$\bar{B} \to D^{\star}\ell\bar{\nu}$ at non-zero recoil

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Outline

- Introduction and motivation
- Form factors and the $|V_{cb}|$ CKM matrix element
- Available data and simulations
- Status of the analysis
  - Two-point functions
  - Three-point functions
    - Ratio X_f and the recoil parameter w
    - Double ratio R_{A_1} and h_{A_1}
    - Ratios R_0, R_1 and h_{A_2}, h_{A_3}
    - Ratio X_V and h_V
- Summary and future work

Introduction and motivation

- Precision test of the standard model, looking into new physics
- Weak decays and CKM matrix

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- Tension between current inclusive and exclusive V_{cb} determinations

$$|V_{cb}|_{inc} = (42.2 \pm 0.8) \times 10^{-3}, \qquad |V_{cb}|_{exc} = (39.2 \pm 0.7) \times 10^{-3}$$

PDG 2016

- Forthcoming experiments (LHCb, Belle-II) aim to reduce the uncertainty in the determination of the CKM matrix elements
- Imperative to reduce errors coming from lattice determination

Form factors and the $|V_{cb}|$ CKM matrix element

- Form factors

$$\frac{\langle D^\star(p_{D^\star},\epsilon^\nu)|\,\mathcal{V}^\mu\,|\bar{B}(p_B)\rangle}{2\sqrt{m_B\,m_{D^\star}}} = \frac{1}{2}\epsilon^{\nu*}\varepsilon^{\mu\nu}_{\rho\sigma}v^\rho_B v^\sigma_{D^\star}h_V(w)$$

$$\frac{\langle D^\star(p_{D^\star},\epsilon^\nu)|\,\mathcal{A}^\mu\,|\bar{B}(p_B)\rangle}{2\sqrt{m_B\,m_{D^\star}}} = \frac{i}{2}\epsilon^{\nu*}\left[g^{\mu\nu}\,(1+w)\,h_{A_1}(w) - v^\nu_B\left(v^\mu_B h_{A_2}(w) + v^\mu_{D^\star}h_{A_3}(w)\right)\right]$$

- From the differential decay rate and the form factors (encoded in $\mathcal{F}(w)$) we can extract V_{cb}
- Zero recoil suppression limit experimental measurements

$$\frac{d\Gamma}{dw} = \frac{G_F^2 M_B^5}{4\pi^3}r^3(1-r^2)\,|\eta_{EW}|^2\,|V_{cb}|^2\,(w^2-1)^{\frac{1}{2}}\chi(w)\,|\mathcal{F}(w)|^2$$

$$r = \frac{M_{D^\star}}{M_B}, \qquad w = v_{D^\star}\cdot v_B = \frac{E_{D^\star}}{M_{D^\star}}$$

Form factors and the $|V_{cb}|$ CKM matrix element

$$\frac{d\Gamma}{dw} = \frac{G_F^2 M_B^5}{4\pi^3} r^3 (1-r^2) \left|\eta_{EW}\right|^2 \left|V_{cb}\right|^2 (w^2-1)^{\frac{1}{2}} \chi(w) \left|\mathcal{F}(w)\right|^2$$

- Kinematic factors

$$\left.\begin{aligned} t^2(w) &= \tfrac{1-2wr+r^2}{(1-r)^2} \\ \lambda(w) &= \tfrac{1+\frac{4w}{w+1}t^2(w)}{12} \end{aligned}\right\} \longrightarrow \quad \chi(w) = (1+w)^2\,\lambda(w),$$

- Definition of $\mathcal{F}(w)$

$$\mathcal{F}(w) = h_{A_1}(w)\sqrt{\frac{H_0^2(w) + H_+^2(w) + H_-^2(w)}{\lambda(w)}},$$

