The $B_{(s)} \to D_{(s)} l\bar{\nu}$ Decay with Highly Improved Staggered Quarks & Improved NRQCD

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Motivation

- $B_{(s)} \to D_{(s)} l\bar{\nu}$ contains a $b \to c$ transiton $\implies$ can be used to determine $|V_{cb}|$.
- $\exists$ many related tensions between prediction & experiment, including in

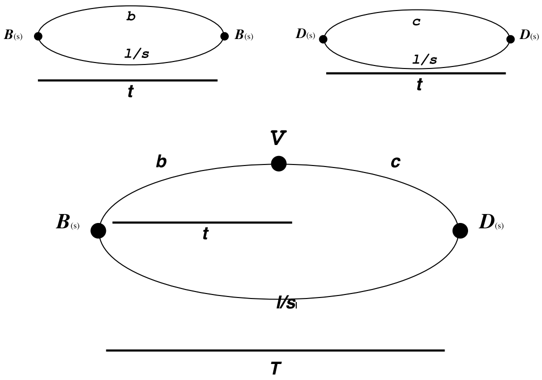
$$R(D) = \frac{\mathcal{B}(B \to D\tau\bar{\nu}_\tau)}{\mathcal{B}(B \to Dl\bar{\nu}_l)}$$

Calculation Setup

- Goal: Deduce $B_{(s)} \to D_{(s)}$ form factors $f_0(q^2)$,$f_+(q^2)$ for $0 < q^2 < q^2_{\text{max}}$. ($q^2 \equiv (p_{B_{(s)}} - p_{D_{(s)}})^2$)
- We use the Darwin cluster @ Cambridge, part of STFC's DiRAC II facility.
- 2nd Generation MILC Gluon Ensembles [1212.4768]:
  - Lüscher-Weisz Gauge action, improved up to $\mathcal{O}(N_f \alpha_s a^2)$.
  - 2+1+1 flavours in the sea, using HISQ action (Highly Improved Staggered Quark).
  - Many Lattice Spacings and quark masses, down to the physical point.

Correlation Functions

- Calculate 2 and 3-point correlation functions.
- Use exponential smearing on both $B_{(s)}$ and $D_{(s)}$ operators.
- Extract $\langle D_{(s)}|V_\mu|B_{(s)}\rangle$ from multiexponential fit using Bayesian constraints.



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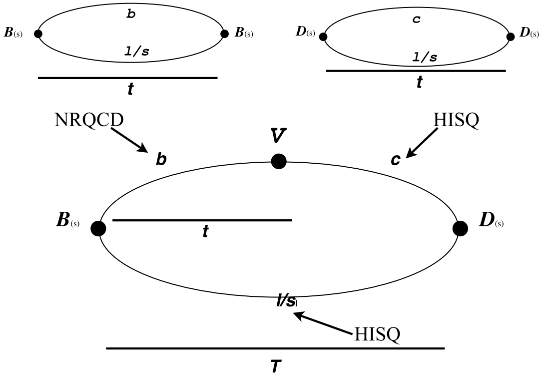


Improved Non-Relativistic QCD (NRQCD)

- Lattice artifacts grow as $(am_b)^n, n > 0$, problem when $am_b > 1$. Solution: non-relativistic formalism for b.
- NRQCD = action expanded in b velocity v, rest mass removed.

$$\begin{aligned} G(\underline{x}, t+1) &= e^{-a(H_0+\delta H)} G(\underline{x}, t) \\ aH_0 &= -\frac{\nabla^{(2)}}{2am_b}, \\ a\delta H &= -c_1 \frac{(\nabla^{(2)})^2}{8(am_b)^3} - c_4 \frac{1}{2am_b} \underline{\sigma} \cdot \underline{\tilde{B}} + ... \end{aligned}$$

- $\{c_i\}$ determined from perturbative matching to full continuum QCD. [1408.5768]
- Improved action: accurate through $\mathcal{O}(\alpha_s v^4)$.

Continuum Vector Current

- Continuum Vector current is expanded in terms of NRQCD-HISQ currents:

$$V_\mu = (1 + z_\mu \alpha_s)\left[V_\mu^{(0)} + V_\mu^{(1)}\right] + \dots$$

$$V_\mu^{(0)} = \bar{c}\gamma_\mu b \quad , \quad V_\mu^{(1)} = -\frac{1}{2m_b}\bar{c}\gamma_\mu \underline{\gamma} \cdot \underline{\nabla} b$$

- z_μ calculated by perturbative matching to full continuum QCD [1211.6966]
- Also $\exists$ at $\mathcal{O}(\alpha_s, v)$:

$$V_\mu^{(2)} = -\frac{1}{2m_b}\bar{c}\gamma_\mu \underline{\gamma} \cdot \overleftarrow{\underline{\nabla}} b$$

$$V_k^{(3)} = -\frac{1}{2m_b}\bar{c}\underline{\nabla}_\mu b \quad , \quad V_k^{(4)} = -\frac{1}{2m_b}\bar{c}\overleftarrow{\underline{\nabla}}_\mu b$$

