Topological Susceptibility under Gradient Flow

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Topological sectors on the lattice

In some models, configurations fall into equivalence classes, characterized by a topological charge $Q \in \mathbb{Z}$

Strictly speaking: no topological sectors on the lattice. However, configurations appear in sectors with local minima of the action

Monte Carlo simulations with small-step updates tend to get stuck in a single sector for a long CPU-time. More problematic for small lattice spacing

Models considered

- 2-flavor QCD with twisted mass fermions
- 2d O(3) model, with standard action

$$S[\vec{e}] = \beta \sum_{\langle xy \rangle} (1 - \vec{e}_x \cdot \vec{e}_y), \quad |\vec{e}\,| = 1$$

  on a square volume $V = L \times L$

- 1d XY model

For the O(N) models we use the geometric definition of the top. charge [Berg/Lüscher '81]. Parity symmetry implies

$$\chi_t = \frac{\langle Q^2 \rangle - \langle Q \rangle^2}{V} = \frac{\langle Q^2 \rangle}{V}$$

The Slab Method [Bietenholz/de Forcrand/Gerber '15]

The simplest variant assumes Gaussian distribution for the topological charge,

$$p(Q) \propto \exp\left(-\frac{Q^2}{2\chi_t V}\right)$$

Split the volume into two slabs:

- $xV \rightarrow$ top. charge q $(0 \leq x \leq 1)$
- $(1-x)V \rightarrow$ top. charge $Q - q$

The Slab Method

If x, V and Q are fixed,

$$p_1(q)p_2(Q-q)\big|_Q \propto \exp\left(-\frac{1}{2\chi_t V}\frac{q'^2}{x(1-x)}\right), \quad q' := q - xQ$$

Where $q, q' \in \mathbb{R}$

Since $\langle q'^2\rangle = \langle q^2\rangle - x^2Q^2$, measuring $\langle q^2\rangle$, $\langle q'^2\rangle$ at each x, one can fit a parabola to determine χ_t

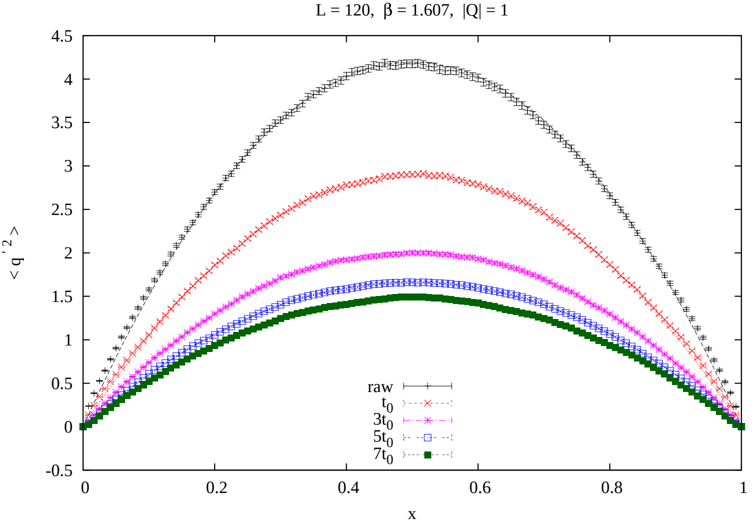
Similar application by [Aoki/Cossu/Fukaya/Hashimoto/Kaneko '17]

The Gradient Flow (GF) e.g. [Lüscher '10]