Form factors and the $|V_{cb}|$ CKM matrix element

- HQET helicity amplitudes

$H_0(w) = \frac{w-r-X_3(w)-rX_2(w)}{1-r}$, longitudinal polarization

$H_\pm(w) = t(w)\left(1 \mp X_V(w)\right)$, transversal polarization

where

$$X_V(w) = \sqrt{\frac{w-1}{w+1}}\frac{h_V(w)}{h_{A_1}(w)},$$

$$X_2(w) = (w-1)\frac{h_{A_2}(w)}{h_{A_1}(w)}, \qquad X_3(w) = (w-1)\frac{h_{A_3}(w)}{h_{A_1}(w)}$$

Form factors and the $|V_{cb}|$ CKM matrix element

- Playing with the polarization/momentum of the $D^\star$ we can calculate the different helicity form factors

$$\langle D^\star(p_\perp)|\, A_1 \,|\bar{B}(0)\rangle = (1+w)\, h_{A_1}(w)$$

Double ratio

$$|R_{A_1}(w)|^2 = \frac{\langle D^\star(p_\perp)|\, A_1 \,|\bar{B}(0)\rangle \langle \bar{B}(0)|\, A_1 \,|D^\star(p_\perp)\rangle}{\langle D^\star(0)|\, V_0 \,|D^\star(0)\rangle \langle \bar{B}(0)|\, V_0 \,|\bar{B}(0)\rangle} = \left(\frac{1+w}{2}\right)^2 |h_{A_1}(w)|^2$$

Axial form factors:

$$R_0(p) = \frac{\langle D^\star(p_\parallel)|\, A_4 \,|\bar{B}(0)\rangle}{\langle D^\star(p_\perp)|\, A_1 \,|\bar{B}(0)\rangle} = \frac{\sqrt{w^2-1}\,(1 - h_{A_2}(w) + w h_{A_3}(w))}{(1+w)\, h_{A_1}(w)}$$

$$R_1(p) = \frac{\langle D^\star(p_\parallel)|\, A_1 \,|\bar{B}(0)\rangle}{\langle D^\star(p_\perp)|\, A_1 \,|\bar{B}(0)\rangle} = w - \frac{(w^2-1)\, h_{A_3}(w)}{(1+w)\, h_{A_1}(w)}$$

Form factors and the $|V_{cb}|$ CKM matrix element

Vector form factor:

$$X_V(p) = \frac{\langle D^\star(p_\perp)| V_1 |\bar{B}(0)\rangle}{\langle D^\star(p_\perp)| A_1 |\bar{B}(0)\rangle} = \sqrt{\frac{w-1}{w+1}} \frac{h_V(w)}{h_{A_1}(w)}$$

Recoil parameter:

$$w^2 = 1 + v_{D^\star}^2$$

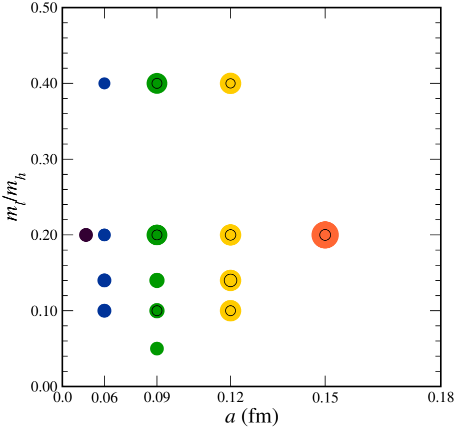
It is measured as a dynamical quantity with the ratio

$$X_f(p) = \frac{\langle D^\star(p)| \mathbf{V} |D^\star(0)\rangle}{\langle D^\star(p)| V_4 |D^\star(0)\rangle} = \frac{\mathbf{v}_{D^\star}}{w+1}$$

From here

$$w(p) = \frac{1+\mathbf{x}_f^2}{1-\mathbf{x}_f^2}$$

Available ensembles



- $N_f = 2 + 1$ staggered asqtad sea quarks
- Size of the point proportional to the statistics (min 2372, max 15072)