Form Factors

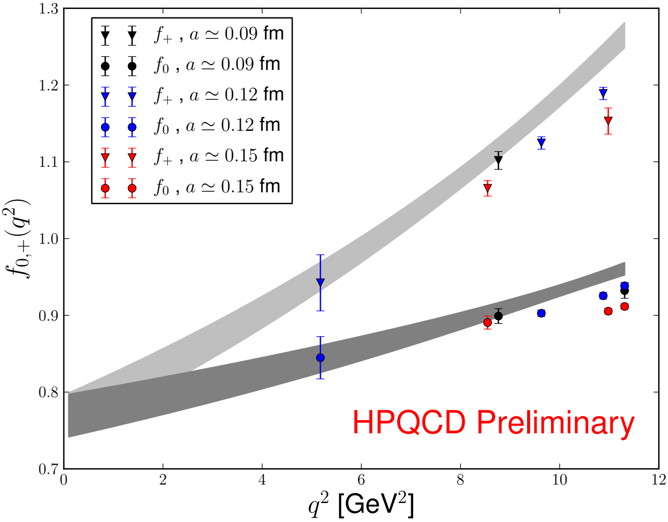
Transition element is parameterized in terms of form factors:

$$\langle D_{(s)}(p_{D_{(s)}})|V^\mu|B_{(s)}(p_{B_{(s)}})\rangle = f_+(q^2)\left[p_1^\mu + p_2^\mu - \frac{M_{B_{(s)}}^2 - M_{D_{(s)}}^2}{q^2}q^\mu\right] + f_0(q^2)\frac{M_{B_{(s)}}^2 - M_{D_{(s)}}^2}{q^2}q^\mu$$

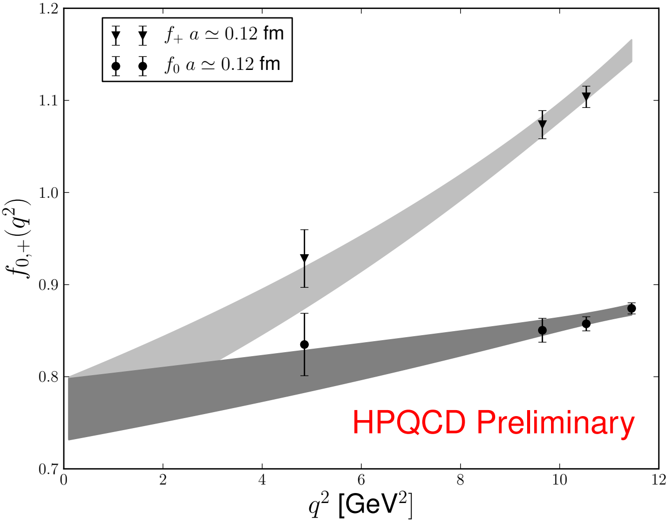
We computed $\langle D_{(s)}|V^\mu|B_{(s)}\rangle$ at varying $q^2 = (p_{B_{(s)}} - p_{D_{(s)}})^2$. We plan on fitting results to a z-fit.

$$f_{0,+}(z(q^2)) \propto \sum_k a_k^{(0,+)} z^k \quad , \quad z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

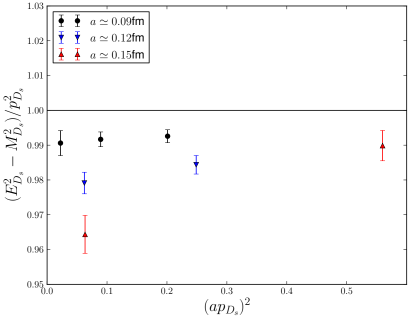
Preliminary Results: $B_s \to D_s$

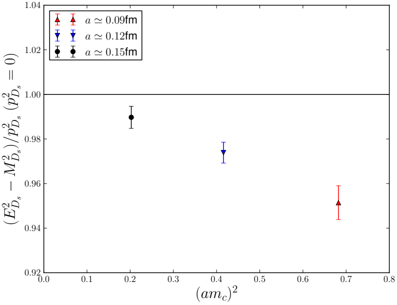


Preliminary Results: $B \to D$



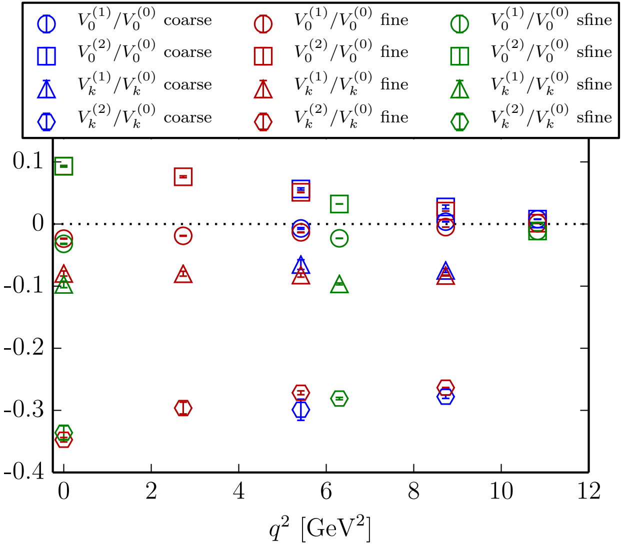
c^2(speed of light) for D_s





See also 1208.2855.

