Equation of State and Freezeout in QCD with Staggered Quarks

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Slowly convergent series

$$\mathcal{R} = 1 + 0.06 + 0.004 + 0. + \cdots$$

$$\mathcal{S} = 1 + 0.72 + 0.52 + 0.38 + \cdots$$

$$\mathcal{T} = 1 + 0.81 + 0.66 + 0.53 + \cdots$$

$$\mathcal{U} = 1 + 0.90 + 0.81 + 0.73 + \cdots$$

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$$1 + x^2 + x^4 + x^6 + \cdots$$

Critical EOS and Padé Approximants

Our estimate of the QCD critical point:

$$\mu_B^E = (1.85 \pm 0.04)\, T^E, \qquad T^E = (0.94 \pm 0.01)\, T_c.$$

Close to a critical point a branch cut

$$\chi_B^2 \propto [\mu_B^2 - (\mu_B^E)^2]^{-\psi} + \mathrm{non-singular}$$

Implies pole in the DLOG

$$m_1 \simeq \frac{\partial \log \chi_B^2}{\partial \mu_B} \quad \longrightarrow \quad \frac{2\psi\mu_B}{\mu_B^2 - (\mu_B^E)^2}.$$

Convert series expansion for χ_B^2 to series for m_1 and then integrate to get equation of state. **ILGTI, 2013, 2016, 2017**

BONUS: m_1 directly measured in experiment, so use this to understand freezeout. **ILGTI 2010**

Using a DLOG Padé

Error in series coefficients give rise to errors in ψ and μ_B^E. As a result finite probability of infinite value of m_1 at any μ!

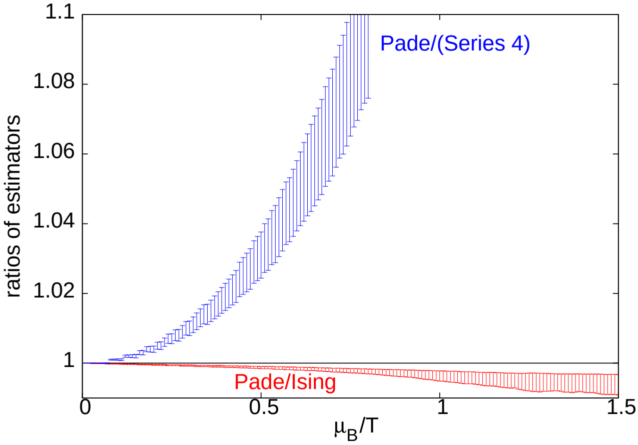
Regularization due to bootstrap with finite statistics. Goes to reasonable limit with infinite statistics, but with critical slowing down!

SG, Karthik, Majumdar (2014)

Is the Padé a good resummation? Use higher order terms in the series to check this. At present only one term can give a reasonable check. In future?

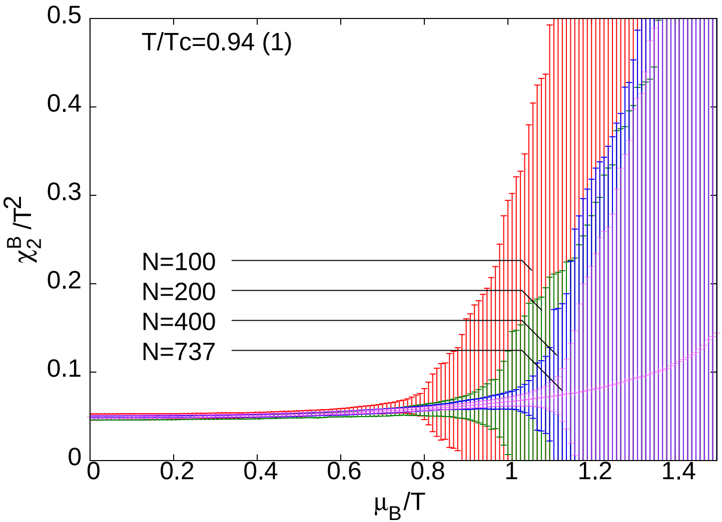
If we assume Ising critical exponent, then only one free parameter: μ_B^E. Two terms serve as tests.

