

The $K_L - K_S$ Mass Difference

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(for Ziyuan Bai)

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- $\Delta m_K = m_{K_L} - m_{K_S} = 3.483(6) \times 10^{-12}$ MeV is tiny on the scale of Λ_{QCD} .
- This FCNC quantity is therefore an excellent one in which to search for new-physics effects.
- It is frequently said that Flavour Physics can probe scales which are unreachable in colliders.
 - Here, if we could reproduce the experimental Δm_K in the SM to 10% accuracy and if we imagine an effective new-physics $\Delta S = 2$ contribution $\frac{1}{\Lambda^2} (\bar{s} \cdots d)(\bar{s} \cdots d)$ then $\Delta \gtrsim (10^3 - 10^4)$ TeV.
- The calculation of Δm_K is one component of the RBC-UKQCD collaborations' programme of long-distance contributions in kaon physics, requiring the evaluation of matrix elements of bilocal operators of the form

$$\int d^4x \langle f | T [Q_1(x) Q_2(0)] | i \rangle.$$

This includes rare-kaon decays and ϵ_K .

X.Feng plenary, Tuesday 20/6 10.00,
A.Lawson, following talk

- As well as computing the non-perturbative long-distance contributions from scales of $O(\Lambda_{\text{QCD}})$, we aim to avoid the necessity of performing perturbation theory at the scale of m_c . For Δm_K this has proved particularly slowly convergent.

J.Brod & M.Gorbahn, arXiv:1108.2036

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- Physical value:

$$\Delta m_K = 3.483(6) \times 10^{-12} \text{ MeV}$$

1 "Long-distance contribution to the $K_L - K_S$ mass difference,"

N.H. Christ, T. Izubuchi, CTS, A. Soni and J. Yu,

Phys.Rev. D88 (2013) 014508 (arXiv:1212.5931)

Development of techniques and exploratory calculation on a $16^3 \times 32$ lattice with unphysical masses ($m_\pi = 421$ MeV) including only connected diagrams \dots but results were encouragingly in the right ball park.

2 " $K_L - K_S$ mass difference from Lattice QCD,"

Z. Bai, N.H. Christ, T.Izubuchi, CTS, A.Soni and J. Yu,

Phys.Rev.Lett. 113 (2014) 112003 (arXiv:1406.0916)

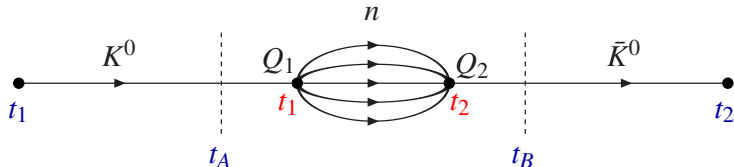
All diagrams included on a $24^3 \times 64$ lattice with $a^{-1} = 1.729(28)$ GeV,

$m_\pi = 330$ MeV, $m_K = 575$ MeV, $m_c^{\overline{\text{MS}}}(2 \text{ GeV}) = 949$ MeV

$$\Rightarrow \Delta m_K = 3.19(41)(96) \text{ MeV.}$$

- Here I present an update of the computations and results at physical masses!
- Thanks to all my colleagues from RBC-UKQCD for helpful discussions and in particular to Ziyuan Bai for the analysis presented here.

Z.Bai, Ph.D. thesis (to be published)



- Δm_K is given by

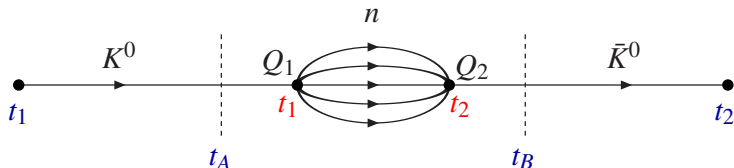
$$\Delta m_K \equiv m_{K_L} - m_{K_S} = 2\mathcal{P} \sum_{\alpha} \frac{\langle \bar{K}^0 | \mathcal{H}_W | \alpha \rangle \langle \alpha | \mathcal{H}_W | K^0 \rangle}{m_K - E_{\alpha}} = 3.483(6) \times 10^{-12} \text{ MeV}.$$

- The above correlation function gives ($T = t_B - t_A + 1$)

$$C_4(t_A, t_B; t_i, t_f) = |Z_K|^2 e^{-m_K(t_f - t_i)} \sum_n \frac{\langle \bar{K}^0 | \mathcal{H}_W | n \rangle \langle n | \mathcal{H}_W | K^0 \rangle}{(m_K - E_n)^2} \times \left\{ e^{(M_K - E_n)T} - (m_K - E_n)T - 1 \right\}.$$