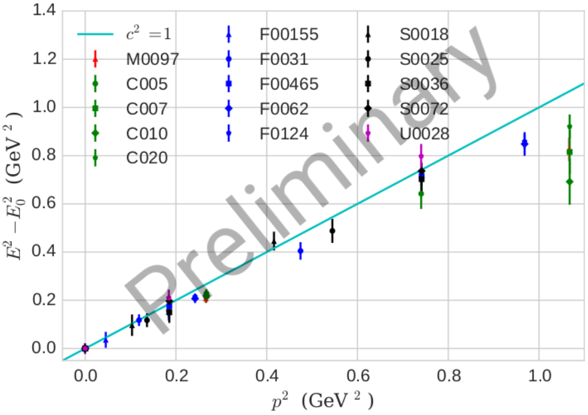
Analysis: two-point functions

- Light spectator quark modeled with the asqtad action
- Heavy quarks modeled with the Fermilab action
- Smeared $1S$ at source, sink, both or none
- Joint fits:
  - Three different smearings $\longrightarrow$ point, $1S$, symmetric average point-$1S$ (3 corr.)
  - If $p \neq 0$, two different momenta $p_{\|}$ and $p_{\perp}$ (6 corr.)
- Fit ranges: t_{Min} the same for all the ensembles, t_{Max} depends on statistics
  - t_{Min} the same for all the ensembles
  - t_{Max} depends on statistics and the size of the correlation matrix

		Zero momentum (3 corr.)		Non-zero momentum (6 corr.)	
Ensemble	a (fm)	$t_{Min}^{D^\star}$ / t_{Min}^{B}	$t_{Min}^{D^\star}$ / t_{Min}^{B} in (fm)	t_{Min}	t_{Min} in (fm)
Medium coarse	0.150	9 / 5	1.350 / 0.750	7	1.050
Coarse	0.120	12 / 6	1.440 / 0.720	9	1.020
Fine	0.090	16 / 8	1.440 / 0.720	11	0.990
Superfine	0.060	24 / 12	1.440 / 0.720	17	1.020
Ultrafine	0.045	31 / 16	1.395 / 0.720	23	1.035

Analysis: two-point functions

- Try $1+1$, $2+2$ and $3+3$ fits and check stability against t_{Min}, kept $2+2$
- Momenta available $|p| = 0, 1, 2$ in lattice units
- Dispersion relation and speed of light



Analysis: three-point functions

- Joint fit of all the available smearings
  - Parent meson $\bar{B}$ always smeared
  - Daughter meson $D^\star$ could be $1S$ or unsmeared
- Oscillating states removed with an average over different sinks

$$\bar{R}(t,T) = \frac{1}{2}R(t,T) + \frac{1}{4}R(t,T+1) + \frac{1}{4}R(t+1,T+1)$$

- In general, fit taking into account first excited state to

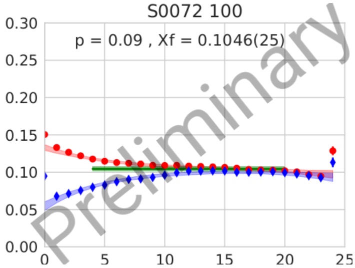
$$R(t) = r\left(1 + Ae^{-\Delta E_{M_0} t} + Be^{-\Delta E_{M_T}(T-t)}\right)$$

- Tight priors (data) for ΔE coming from the two-point fits
- Loose priors for r, A and B
- The Z factors were applied per jackknife bin
- For each ratio, common fit ranges for all the ensembles

Results: X_f and the recoil parameter w

- Ratio

$$\frac{\langle D^\star_{s_1}(p_\perp)|\,\mathbf{V}\,|D^\star_{1S}(0)\rangle}{\langle D^\star_{s_1}(p_\perp)|\,V_4\,|D^\star_{1S}(0)\rangle} = \mathbf{x}_f, \quad \rightarrow \quad w = \frac{1+\mathbf{x}_f^2}{1-\mathbf{x}_f^2}$$



- Meson at source smeared (blue) and unsmeared (red)
- Meson at sink always smeared, set $B = 0$ in the fit ansatz

$$R(t) = r\left(1 + Ae^{-\Delta E_{M_0} t} + Be^{-\Delta E_{M_T}(T-t)}\right)$$

Results: double ratio R_{A_1} and h_{A_1}

- Excited states at source and sink coming from $D^\star$
- Missing $\langle \bar{B}(0) | A_1 | D^\star(p_\perp) \rangle$, used T-reversal
- Compare double ratio against single ratio