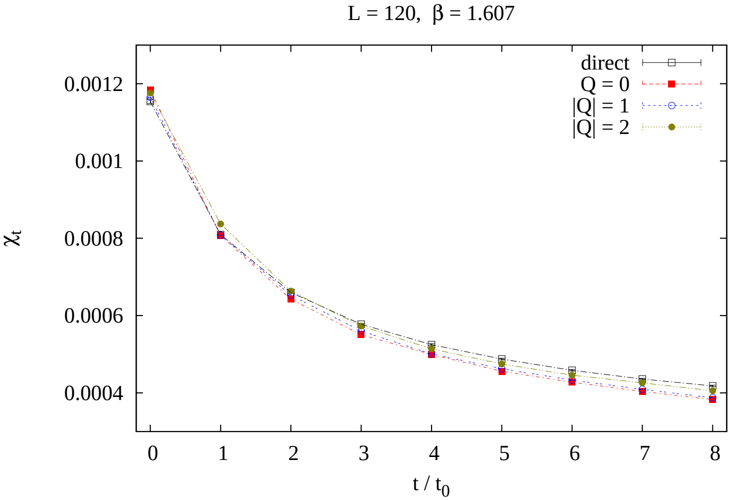
- Renormalization scheme that smoothens the fields
- As the flow-time t grows, the action tends to decrease monotonically
- Allows to understand the emergence of topological sectors in the continuum limit

Results for the 2d O(3) model

Slab Method at multiples of GF time unit $t_0 = 0.083$



χ_t^{direct} (from cluster algorithm) consistent with results from Slab Method

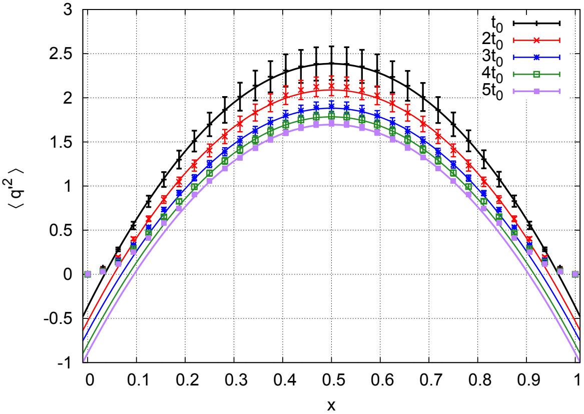


Results in 2-flavor QCD

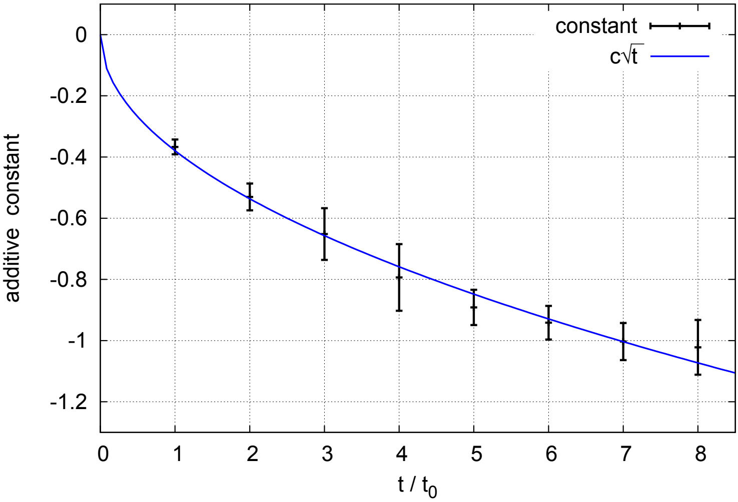
Slab Method at multiples of $t_0 = 2.42$

$V = 16^3 \times 32$, $M_\pi \simeq 650\,\mathrm{MeV}$, $a \simeq 0.079\,\mathrm{fm}$

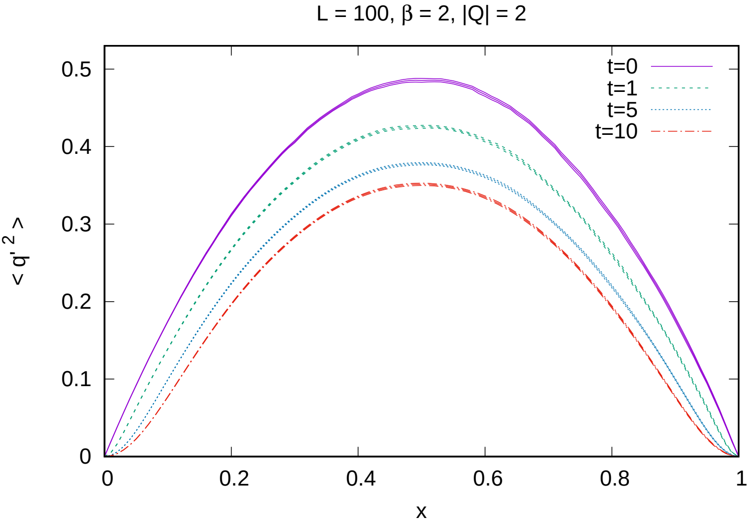
$\longrightarrow \chi_t = 7.63(14) \times 10^{-5}$ consistent with other methods



