

# HQET form factors for $B_s \rightarrow K\ell\nu$ decays beyond leading order

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# B physics without Perturbation Theory

- The goal: Calculate the form factor of the  $B_s \rightarrow K\ell\nu$  decay.
- The QCD matrix element in the rest-frame of the  $B_s$ :

$$(2m_{B_s})^{-1/2} \langle K(p_K) | V^0(0) | B_s(0) \rangle = h_{\parallel}(E_K)$$

$$(2m_{B_s})^{-1/2} \langle K(p_K) | V^k(0) | B_s(0) \rangle = p_K^k h_{\perp}(E_K)$$

- And our way to obtain it:

① Use *HQET* for the b quark.

- Order by order in 1/m<sub>h</sub> {
- ② Obtain the bare ground-state matrix elements  $\langle K | V^k | B_s \rangle$ .
  - ③ *Non-perturbatively* renormalize the currents in EFT and relate to QCD (“matching”).
  - ④ Take their *continuum limit*.
  - ⑤ Extrapolate to physical quark masses in Nature.
  - ⑥ Map out the  $q^2$  dependence.

# HQET expansion

- $\mathcal{O}(1/m_h)$  terms in the heavy-quark expansion treated as operator insertions in static correlation functions:

$$\langle O \rangle = \langle O \rangle_{\text{stat}} + \omega_{\text{kin}} \sum_x \langle O \mathcal{O}_{\text{kin}}(x) \rangle_{\text{stat}} + \omega_{\text{spin}} \sum_x \langle O \mathcal{O}_{\text{spin}}(x) \rangle_{\text{stat}}$$

$$\mathcal{O}_{\text{kin}}(x) = \bar{\psi}_h(x) \mathbf{D}^2 \psi_h(x), \quad \mathcal{O}_{\text{spin}}(x) = \bar{\psi}_h(x) \boldsymbol{\sigma} \cdot \mathbf{B} \psi_h(x)$$

- Additional terms at  $\mathcal{O}(1/m_h)$  to vector current:

$$V_0^{\text{HQET}}(x) = Z_{V_0}^{\text{HQET}} (V_0^{\text{stat}}(x) + \sum_{l=1}^2 \omega_{0,l} V_{0,l}(x)),$$

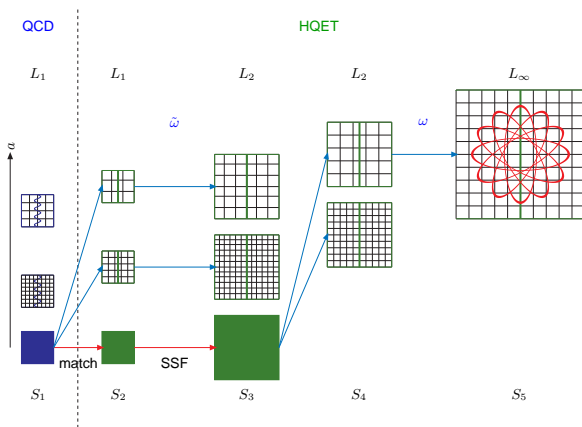
$$V_k^{\text{HQET}}(x) = Z_{V_k}^{\text{HQET}} (V_k^{\text{stat}}(x) + \sum_{l=1}^4 \omega_{k,l} V_{k,l}(x))$$

where e.g.  $V_{k,1}(x) = \bar{\psi}_\ell(x) \frac{1}{2} (\nabla_i^S - \overleftarrow{\nabla}_i^S) \gamma_i \gamma_k \psi_h(x)$ .

- For the final result: *large-volume matrix elements* and *matching coefficients* needed

# Non-perturbative matching to QCD

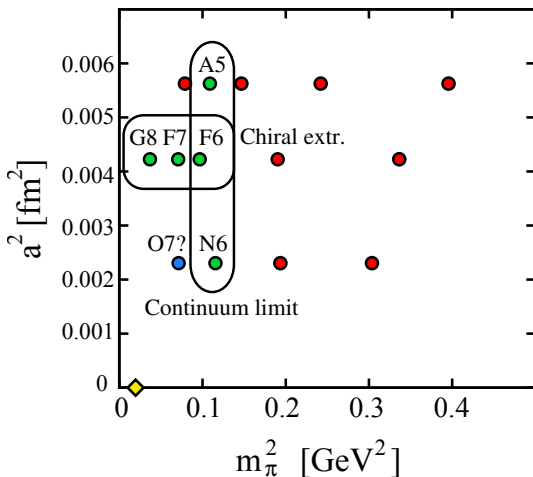
A finite-volume, recursive strategy:



Matching volume  $L_1 \approx 0.5 fm \Rightarrow$  relativistic b-quark feasible

[Della Morte et al., PoS (LATTICE2016) 199 and references therein]