Double ratio

$$\frac{\langle D^\star_{s_1}(p_\perp) | A_1 | \bar{B}_{1S}(0) \rangle \langle \bar{B}_{1S}(0) | A_1 | D^\star_{s_2}(p_\perp) \rangle}{\langle D^\star_{s_3}(0) | V_0 | D^\star_{1S}(0) \rangle \langle \bar{B}_{1S}(0) | V_0 | \bar{B}_{1S}(0) \rangle} =$$

$$\sqrt{\frac{Z_{s_1}(p_\perp) Z_{s_2}(p_\perp)}{Z_{1S}(0) Z_{s_3}(0)}} \frac{m_{D^\star}}{E_{D^\star}} e^{-(E_{D^\star} - m_{D^\star})T} \left(\frac{1+w}{2} \right)^2 |h_{A_1}(w)|^2$$

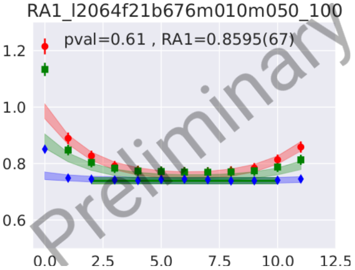
Single ratio

$$\frac{\langle D^\star_{1S}(p_\perp) | A_1 | \bar{B}_{1S}(0) \rangle}{\langle D^\star_{1S}(0) | A_1 | \bar{B}_{1S}(0) \rangle} = \sqrt{\frac{Z_{1S}(p_\perp)}{Z_{1S}(0)} \frac{m_{D^\star}}{E_{D^\star}}} e^{-(E_{D^\star} - m_{D^\star})t} \frac{1+w}{2} \frac{h_{A_1}(w)}{h_{A_1}(0)}$$

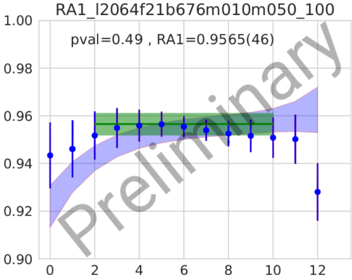
Results: double ratio R_{A_1} and h_{A_1}

- Compare double ratio against single ratio
- Blue: smeared. Red: unsmeared. Green: mix smeared-unsmeared.

