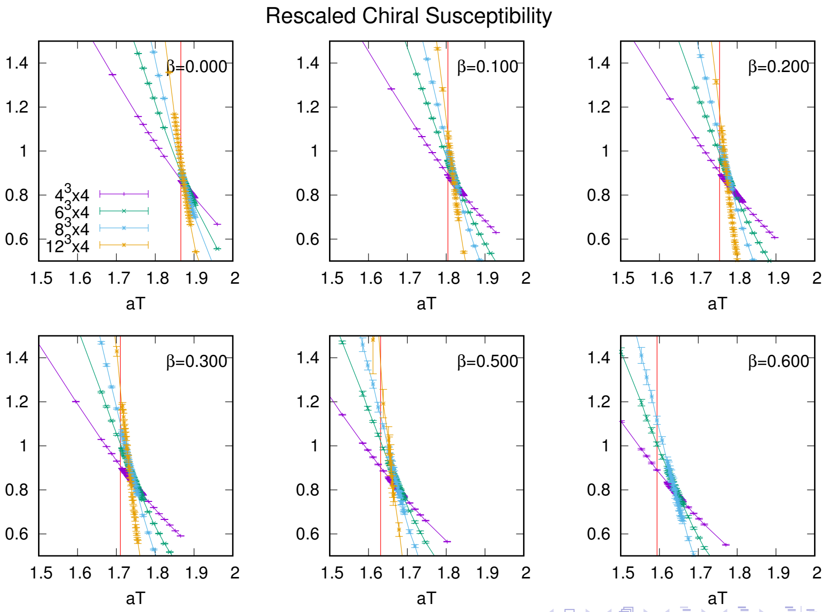
- At finite β, $U(N)$ ensembles have sign problem.