Results in 2-flavor QCD



Results for the 1d XY model



$\chi_t \xi^2$ under GF in the 2d O(3) model

The scaling quantity $\chi_t \xi^2$ is known to diverge in the continuum limit, $\xi \to \infty$

Does it mean that the model's topology is ill-defined?

Goal: determine whether $\chi_t \xi^2$ has a finite continuum limit if one considers GF-renormalized fields

We fix

$$\frac{L}{\xi} \simeq 6,$$

as in [Blatter/Burkhalter/Hasenfratz/Niedermayer '96], where the divergence persists using "classically perfect action"

Setting a flow-time scale

Defining

$$\langle E \rangle := \langle \nabla_\mu \vec{e}_x \cdot \nabla_\mu \vec{e}_x \rangle$$

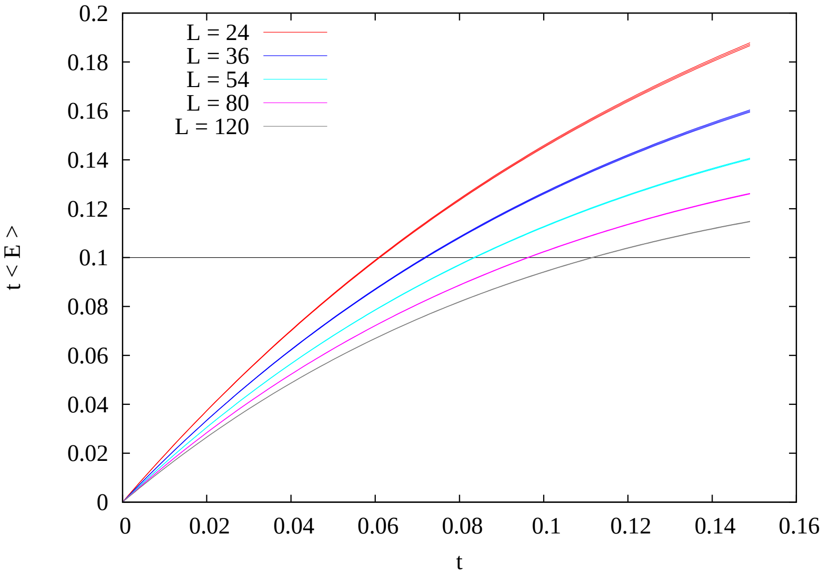
a characteristic flow-time unit t_0, can be set

$$t_0^{d/2} \langle E \rangle = const., \quad \text{here} \quad t \langle E \rangle = 0.1, \quad \text{with } [t] = \mathrm{length}^2$$

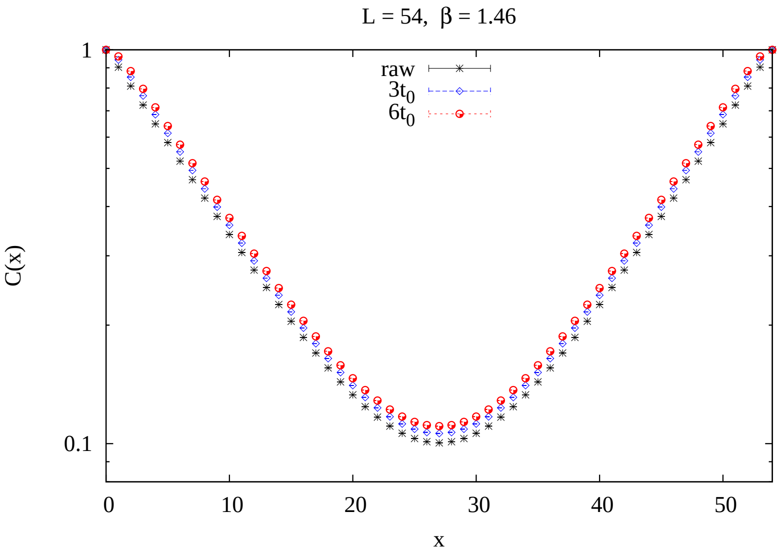
such that t/t_0 is the scale for any V and β

Next: χ_t and ξ under GF $\longrightarrow$ does $\chi_t \xi^2$ reach a finite continuum limit?