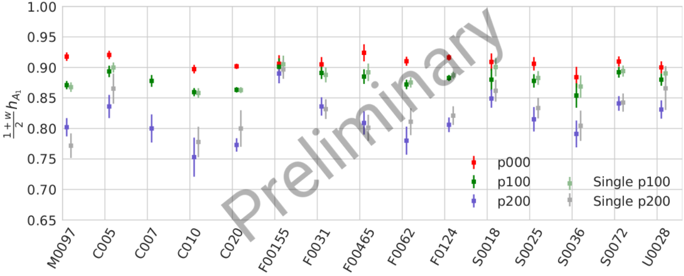
Double ratio



Single ratio

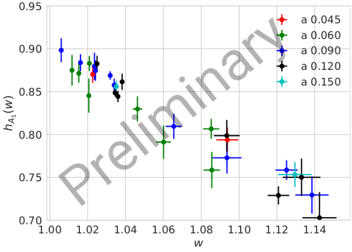


- Consistent single-double ratio

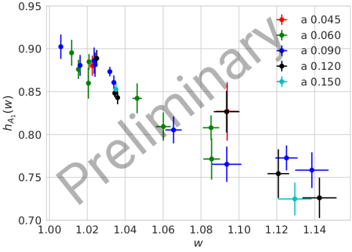


Results: double ratio R_{A_1} and h_{A_1}

Double ratio



Single ratio



Results: ratios R_0, R_1 and h_{A_2}, h_{A_3}

- Ratios

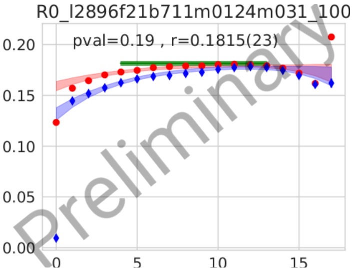
$$R_0(p) = \frac{\left\langle D^\star(p_\parallel)\right| A_4 \left|\bar{B}(0)\right\rangle}{\left\langle D^\star(p_\perp)\right| A_1 \left|\bar{B}(0)\right\rangle} = \sqrt{\frac{Z(p_\parallel)}{Z(p_\perp)}} \frac{\sqrt{w^2-1}\,(1 - h_{A_2}(w) + w h_{A_3}(w))}{(1+w)\, h_{A_1}(w)}$$

$$R_1(p) = \frac{\left\langle D^\star(p_\parallel)\right| A_1 \left|\bar{B}(0)\right\rangle}{\left\langle D^\star(p_\perp)\right| A_1 \left|\bar{B}(0)\right\rangle} = \sqrt{\frac{Z(p_\parallel)}{Z(p_\perp)}} \left(w - \frac{\left(w^2-1\right) h_{A_3}(w)}{(1+w)\, h_{A_1}(w)}\right)$$

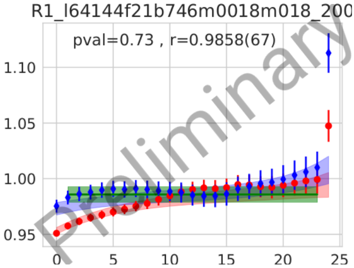
- Excited states at source from $D^\star$ and at sink from $\bar{B}$

Results: ratios R_0, R_1 and h_{A_2}, h_{A_3}

Fit for R_0

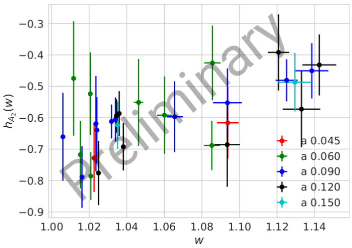


Fit for R_1

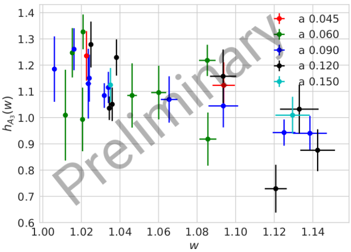


Results: ratios R_0, R_1 and h_{A_2}, h_{A_3}

$h_{A_2}(w)$



$h_{A_3}(w)$

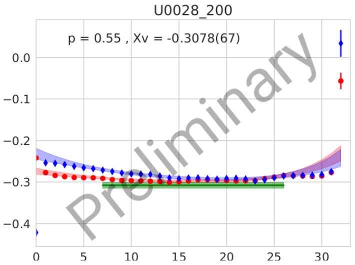


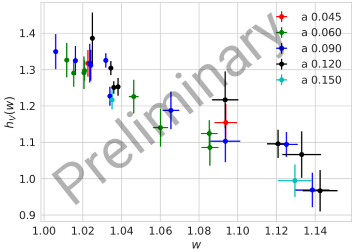
Results: X_V and h_V

- Ratio

$$\frac{\langle D^\star(p_\perp)|\, V_1 \,|\bar{B}(0)\rangle}{\langle D^\star(p_\perp)|\, A_1 \,|\bar{B}(0)\rangle} = \sqrt{\frac{w-1}{w+1}}\frac{h_V(w)}{h_{A_1}(w)}$$

- Blue: smeared daughter meson. Red: unsmeared daughter meson.





Summary and future work

- The CKM matrix elements are a good place to look for new physics
- As experiments improve, we *must* reduce our uncertainty in order to validate the SM/rule out BSM scenarios
- Data at zero and small recoil much necessary to complete experimental data
- This ongoing analysis is an important piece in the puzzle, but much to be done yet
  - Improve fits and reduce errors
  - HQET and chiral fits
  - Continuum limit
  - z-expansion
  - Combined fit with experimental data
  - ...

Thank you for your attention