# Ensembles: CLS $N_f = 2$

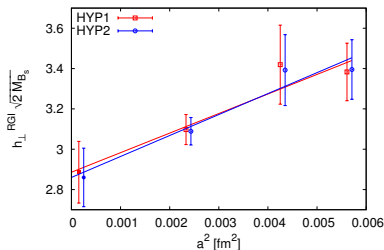
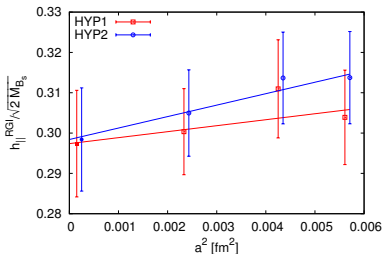


- Twisted b.c. to stay at fixed  $q^2$ ,  $p_K = (1, 0, 0) \times 0.535\text{GeV}$

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# Static results: reminder



## Continuum limit with renormalized static form factors

[Bahr, MK et al. 2016]:

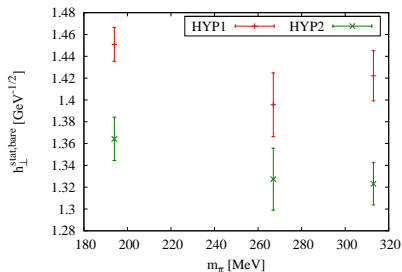
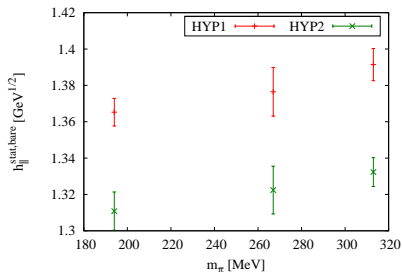
$$h_{\parallel}(E_K) = C_{V_0}(M_b/\Lambda_{\overline{MS}})h_{\parallel}^{stat,RGI}(E_K)[1 + \mathcal{O}(1/m_b)]$$

$$h_{\perp}(E_K) = C_{V_k}(M_b/\Lambda_{\overline{MS}})h_{\perp}^{stat,RGI}(E_K)[1 + \mathcal{O}(1/m_b)]$$

with  $C_x$  being perturbative conversion functions (*to be removed when full matching available*)



# Static results: Light quark mass dependence



- Chiral trajectory taken so that  $m_K/f_K = \text{Const.}$
- No signs of strong  $m_{\text{light}}$  dependence in the static case

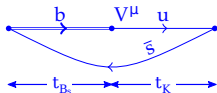
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# Ratios at order 1/m<sub>h</sub>

- For simplicity only analyze  $t_K = t_{B_s} = t$ :

$$\mathcal{R}_I^\mu(t) = \mathcal{Z} \frac{\mathcal{C}_\mu^{3\text{pt}}(t, t)}{[\mathcal{C}^K(2t)\mathcal{C}^{B_s}(2t)]^{1/2}}$$



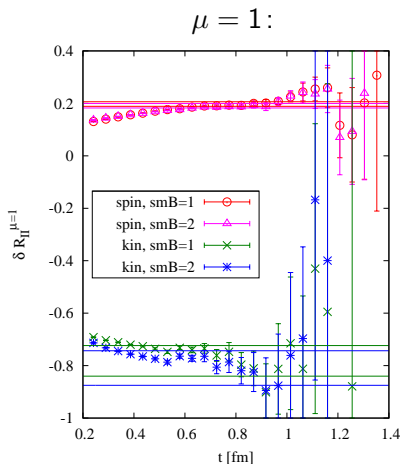
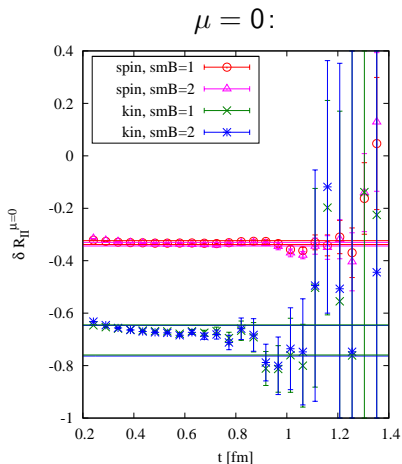
- HQET expansion:

$$\begin{aligned} \mathcal{R}_I^\mu(t) &= \mathcal{Z} \frac{\mathcal{C}_{\text{stat}}^{3\text{pt},\mu}(t, t) + \omega_\zeta \mathcal{C}_\zeta^{3\text{pt},\mu}(t, t)}{[\mathcal{C}^K(2t)[\mathcal{C}_{\text{stat}}^{B_s}(2t) + \omega_\zeta \mathcal{C}_\zeta^{B_s}(2t)]]^{1/2}} \\ &= \mathcal{Z} \mathcal{R}_I^{\text{stat},\mu}(t) \left[ 1 + \omega_\zeta \left( \frac{\mathcal{C}_\zeta^{3\text{pt},\mu}(t, t)}{\mathcal{C}_{\text{stat}}^{3\text{pt},\mu}(t, t)} - \frac{1}{2} \frac{\mathcal{C}_\zeta^{B_s}(2t)}{\mathcal{C}_{\text{stat}}^{B_s}(2t)} \right) \right] \\ &= \mathcal{Z} \mathcal{R}_I^{\text{stat},\mu}(t) [1 + \omega_\zeta \delta^S \mathcal{R}_I^\mu(t)] \end{aligned}$$

