The thermal photon rate from dynamical lattice QCD – part 2

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Padé fit to the spectral function

- so far model independent approach (BG),

  now explore a maximum likelihood fit ansatz:

- $\tanh$-regulated spectral function can be modeled as

$$\frac{\rho(\omega,k)}{\tanh(\omega\beta/2)} = \frac{A(1+B\omega^2)}{[\omega^2+a^2]\,[(\omega+\omega_0)^2+b^2]\,[(\omega-\omega_0)^2+b^2]}$$

  two linear parameters A and B,
  three nonlinear parameters (a, ω_0, b)

- inspired by a superconvergent sum rule derived from Lorentz invariance, charge conservation and the operator product expansion (OPE)

Padé fit to the spectral function

$$\frac{\rho(\omega,k)}{\tanh(\omega\beta/2)} = \frac{A}{[\omega^2 + a^2]}$$

- inspired by the diffusion pole as it arises in hydrodynamics prediction in the infrared limit
- with $a \leftrightarrow Dk^2$ for small k

$$\frac{\rho(\omega,k)}{\omega} \approx \frac{4\chi_s Dk^2}{\omega^2 + (Dk^2)^2}, \qquad \omega, k \ll D^{-1} \qquad \mathrm{hep-th/0607237v2}$$

Padé fit to the spectral function

$$\frac{\rho(\omega,k)}{\tanh(\omega\beta/2)} = \frac{A(1+B\omega^2)}{[\omega^2+a^2]\;[(\omega+\omega_0)^2+b^2]\,[(\omega-\omega_0)^2+b^2]}$$

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- satisfies the large-ω behaviour known from an OPE:

$$\rho(\omega,k) \propto k^2/\omega^4, \qquad \omega \gg \pi T, k$$

- satisfies the superconvergent sum rule

$$\int_0^\infty d\omega\omega\rho(\omega,k) = 0$$

$\Rightarrow$second linear parameter B becomes a function of (a, ω_0, b)

Padé fit to the spectral function

in total:

$$\frac{\rho(\omega,k)}{\tanh(\omega\beta/2)} = \frac{A(1+B\omega^2)}{[\omega^2+a^2]\,[(\omega+\omega_0)^2+b^2]\,[(\omega-\omega_0)^2+b^2]}$$

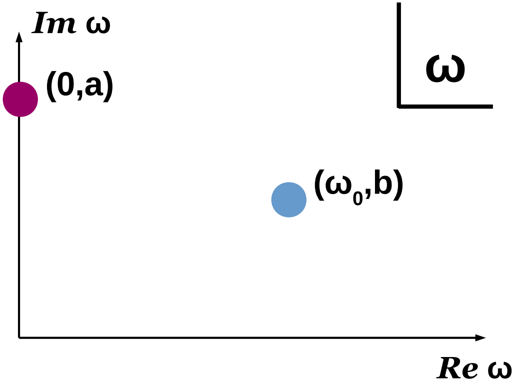
- $a \simeq Dk^2$ for small k
- second pole of thermal size ($\sim T$)
- satisfies UV behaviour and sum rule
- we know there is spectral positivity below the light cone:

  $$\rho(\omega) \geq 0, \qquad \omega \leq k$$

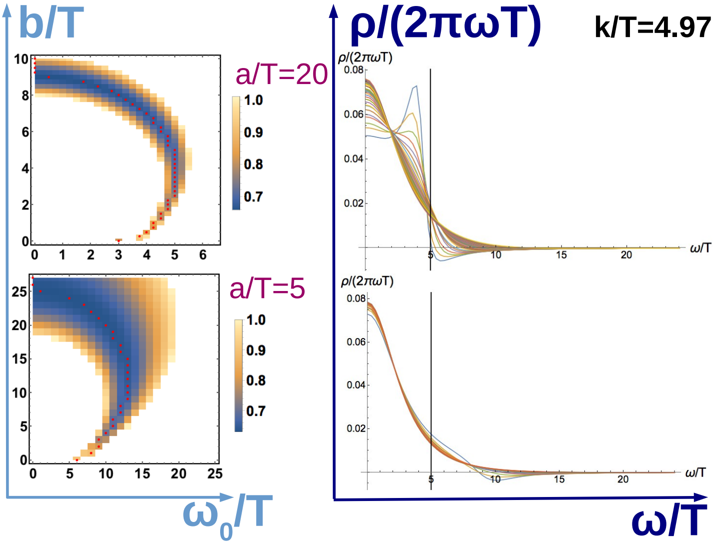
  this is checked *a posteriori*.