- From the coefficient of T we can therefore obtain

$$\Delta m_K^{\text{FV}} \equiv 2 \sum_n \frac{\langle \bar{K}^0 | \mathcal{H}_W | n \rangle \langle n | \mathcal{H}_W | K^0 \rangle}{(m_K - E_n)}.$$



$$C_4(t_A, t_B; t_i, t_f) = |Z_K|^2 e^{-m_K(t_f - t_i)} \sum_n \frac{\langle \bar{K}^0 | \mathcal{H}_W | n \rangle \langle n | \mathcal{H}_W | K^0 \rangle}{(m_K - E_n)^2} \times \left\{ e^{(M_K - E_n)T} - (m_K - E_n)T - 1 \right\}.$$

- The presence of terms which grow exponentially in T is a generic feature of calculations of matrix elements of bilocal operators.
 - The freedom of adding $c_S \bar{s}d$ or $c_P \bar{s}\gamma^5 d$ can be used to remove two of the contributions.
- Finite-volume corrections: the extension of the Lüscher & Lellouch-Lüscher formalism to relate Δm_K^{FV} to Δm_K has been derived (see below).

N.H. Christ, X. Feng, G. Martinelli and C.T. Sachrajda, arXiv:1504.01170

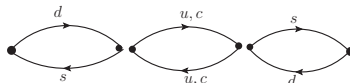
- The $\Delta S = 1$ effective Weak Hamiltonian takes the form:

$$H_W = \frac{G_F}{\sqrt{2}} \sum_{q,q'=u,c} V_{qd} V_{q's}^* (C_1 Q_1^{qq'} + C_2 Q_2^{qq'})$$

where the $\{Q_i^{qq'}\}_{i=1,2}$ are current-current operators, defined as:

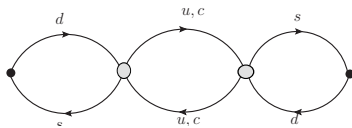
$$\begin{aligned} Q_1^{qq'} &= (\bar{s}_i \gamma^\mu (1 - \gamma^5) d_i) (\bar{q}_j \gamma^\mu (1 - \gamma^5) q'_j) \\ Q_2^{qq'} &= (\bar{s}_i \gamma^\mu (1 - \gamma^5) d_j) (\bar{q}_j \gamma^\mu (1 - \gamma^5) q'_i). \end{aligned}$$

- As the two H_W approach each other, we have the potential of new ultraviolet divergences.
 - Taking the u -quark component of the operators \Rightarrow a quadratic divergence.

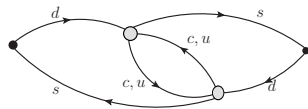


- GIM mechanism & $V - A$ nature of the currents \Rightarrow elimination of both quadratic and logarithmic divergences.
- This is not the case for ϵ_K or for $K \rightarrow \pi \nu \bar{\nu}$ rare kaons decays.

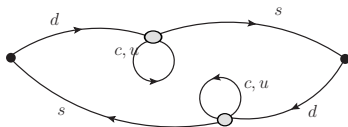
- There are four types of diagram to be evaluated:



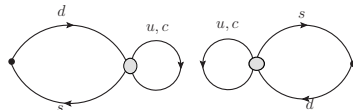
Type 1



Type 2



Type 3



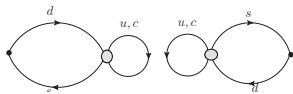
Type 4

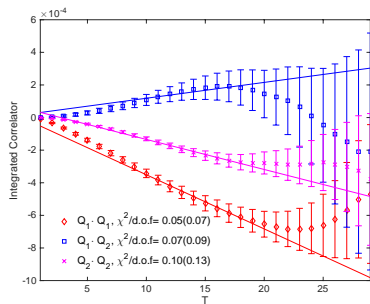
- The calculation was performed on a $64^3 \times 128 \times 12$ lattice with Möbius DWF and the Iwasaki gauge action. The inverse lattice spacing is $2.359(7)$ GeV, $m_\pi = 135.9(3)$ MeV and $m_K = 496.9(7)$ MeV.

T.Blum et al., RBC-UKQCD Collabs., arXiv:1411.7017

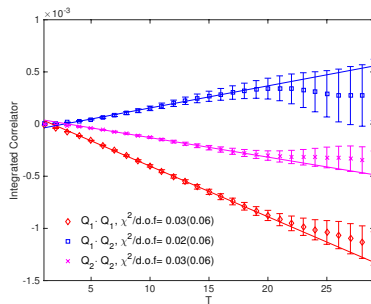
Charm-physics studies with this action $\Rightarrow am_c \simeq 0.32 - 0.33$. We have used $am_c \simeq 0.31$ and studied the dependence on m_c (see below).