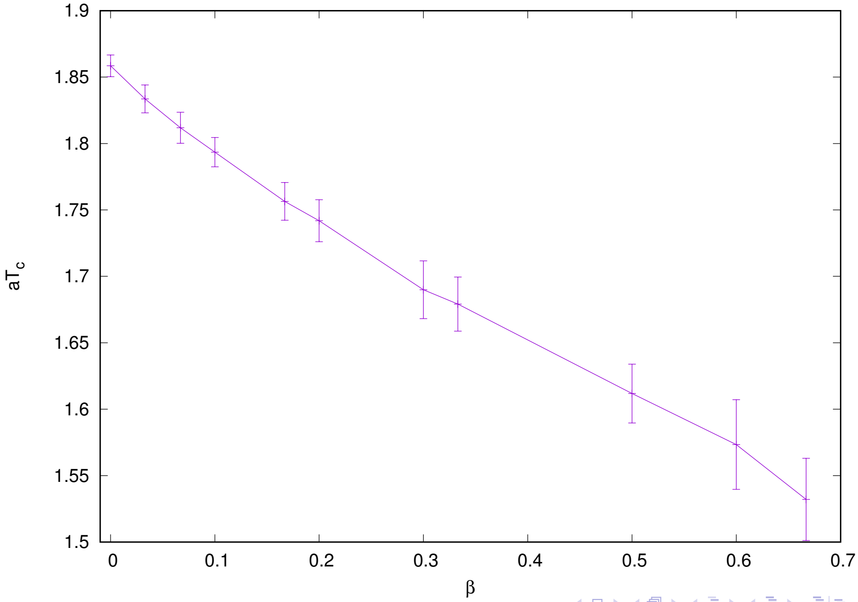
Induced sign problem for $U(3)$



Chiral susceptibility for $U(3)$

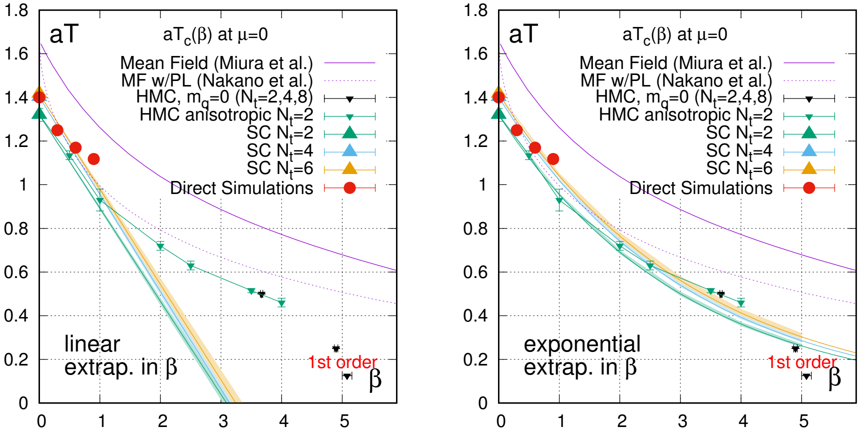


Critical temperature aT_c in β for $U(3)$



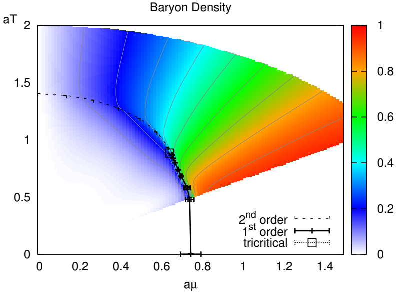
Chiral Transition at finite β and $\mu = 0$

Via direct sampling of plaquettes: $\mathcal{O}(\beta^2)$ corrections for SU(3)

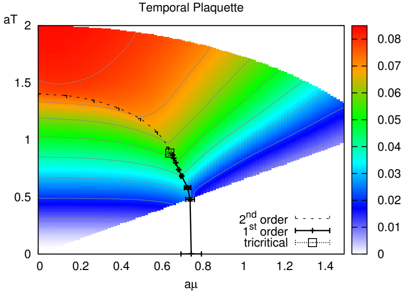


- Comparison to mean field and reweighting [Nakano, Miura, Ohnishi, PRD 83,2011]
- Exponential extrapolation slightly favored

