Weak hamiltonian Wilson Coefficients from Lattice QCD

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Introduction

Weak decays of hadrons

rich phenomenology
(e.g. CP violation in $K \to \pi\pi$)

QCD $\to$ confinement, light objects

Weak interactions $\to$ short range, heavy mediators

There is a natural scale separation in these decays

$\downarrow$

build an **effective low-energy theory**

Integrate out heavy degrees of freedom: heavy quarks, weak bosons

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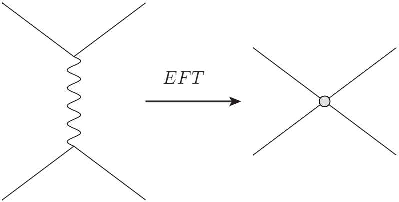
Effective theory

Integrating out weak bosons generates **four-quark vertices**



Current-current diagrams:

$c \to s u \bar{d}$

new divergences in the EFT

$\downarrow$

operator mixing

$$\mathcal{H}_{\text{eff}} \propto G_{\text{F}} \sum_i C_i Q_i \quad \text{with} \quad i = 1, 2 \text{ in our example}$$

Long distance matrix elements $\langle Q_i \rangle \to$ Lattice

Wilson Coefficients $C_i \to$ PT

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OPE

We use W boson propagator in unitary gauge (Euclidean)

$$W_{\mu\nu}(q) = \frac{1}{q^2 + m_{\rm W}^2}\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{m_{\rm W}^2}\right) \overset{m_{\rm W}\to\infty}{\approx} \frac{1}{m_{\rm W}^2}\left[\delta_{\mu\nu} + O\left(\frac{q^2}{m_{\rm W}^2}\right)\right]$$

Four-quark operators Q_i are first terms in the expansion

$$\mathcal{H}_{\rm eff} \propto G_{\rm F}\left[\sum_i C_i Q_i + \sum_i \frac{c_i^{(d)}}{m_{\rm W}^{d-6}} O_i^{(d)}\right] \quad , \quad d \geq 8$$

$O_i^{(d)}$ can be gauge-invariant operators

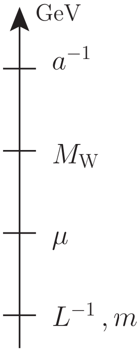
if we fix the (QCD) gauge $O_i^{(d)}$ can be gauge-noninvariant operators

$O_i^{(d)}$ depend on momenta p_i of external states

In the limit $p_i/m_{\rm W} \to 0$, $\forall i$, only Q_1 and Q_2 survive

Window problem

μ is the matching scale:



$am_{\rm W} \ll 1$ for discretization effects

$\mu \ll m_{\rm W}$ for higher order operators

$\mu \gg m\,, \mu L \gg 1$ for infrared effects

Present study is focused on unphysically small $m_{\rm W} \approx 2\;$ GeV

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Matching theories

Caveats:

current available lattices $m_{\mathrm{W}} \approx 2$ GeV

disconnected ($\rightarrow$penguin) diagrams for full operator basis

What can we learn from a calculation with $m_{\mathrm{W}} \approx 2$ GeV ?

we need the limit $p^2/m_{\mathrm{W}}^2 \rightarrow 0 \rightarrow$ infrared scales

study finite volume effects

study finite quark mass effects

Explore different renormalization schemes:

study properties of RI/MOM schemes (e.g. pole structures)

[Dawson, Martinelli, Rossi, Sachrajda, Sharpe, Talevi, Testa '98]

Seminal ideas for a non-perturbatively defined weak hamiltonian

RI/(S)MOM scheme

[Martinelli, Pittori, Sachrajda, Testa, Vladikas 95]

Given renormalized amputated Green's function Λ^R
Regularization Independent conditions (RI-MOM)

$$\Lambda^R|_{p^2=\mu^2} = Z_{\mathrm{q}}^{-n/2}\ Z\ \Lambda^{\mathrm{bare}}|_{p^2=\mu^2} = \Lambda^{\mathrm{tree}}$$

The renormalization scheme is defined by the choice of the external states:

- we use off-shell external quark states
  with momentum p_i, $i = 1, 2, 3, 4$
  with masses $m_i = m$, $\forall i$
  with Projectors P_i to project onto definite spin-color states
- we use Landau gauge

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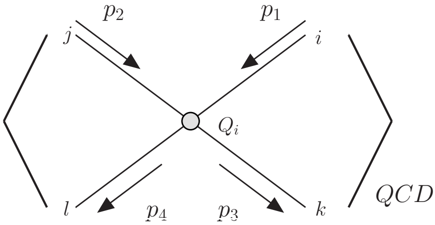
Lattice Observables - EFT