- Preliminary results presented here obtained on a subset of 59 configurations for type 3 and type 4 diagrams, using AMA with “exact” propagators determined on 7 of the configurations and with “sloppy” stopping conditions for the remaining 52.
- The more precise type 1 and type 2 diagrams were calculated on 11 configurations with exact stopping conditions for the propagators.
- For the (disconnected) type-4 diagrams, which are the most noisy, we store the left- and right-hand sides separately in order to be able to vary the source-sink separations.





all diagrams



Type 1 & 2 diagrams only

- Lines here correspond to uncorrelated fits in the range $10 < T < 20$.

- Results given in units of 10^{-12} MeV.

Fit	ΔM_K	ΔM_K (tp 3&4 only)	ΔM_K (tp 1&2 only)
un-corr, 10:20	5.8(17)	-1.1(12)	7.0(13)
corr, 14:20	6.6(18)	-0.4(14)	8.5(9)
corr, 16:22	5.6(25)	-1.6(21)	7.5(15)

We follow the standard procedure:

1 NPR: $O^{\text{latt}} \rightarrow O^{\text{RI-SMOM}}$.

This is performed on a smaller $32^3 \times 64$ with the same action and lattice spacing at $p_1^2 = p_2^2 \simeq 7 \text{ GeV}^2$.

$$Z^{\text{lat} \rightarrow \text{RI-SMOM}} = \begin{pmatrix} 0.6266 & -0.0437 \\ -0.0437 & 0.6266 \end{pmatrix}$$

2 Perturbation theory: $O^{\text{RI-SMOM}} \rightarrow O^{\overline{\text{MS}}}$ at one loop.

Extension of C. Lehner and C. Sturm, arXiv:1104.4948

$$Q_i^{\overline{\text{MS}}} = (I + \Delta r)_{ij} Q_j^{\text{RI-SMOM}}$$

with

$$\Delta r = \begin{pmatrix} -2.2817 \cdot 10^{-3} & 6.8452 \cdot 10^{-3} \\ 6.8452 \cdot 10^{-3} & -2.2817 \cdot 10^{-3} \end{pmatrix}$$

3 Use the perturbatively calculated Wilson coefficients in the $\overline{\text{MS}}$ scheme.

G. Buchalla, A.J. Buras and M.E. Lautenbacher, hep-ph/9512380

- Results for ΔM_K calculated from different charm quark masses (uncorrelated fit in the range $10 \leq T \leq 20$) on 56 configurations:

m_c	0.25	0.28	0.31	0.34
ΔM_K	4.7(19)	5.1(18)	5.5(20)	5.9(21)

- Jackknife differences between ΔM_K calculated using different charm quark masses and ΔM_K from $m_c = 0.25$:

m_c	0.28	0.31	0.34
ΔM_K	0.38(66)	0.78(65)	1.23(80)

- Theoretically the FV correction is given by

N.H. Christ, X. Feng, G. Martinelli and C.T. Sachrajda, arXiv:1504.01170

$$\Delta m_K^{FV} = 2\mathcal{P} \int dE \rho_V(E) \frac{f(E)}{m_K - E} - 2 \sum_n \frac{f(E_n)}{m_K - E_n} = -2 \left(f(m_K) \cot(h) \frac{dh}{dE} \right)_{E=m_K},$$

where

$$f(m_K) = {}_V \langle \bar{K}^0 | H_W | (\pi\pi)_{E=m_K} \rangle_V \quad {}_V \langle (\pi\pi)_{E=m_K} | H_W | K_0 \rangle_V \quad \text{and} \quad h(k) = \delta(k) + \phi(k).$$

- The phase-shift $\delta(k_{m_K})$ (and its derivative) is unknown from this calculation and so we can only estimate the FV correction.
- We do this very approximately by determining the scattering length $a_{\pi\pi}$ and using the linear approximation $\delta(k_{m_K}) = k_{m_K} a_{\pi\pi}$.
- With this approximation the FV correction is found to be \ll statistical uncertainty $\Delta m_K = -0.27(18) \times 10^{-12}$ MeV.
- Further studies are needed to confirm that this is a general feature.

- We have performed the first calculation of the $K_L - K_S$ mass difference with physical quark masses.
- Our preliminary result based on an analysis of 59 configurations is

$$\Delta m_K = (5.5 \pm 1.7) \times 10^{-12} \text{ MeV},$$

to be compared to the physical value

$$(\Delta m_K)^{\text{phys}} = 3.483(6) \times 10^{-12} \text{ MeV}.$$

- We plan to finish the present calculation by performing measurements on 160 configurations, aiming to reduce the uncertainty to about 1.0×10^{-12} MeV.
- Longer term, we plan to develop a strategy which will include an improved determination of Δm_K together with other elements of our kaon physics programme.