Padé fit to the spectral function - pole structure

$$\frac{\rho(\omega, k)}{\tanh(\omega\beta/2)} = \frac{A(1 + B\omega^2)}{[\omega^2 + a^2]\,[(\omega + \omega_0)^2 + b^2]\,[(\omega - \omega_0)^2 + b^2]}$$



Padé fit to the spectral function - uncorrelated $\chi^2(\omega_0, b)$-landscape



Padé fit to the spectral function - uncorrelated χ^2

- 4 fit parameters, 3 degrees of freedom
  for continuum: 7 data points from $t_{\mathrm{min}}/\beta = 0.25$ up to $t_{\mathrm{max}}/\beta = 0.5$
- nonlinear fits are very difficult
- rather than minimizing uncorrelated χ^2:
  in order to bound the photon rate: take the *min* and *max* values of all photon rates with $\chi^2(A, a, \omega_0, b) < 1$
- exclude photon rates which are incompatible with the data $(\chi^2 > 1)$
- another exclusion criterion for (a, b):

$$\min(a, b) > \min(D_{\mathrm{AdS/CFT}} \cdot k^2, D_{\mathrm{PT}}^{-1})$$

where

$$\begin{aligned} D_{\mathrm{AdS/CFT}} &= 1/(2\pi T) \\ D_{\mathrm{PT}}^{-1} &= \mathcal{O}(\alpha_s^2) \cdot T, \qquad \alpha_s = 0.25 \end{aligned}$$

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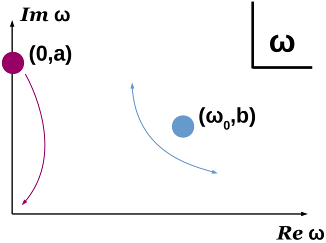
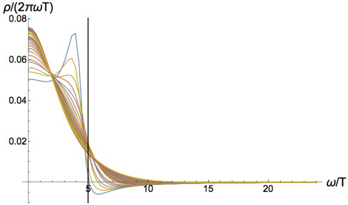
Padé fit to the spectral function - constraining a, b





$$\min(a,b) > \min(D_{\mathrm{AdS/CFT}} \cdot k^2, D_{\mathrm{PT}}^{-1})$$

- corresponds to constraining any additional excitation to be shorter-lived than the largest possible relaxation times in the system (this amounts to the most conservative constraint based on physics):
- $D_{\mathrm{AdS/CFT}} \cdot k^2$ ∼ diffusion of electric charge ($D_{\mathrm{AdS/CFT}} = 1/(2\pi T)$)
- D_{PT}^{-1} ∼ damping of static current ($D_{\mathrm{PT}}^{-1} = \mathcal{O}(\alpha_s^2) \cdot T, \quad \alpha_s = 0.25$)