$G(Q_i) =$



Green's function $G(Q_i)$

$Q_1 = (\bar{s}_i c_j)_{\mathrm{V-A}} \otimes (\bar{u}_j d_i)_{\mathrm{V-A}}$

$Q_2 = (\bar{s}_i c_i)_{\mathrm{V-A}} \otimes (\bar{u}_j d_j)_{\mathrm{V-A}}$

RI schemes

$p_1 = p_3 = p$, $p_2 = p_4 = -p$

$p_1 \neq p_2 \neq p_3 \neq p_4$, $p_i^2 = p^2$

$\Lambda(Q_i)$: amputated $G(Q_i)$ with quark propagators $S(p_i, m_i)$

Projectors: $P_1 = \delta_{il}\delta_{kj}\ (\Gamma_1 \otimes \Gamma_2)$, $P_2 = \delta_{ij}\delta_{kl}\ (\Gamma_1 \otimes \Gamma_2)$ [RBC/UKQCD '10]

We define $M_{ij} = P_j\left[\,\Lambda(Q_i)\,\right]$

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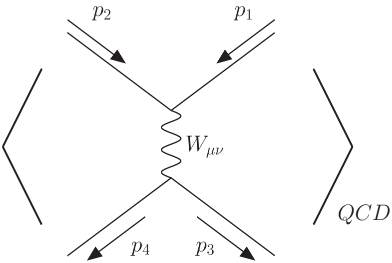
Lattice Observables - Full theory

$G_{\text{SM}} =$



W boson in unitary gauge

RI schemes:

$p_1 = p_3 = p$, $p_2 = p_4 = -p$

$p_1 \neq p_2 \neq p_3 \neq p_4$, $p_i^2 = p^2$

Weak vertex factor $\propto g_2$

Λ_{SM}: amputated G_{SM} with quark propagators $S(p_i, m_i)$

3. Define $W_i = P_i(\Lambda_{\text{SM}})$
4. Note that $W_i^{\text{RI}}(\mu) = Z_{\text{q}}^{-2}(\mu)\ Z_V^2\ W_i^{\text{lat}}|_{p^2=\mu^2}$
   Z_V: vector bilinear operator renormalization factor

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Matching procedure

Matching equation for RI conditions

$$\frac{G_{\mathrm{F}}}{\sqrt{2}}\, C_i^{\mathrm{RI}}(\mu) M_{ij}^{\mathrm{RI}}(\mu) = \frac{g_2^2}{8} W_j^{\mathrm{RI}}(\mu)$$

CKM matrix elements simplify

$G_{\mathrm{F}}/\sqrt{2}$ and $g_2^2/8$ simplification $\rightarrow 1/m_{\mathrm{W}}^2$

$$C_i^{\mathrm{RI}}(\mu) = m_{\mathrm{W}}^2 \left(W_j^{\mathrm{lat}} [M^{\mathrm{lat}}]_{jk}^{-1} \right) \left([Z^{\mathrm{RI}}(\mu)]_{ki}^{-1} Z_V^2 \right)$$

Bare lattice Wilson Coefficients: $C_k^{\mathrm{lat}} = m_{\mathrm{W}}^2 W_j^{\mathrm{lat}} [M^{\mathrm{lat}}]_{jk}^{-1}$

1. The matching procedure on the lattice
   - study effects of higher order operators $O(p^2/m_{\mathrm{W}}^2)$
   - study infrared effects in limit $p^2 \rightarrow 0$
2. Renormalization of the lattice theory to RI (or $\overline{\mathrm{MS}}$)

Lattice Setup

Ensembles $N_{\rm f} = 2 + 1$ Shamir Domain-Wall fermions

$a^{-1} \approx 1.78$ GeV ≈ 0.11 fm — $a^{-1} \approx 2.38$ GeV ≈ 0.08 fm

$L \approx 1.8$ fm and 2.6 fm — $L \approx 2.6$ fm

Bare operators with external p between 0.2 and 1.0 GeV

RI schemes with external p between 1.4 and 2.4 GeV

Artificially small $m_{\rm W} \in [1.4\,, 2.4]$ GeV $\rightarrow 0.6 < am_{\rm W} < 1.2$

Momentum sources and Twisted BC

Can we define a method to safely extract Wilson Coefficients?