Subleading Currents ($B_c \to \eta_c$)



Next Steps

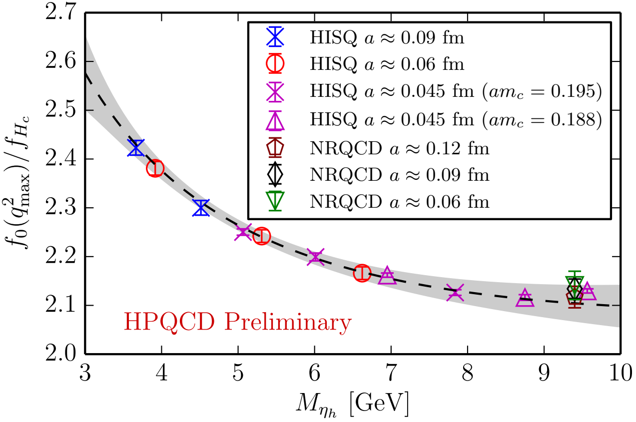
- Aim to extend data to minimum possible q^2 (requires overcoming signal/noise degradation).
- Fix renormalization of spacial currents non-perturbatively?
- New approach of extracting f_0 from scalar current:

$$S^{(0)} = \bar{c}b \quad , \quad m_b(1 - \frac{m_c}{m_b})\langle D_s|S|B_s\rangle = (M_{B_s}^2 - M_{D_s}^2)f_0(q^2)$$

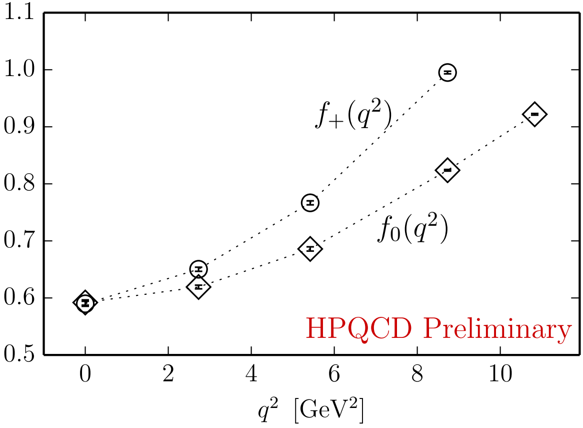
thanks for listening!

Backup Slides

NRQCD-HISQ Comparison



Proof of Principle: $B_c \to \eta_c$



[Colquhoun, Lytle et. al 1611.01987]

HISQ Details

$$\mathcal{F}_\mu = \prod_{\rho\neq\mu}\left(1+\frac{a^2\delta_\rho^{(2)}}{4}\right)$$

$$X_\mu(x) \equiv \mathcal{U}\mathcal{F}_\mu U_\mu(x)$$

$$W_\mu(x) \equiv \left(\mathcal{F}_\mu - \sum_{\rho\neq\mu}\frac{a^2(\delta_\rho)^2}{2}\right)\mathcal{U}\mathcal{F}_\mu U_\mu(x)$$

$$S_{\mathrm{HISQ}} = \sum_x \bar{\psi}(x)\left(\gamma\cdot\left(\nabla_\mu(W) - \frac{a^2}{6}(1+\epsilon)\nabla_\mu^3(X)\right) + m\right)\psi(x)$$

ϵ tuned to satisfy

$$\lim_{\underline{p}\to 0}\frac{E^2(\underline{p}) - m^2}{\underline{p}^2} = 1$$

NRQCD Details

$$
\begin{aligned}
G(\underline{x}, t+1) &= e^{-a(H_0+\delta H)} G(\underline{x}, t) \\
aH_0 &= -\frac{\nabla^{(2)}}{2am_b}, \qquad (1) \\
a\delta H &= -c_1 \frac{(\nabla^{(2)})^2}{8(am_b)^3} + c_2 \frac{i}{8(am_b)^2} (\underline{\nabla} \cdot \underline{\tilde{E}} - \underline{\tilde{E}} \cdot \nabla) \\
&- c_3 \frac{1}{8(am_b)^2} \sigma \cdot (\underline{\nabla} \times \underline{\tilde{E}} - \underline{\tilde{E}} \times \underline{\nabla}) \\
&- c_4 \frac{1}{2am_b} \sigma \cdot \underline{\tilde{B}} + c_5 \frac{\nabla^{(4)}}{24am_b} \\
&- c_6 \frac{(\nabla^{(2)})^2}{16n(am_b)^2} \qquad (2)
\end{aligned}
$$