Baryon Density and Plaquette at strong coupling

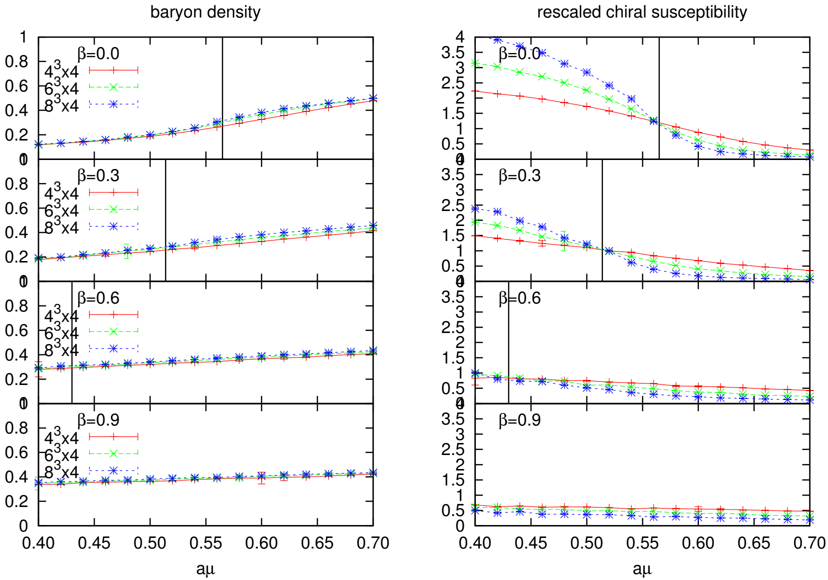


(g) Nuclear transition

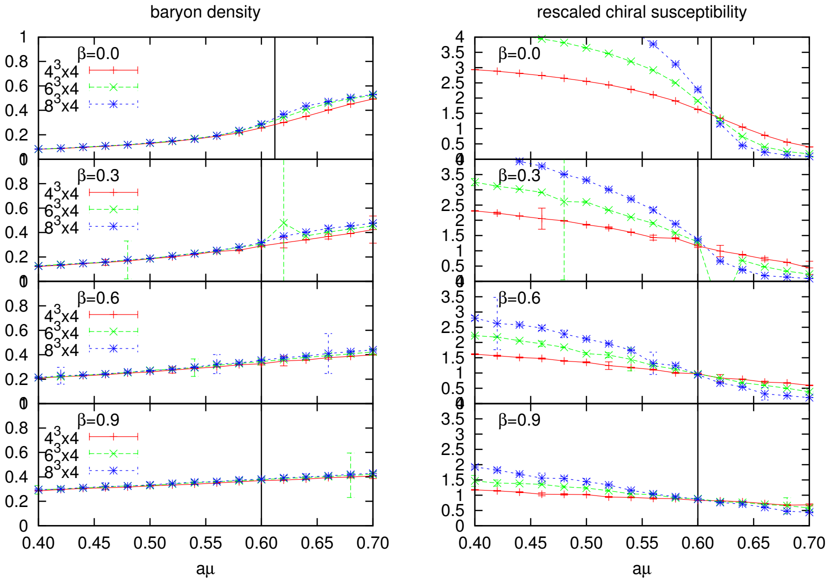


(h)

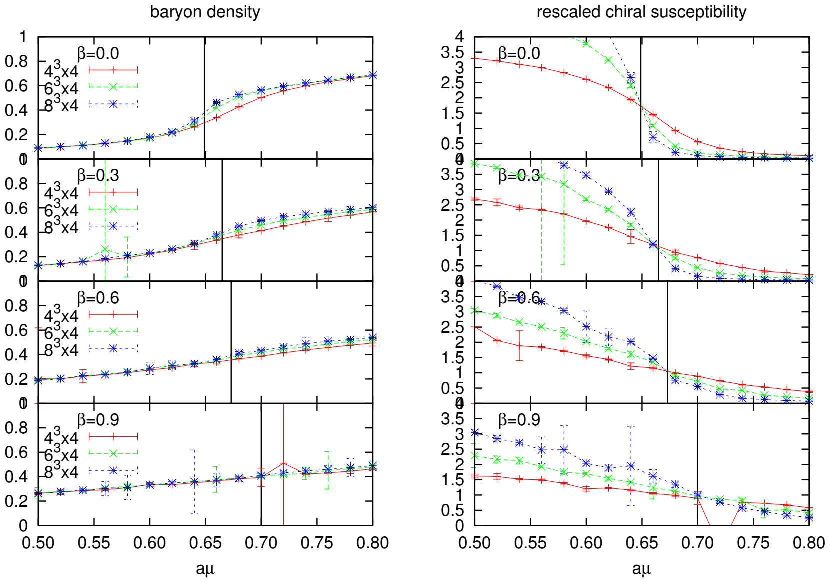
Phase boundary for $SU(3)$ at finite β, finite μ and $aT = 1.1 > aT_{\text{tric}}$



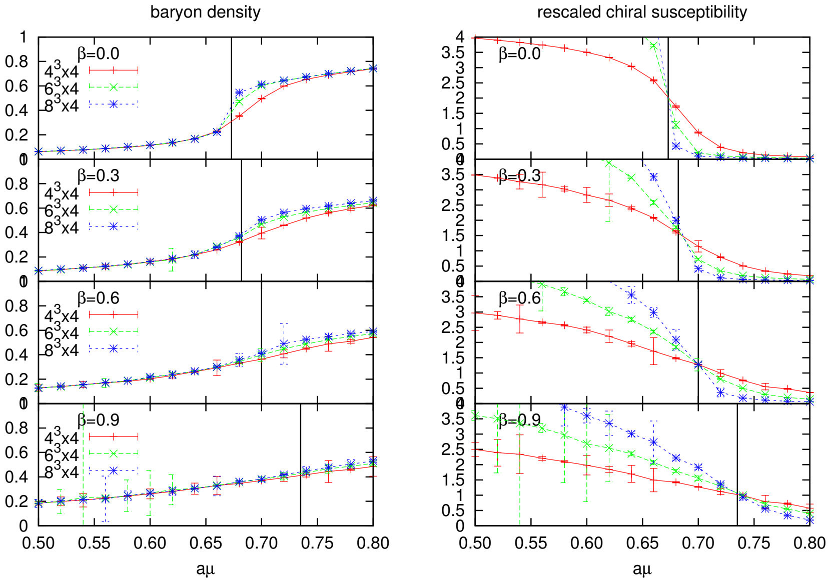
Phase boundary for $SU(3)$ at finite β, finite μ and $aT = 1.0 \sim aT_{\text{tric}}$