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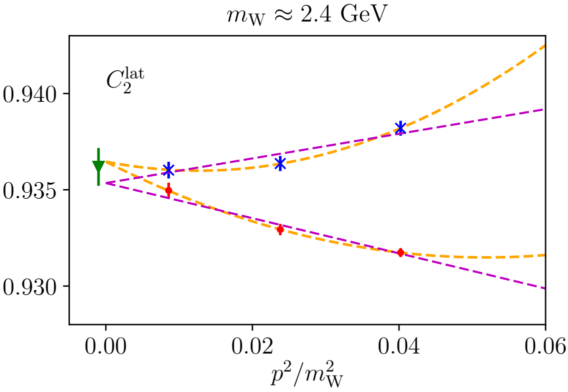
p^2 DEPENDENCE

Different external quark states $\rightarrow$ different p^2 behaviors

C_2^{lat} = unique answer in $p^2 \rightarrow 0$ limit



Except. vs Non-except. kinematics

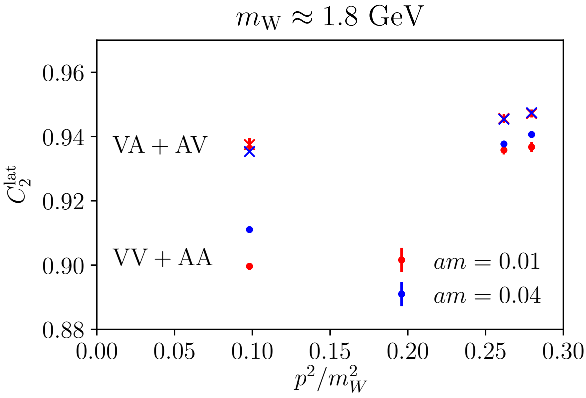
$C_2 = 1 + O(\alpha_s)$

Spin part of Projectors: $VA + AV$

Quark mass dependence - I

Projectors: (parity even and odd) $VV + AA$ and $VA + AV$

suppression of quark mass effects with parity odd Projectors



Except. kinematics

Fourier modes

$C_2 = 1 + O(\alpha_s)$

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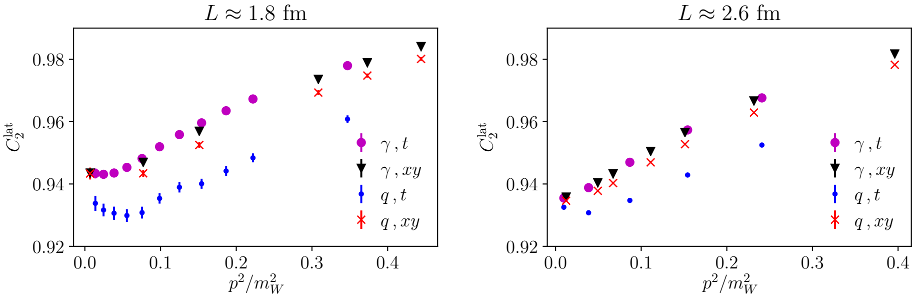
Finite volume effects

Momentum injected along time (t) or spatial (xy) directions

time extent is $2\times$ spatial extent

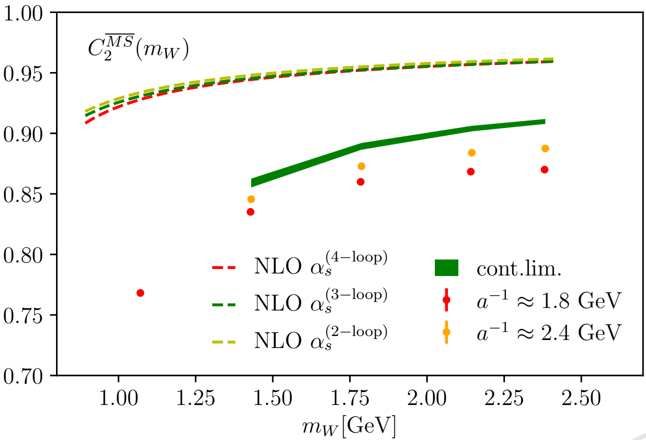
Projectors $VA + AV$: γ and $\not{q}$ schemes [RBC/UKQCD '10]



Breaking of universality at $p^2 = 0$
is a finite volume effect

PRELIMINARY RESULTS - C_2

Warning: $\mathrm{RI} \to \overline{\mathrm{MS}}$ missing (small for $\mathrm{RI} - \mathrm{SMOM}\text{-}\gamma$)