- In practice, better signal obtained with

$$\mathcal{R}_{II}^\mu(t) = \mathcal{Z} \frac{\mathcal{C}_\mu^{3\text{pt}}(t, t)}{[\mathcal{C}^K(t)\mathcal{C}^{B_s}(t)]^{1/2}} e^{(E_{B_s}^{\text{eff}} + E_K^{\text{eff}}) \frac{t}{2}}$$

# 1/m<sub>h</sub> ratios, fixed a ≈ 0.048 fm: kin and spin insertions



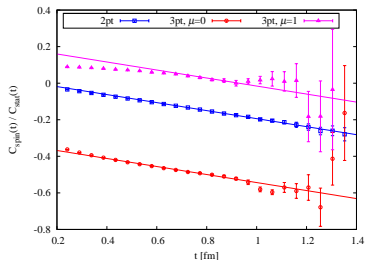
Kin and spin insertions for two B smearings. Fit bands explained on the next slide.

# Kin and spin insertions: fit procedure

- Neglecting excited states one has:

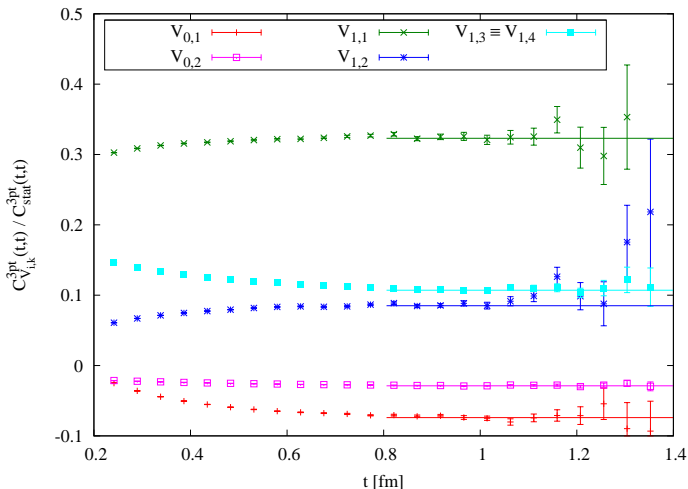
$$\frac{\mathcal{C}_\zeta^{\text{B}_s}(t)}{\mathcal{C}_{\text{stat}}^{\text{B}_s}(t)} = \rho_\zeta^{2\text{pt}} - E_\zeta t + \mathcal{O}(te^{-\Delta E t})$$

$$\frac{\mathcal{C}_\zeta^{3\text{pt},\mu}(t, t)}{\mathcal{C}_{\text{stat},j}^{3\text{pt},\mu}(t, t)} = \rho_\zeta^{3\text{pt},\mu} - E_\zeta t + \mathcal{O}(te^{-\Delta E t})$$



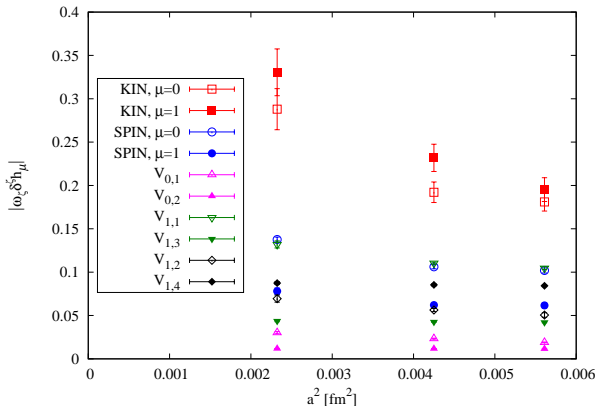
- For increased precision: parameters are obtained by fitting the equations simultaneously for both  $\mu = 0, 1$  in the range [0.8 fm, 1.4 fm].
- Final result is then given by:  $\delta^\zeta \mathcal{R}_{\text{II}}^\mu = \rho_\zeta^{3\text{pt},\mu} - \frac{1}{2} \rho_\zeta^{2\text{pt}}$

# $1/m_h$ ratios, fixed $a \approx 0.048$ fm: current insertions



Current corrections have different size but all can be determined rather precisely.

# Ratios, lattice spacing dependence



Absolute values of  $1/m_h$  ratios with tree-level matching coefficients.

Divergence in  $a$  will be removed with the non-perturbative **matching results**:

$$\log h_\mu = \log Z_V^{\text{HQET}} + \log h_\mu^{\text{stat}} + \omega_\zeta \delta^S h_\mu$$

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# Summary & Outlook

- $N_f = 2$  large-volume matrix elements now available including  $1/m_h$  terms
- Precision limited by signal to noise problem
- Most challenging:  $\mathcal{O}_{kin}$  insertions, contributing 2.5% error at  $a \approx 0.05$  fm

## TODO:

- Finish matching
- Then: continuum limit + physical mass extrapolation
- Outlook on future improvements:
  - Better smearing [Bali et al., 2016]
  - Multilevel [Ce, Giusti, Schaefer 2016]

*Thank you for your attention!*