Comparing extrapolations

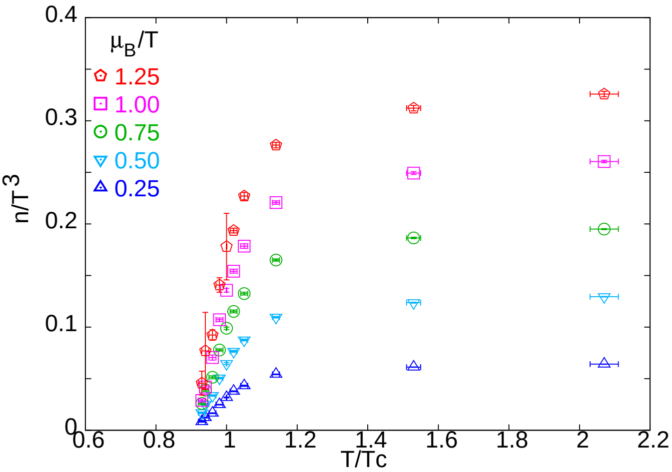


Measurements consistent with Ising exponents (also with mean field, at present). Not consistent with truncated series expansion.

Extrapolation and critical slowing down

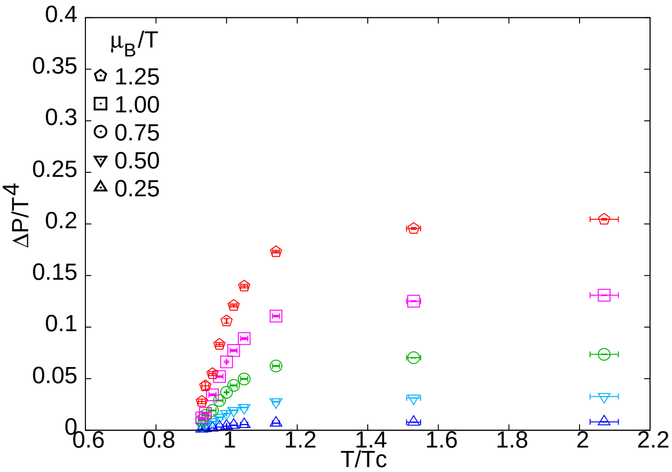


Equation of state



$\Delta P = P(T, \mu_B) - P(T, 0)$, and $n = \partial \Delta P / \partial \mu_B$.

Equation of state



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Freezeout

$$m_1 = \frac{\partial \log(\chi^2_B/T^2)}{\partial(\mu_B/T)} = \frac{\chi^3_B/T}{\chi^2_B/T^2}$$

Suggested as an experimental measurement in **SG (2009)**, **Gavai and SG (2010)**. First data from RHIC in **STAR (2010)**.

Freezeout is a signal of non-equilibrium evolution. Why compare data to equilibrium statistics determined on lattice?

Compare different signals with lattice data and extract T and μ_B. Different signals will be slightly different. Degree of difference tells us about how close to equilibrium the system was.

Much later work including important measurements of m_1 by **Bazavov et al (2012)**, **Borsanyi et al (2013)**, and several others.

Sources of error

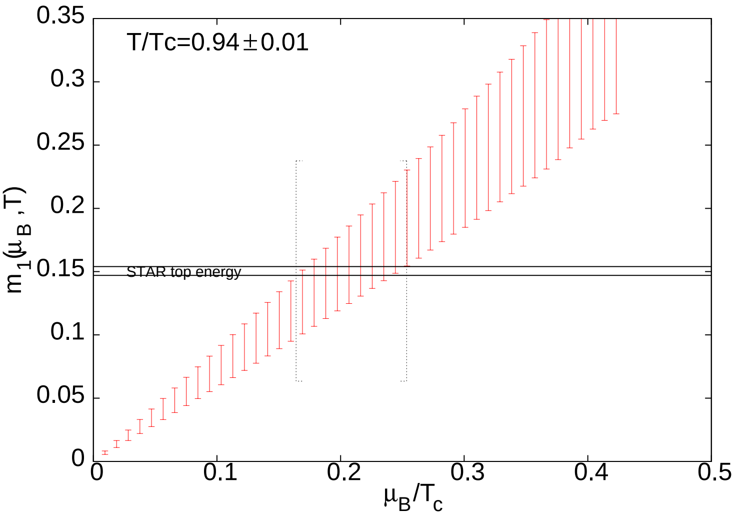
Statistical and lattice systematics

Previously unexamined source of error: series truncation and critical slowing down. Since freezeout occurs close to the critical temperature, lattice computations may suffer from freezeout. Not visible in truncated series analysis.