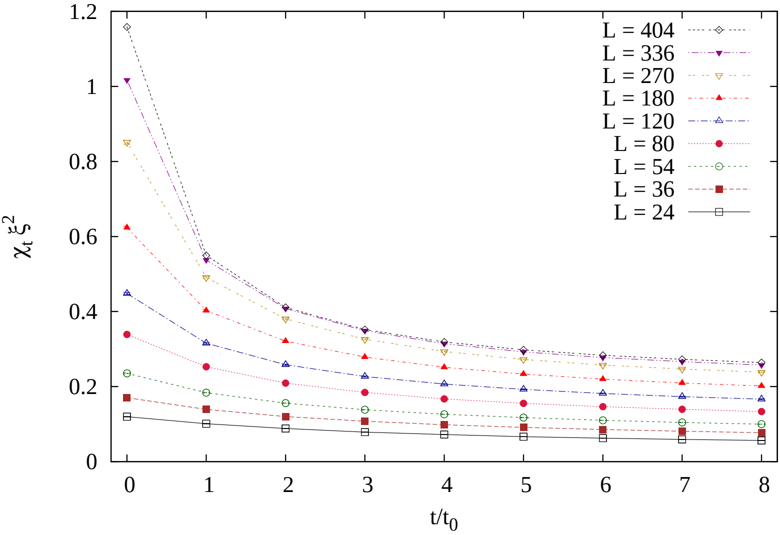
Flow-time scale t_0



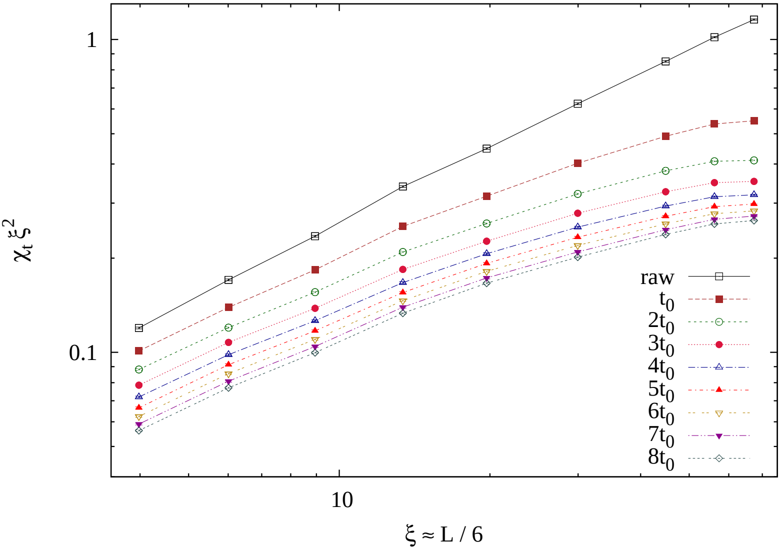
Correlation length



$\chi_t \xi^2$ under GF



Does $\chi_t \xi^2$ have a continuum limit under GF?



Summary and outlook

- The Slab Method
  - Parabolic shape remains in 2d O(3), goes flatter as flow-time grows. No subtractive constant is required.
  - 2-flavor QCD: fit requires subtractive constant, but χ_t is almost unchanged, $\chi_t a^4 = 7.63(14) \times 10^{-5} \simeq \chi_t^{direct} a^4$
  - 1d XY also needs subtractive constant, still unclear why
- Is $\chi_t \xi^2$ still divergent under GF in the 2d O(3) model?
  - Fate of the topology under GF still unclear. Further investigation going on