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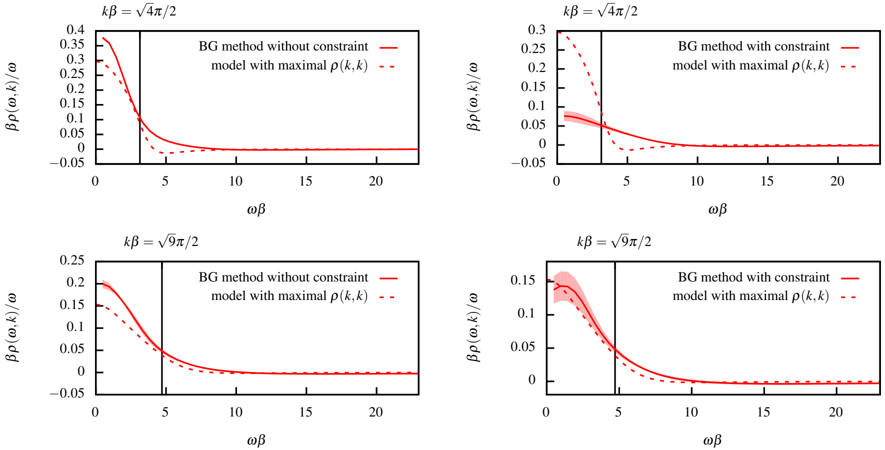
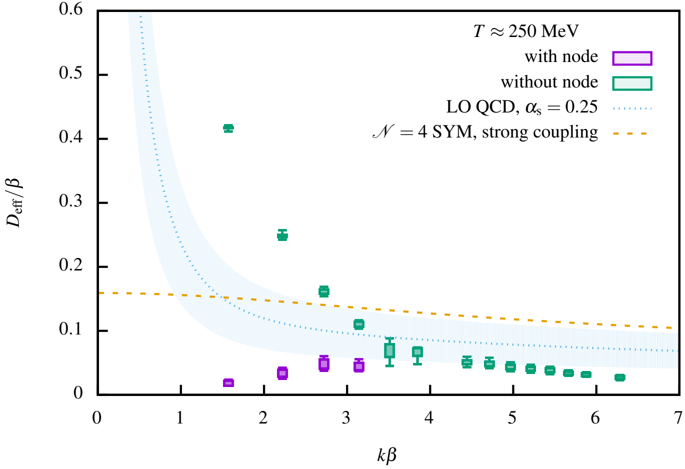
Comparison of BG and model



Plot of the effective diffusion constant $D_{\text{eff}}/\beta = \rho(k,k)/(4\chi_s k)$



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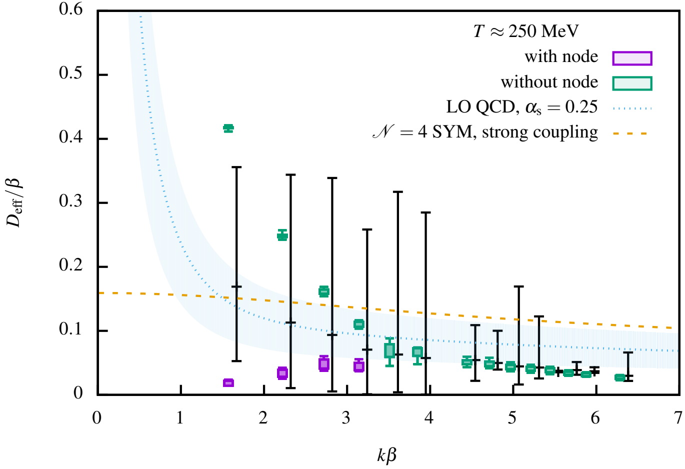

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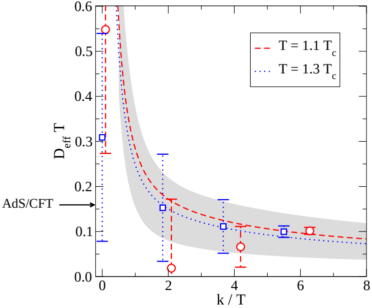