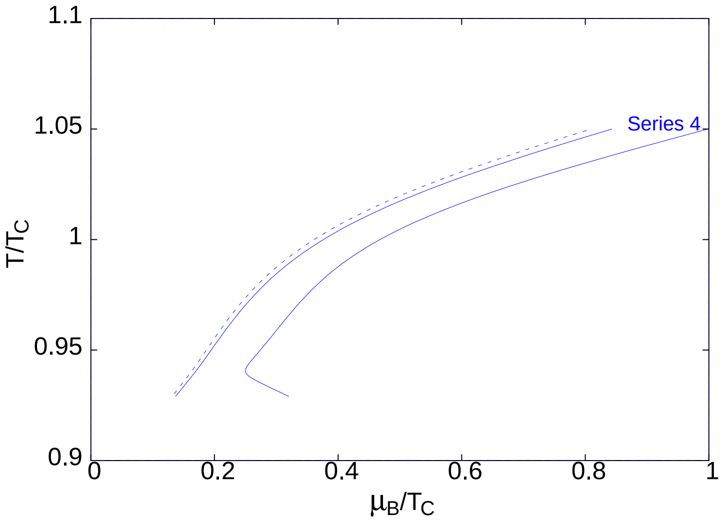
Mathematical

At least two freezeout parameters to be determined: T and μ at each collision energy. Cannot use one measurement to extract them from first principles. One lattice measurement gives an allowed region of parameters.

Critical slowing down



Freezeout parameters



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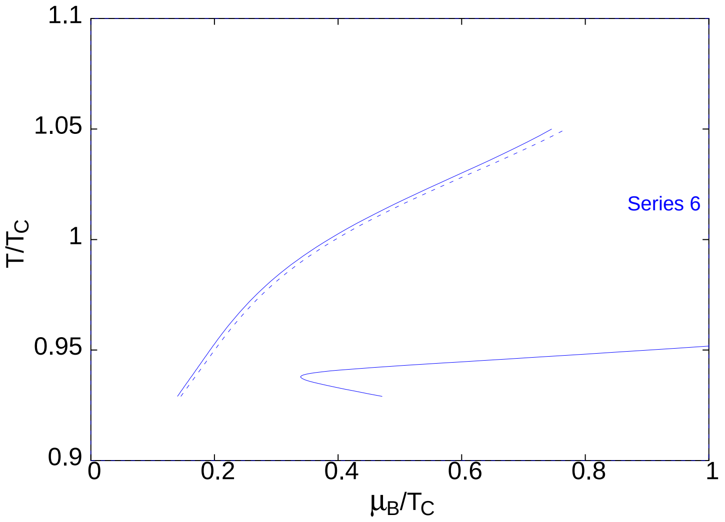

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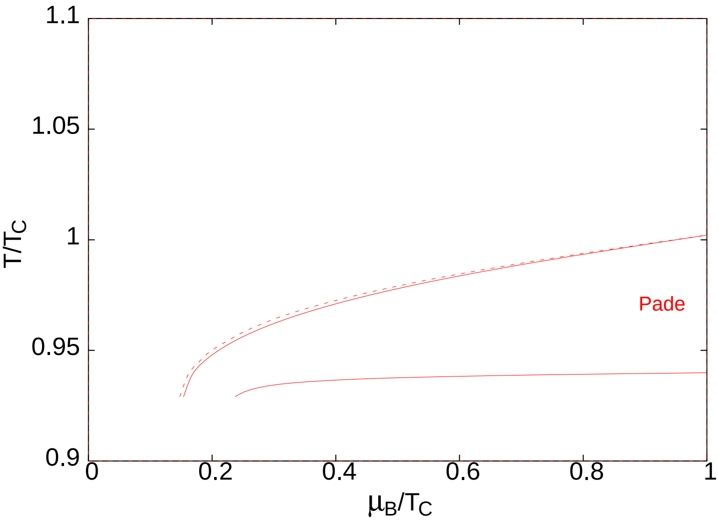

Conclusions

Equation of state

Critical slowing down visible in numerical evaluation of DLOG Padé (m_1). Reconstruction of singular part of free energy and its derivatives straightforward.

Freezeout

The DLOG Padé m_1 is unlikely to be a baryometer. Resummed series shows that freezeout temperature is likely to be below T_c. Extraction of freezeout parameters still error bound.