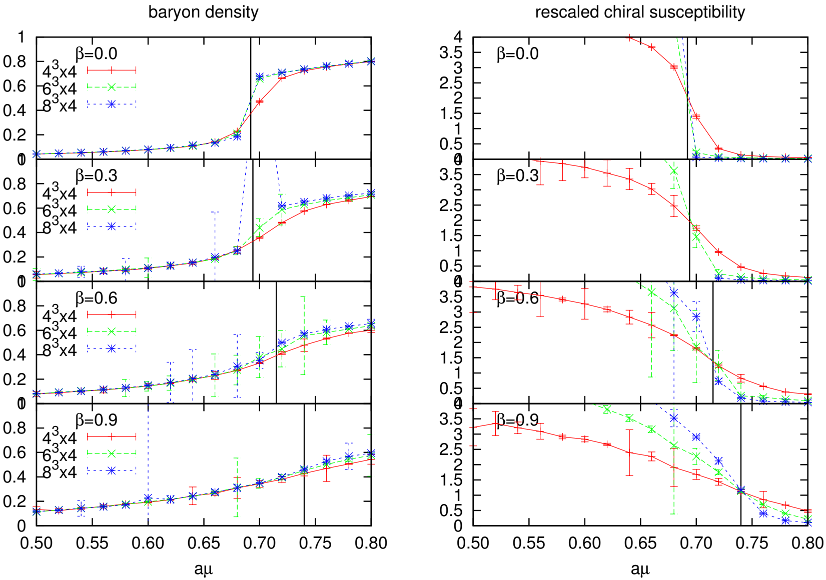
Phase boundary for $SU(3)$ at finite β, finite μ and $aT = 0.9 < aT_{\text{tric}}$



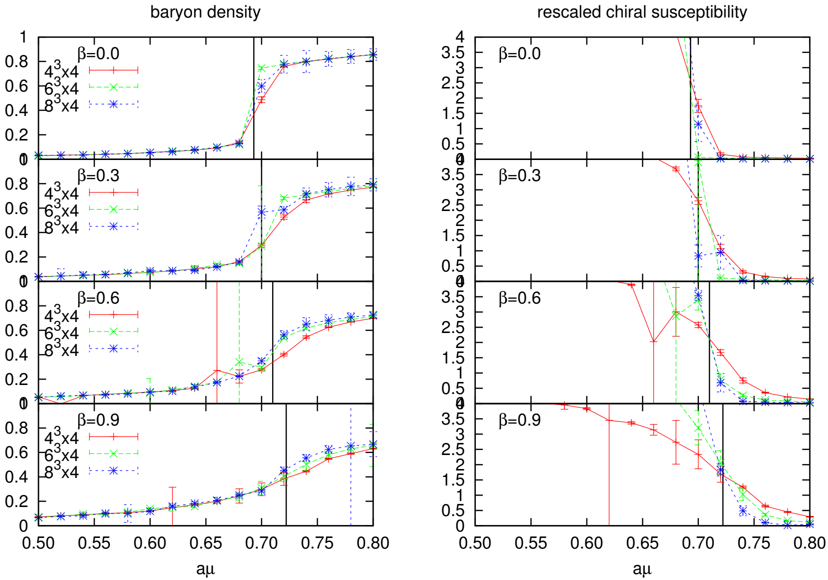
Phase boundary for $SU(3)$ at finite β, finite $\mu$and $aT = 0.81 < aT_{\text{tric}}$