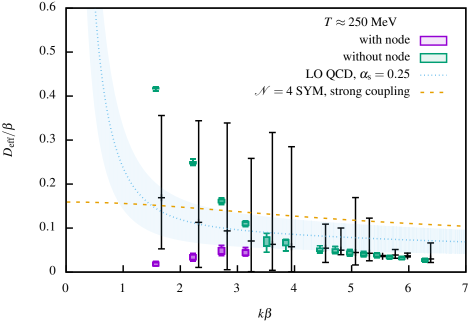

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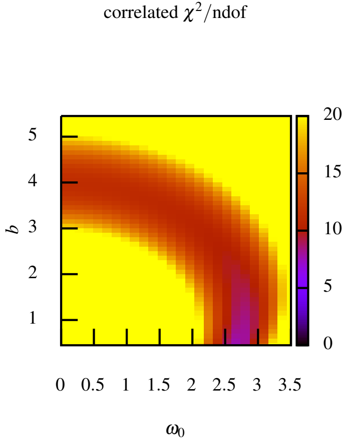
Summary and Outlook

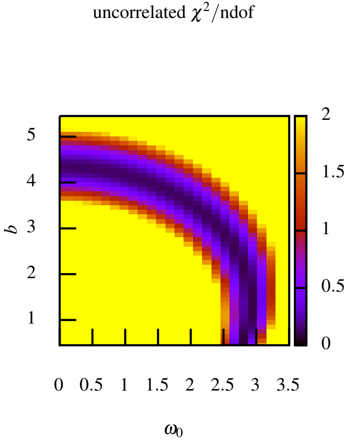
CONCLUDING REMARKS:

- first continuum estimate of the photon rate from dynamical QCD
- employing an alternative linear combination of the vector-vector correlator to eliminate the UV contamination
- exploiting UV behavior known from OPE and imposing a superconvergent sum rule
- applying a model independent method to reconstruct the spectral function
- applying a Padé fit inspired by hydro and physics constraints

AGENDA:

- increase statistics
- examine the correlated χ^2 as a more trustworthy exclusion criterion
- define another effective $D_{\rm eff}(\xi,k) = \frac{\xi\rho(\xi k, k, -2)}{4\chi_s k} \to D$ for $k \to 0$ at fixed $\xi \in [0,1)$
  $\Rightarrow$ more control in small k region
- further examination of systematics including the continuum limit
- extension to higher temperature


correlated χ^2/ndof
b
1
2
3
4
5
0
0.5
1
1.5
2
2.5
3
3.5
ω_0
0
5
10
15
20


uncorrelated χ^2/ndof
b
1
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3.5
ω_0
0
0.5
1
1.5
2

Derivation of a sum rule for $\rho \equiv \rho_{\lambda=-2}$ short version

i. Lorentz invariance and transversity $\Rightarrow \tilde{G}_{\mathrm{E}}(\omega_n, k) = 0$ in vacuum and UV finite at $T > 0$

ii. UV finite correlation admits an OPE $\tilde{G}_{\mathrm{E}}(\omega_n, k) \sim \frac{\mathcal{O}_4}{\omega_n^2}$

Furthermore, charge conservation demands $\tilde{G}_{\mathrm{E}}(\omega_n, k) \to 0$ as $k \to 0$ and $\omega > 0$, so

$$\tilde{G}_{\mathrm{E}}(\omega_n, k) \sim \frac{k^2 \mathcal{O}_4}{\omega_n^4}$$

iii. Matching the large ω_n-behaviour

$$\tilde{G}_{\mathrm{E}}(\omega_n, k) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \omega \frac{\rho(\omega, k)}{\omega^2 + \omega_n^2}$$

$$\xrightarrow{\omega_n \to \infty} \frac{1}{\pi \omega_n^2} \int_0^\infty \mathrm{d}\omega\, \omega\, \rho(\omega, \mathrm{k}) \sim \frac{\mathrm{k}^2 \mathcal{O}_4}{\omega_\mathrm{n}^4}$$

results in the sum rule $\int_0^\infty \mathrm{d}\omega\, \omega \rho(\omega, \mathrm{k}) = 0$