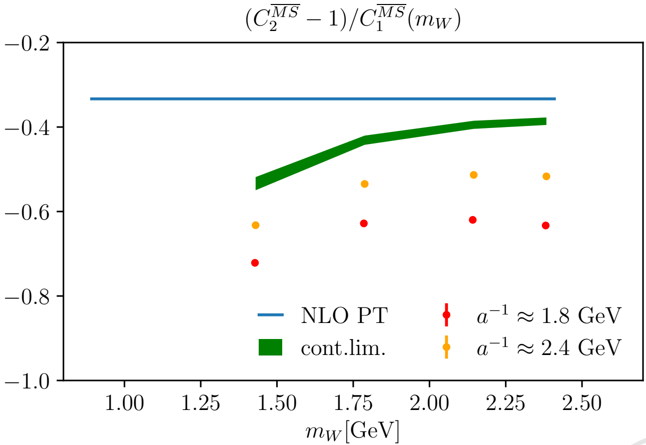


PT questionable at 2 GeV

From these results $\to$ possibility to bound PT error

Preliminary results - $(C_2 - 1)/C_1$

Warning: $\mathrm{RI} \to \overline{\mathrm{MS}}$ missing (small for $\mathrm{RI} - \mathrm{SMOM}\text{-}\gamma$)



PT questionable at 2 GeV

From these results → possibility to bound PT error

Conclusions

We have developed a method to compute (weak) wilson coefficients to all-orders in α_s

controlled quark mass and finite volume errors

discretization effects removed with 2 lattice spacings

excellent statistical precision

possibility to bound perturbative error with current data

Outlook:

complete current study with $\mathrm{RI} \to \overline{\mathrm{MS}}$

extend the basis of operators

push towards higher values of m_{W}

Thanks for the attention!

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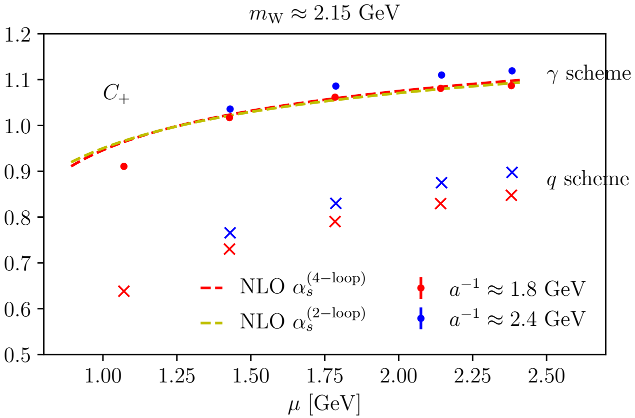
Running of C_+

We test also the running: $C_+(\mu) = Z_+(\mu, m_W) C_+(m_W)$

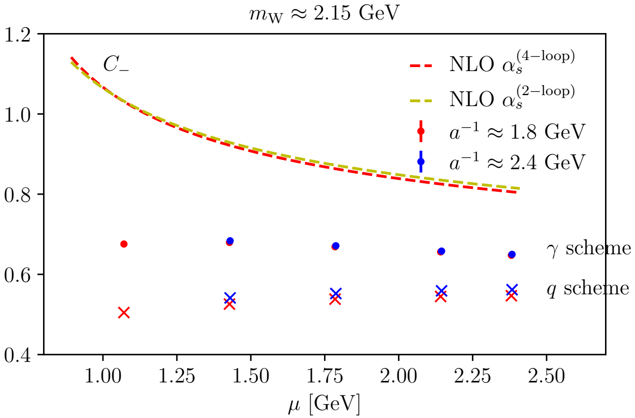


Warning:
RI $\to \overline{\text{MS}}$ missing (expected large for $\not{q}$)

Convergence of RI schemes at large μ^2

Running of C_-

We test also the running: $C_-(\mu) = Z_-(\mu, m_W) C_-(m_W)$



Warning:
RI $\rightarrow \overline{\text{MS}}$ missing (expected large for $\not{q}$)

Convergence of RI schemes at large μ^2

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Quark mass dependence - II

$$\overline{Z} \equiv [Z^{\mathrm{RI}}(\mu)]^{-1} Z_V^2$$

Implement mass-less renormalization condition

$$\lim_{m \to 0} [Z^{\mathrm{RI}}(\mu)]^{-1} Z_V^2$$

1. compute $\overline{Z}$ for $am = 0.02$ and 0.04 at one value of μ
2. Estimate first derivative w.r.t. quark mass $d\overline{Z}/dm$
3. Assumption

$$\frac{d}{d\mu}\left(\frac{d\overline{Z}}{dm}\right) = 0$$

$$\frac{d\overline{Z}_{1,1}}{dm} \approx -0.35 \quad \frac{d\overline{Z}_{1,2}}{dm} \approx -0.07$$

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