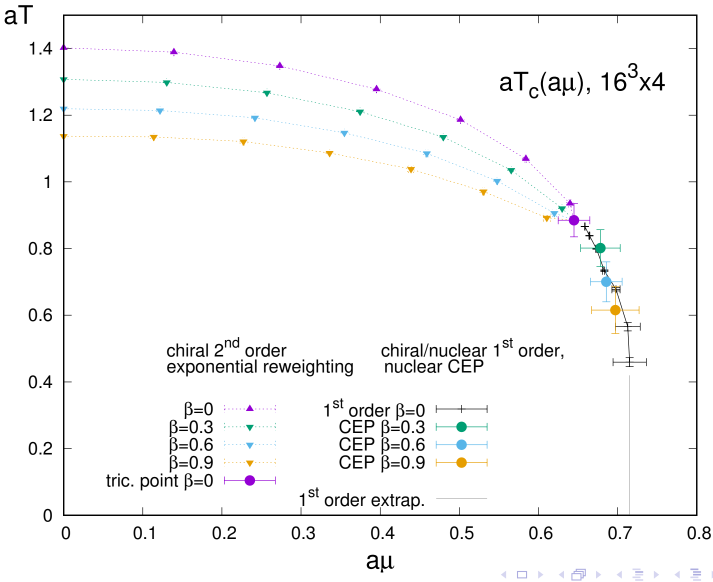
Phase boundary for $SU(3)$ at finite β, finite μ and $aT = 0.72 < aT_{\text{tric}}$



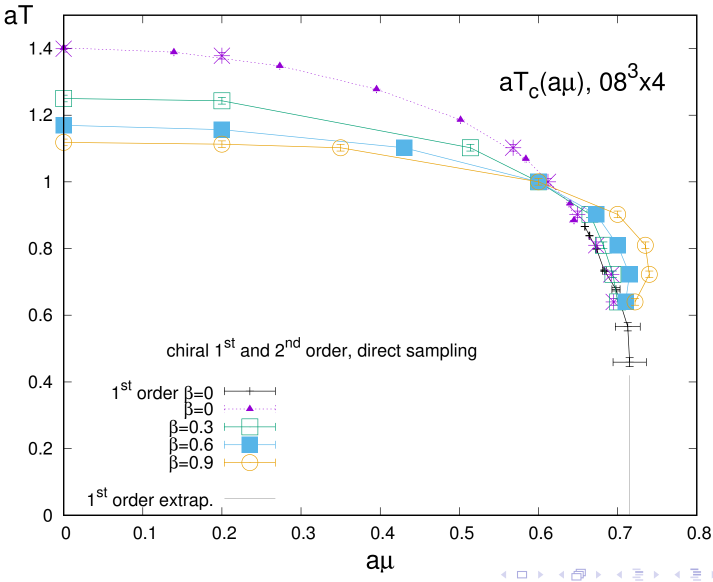
Phase boundary for $SU(3)$ at finite β, finite μ and $aT = 0.64 < aT_{\text{tric}}$



Phase Diagram in the Strong Coupling Regime
[Langelage, de Forcrand, Philipsen &U.,PRL 113(2014)]



Phase Diagram in the Strong Coupling Regime, New Preliminary data



Conclusions

Results

- We obtain the phase boundary in the stong coupling regime via direct simulations.
- $\mathcal{O}(\beta)^2$ corrections are included
- Below aT_{tric}, $a\mu_c^{\mathrm{chiral}}(aT)$ increases with β, in contrast to reweighting.
- sign problem severe for $\beta > 1$.

Questions to be addressed next

- indication that the tri-critical point moves to larger μ?
- β dependance of nuclear transition $a\mu_c^{\mathrm{chiral}}(aT)$ are not settled yet.

Backup

Backup: Connection Between Strong Coupling and Continuum Limit?

One of several possible scenarios for the extension to the continuum: