Glueball Spectrum from $N_f = 2$ Lattice QCD Study on Anisotropic Lattices

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Contents

Non-$q\bar{q}$ meson states

- Conventional quark model classifies $q\bar{q}$ bound states into $SU(N_f)$ flavor multiplets.
- Non-Abelian nature of QCD suggests other non-$q\bar{q}$ meson states.

- Glueball:
  bound state of gluons
- Hybrid:
  $q\bar{q}$ with excited gluon
- Tetraquark or Molecular:
  color-singlet of $qq\bar{q}\bar{q}$
- Six-quark or Baryonium:
  color-singlet of $qqq\bar{q}\bar{q}\bar{q}$

Glueball studies in Lattice QCD

Quenched approximation:

- B.Berg and A.Billoire, *Nuclear Physics* **B221** (1983):109-140
- C.Morningstar and M.Peardon, *Phys.Rev.***D56**(1997):4043-4061
- C.Morningstar and M.Peardon, *Phys.Rev.***D60**(1999)034509
- H.B.Meyer and M.J.Teper, *Phys.Lett.***B605**(2005)344-345
- Y.Chen et al, *Phys.Rev.***D73**(2006)014516
- ...

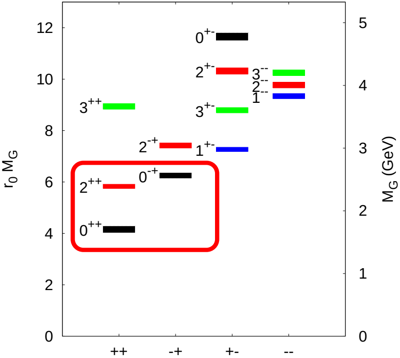
Dynamical sea quark:

- G.S.Bali et al,*Phys.Rev.***D62**(2000)054503
- A.Hart and M.Teper, *Phys.Rev.***D65**(2002)034502
- **UKQCD**, *Phys.Rev.***D82**(2010)034501
- E.Gregory et al, *JHEP* **10**(2012)170
- ...

Lowest-lying glueballs in quenched LQCD

- Lowest states with $J^{PC} = 0^{++},\ 2^{++},\ 0^{-+}$
- Masses around 1.7 GeV, 2.4 GeV and 2.6 GeV respectively.

[Y. Chen et al, *Phys. Rev.* **D 73**, 014516(2006)]

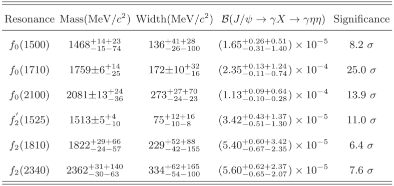


BESIII results on J/ψ radiative decay

- Radiative decay of J/ψ is a gluon rich process.
- BESIII Collaboration observed isosinglet scalar, pseudoscalar and tensor resonances in

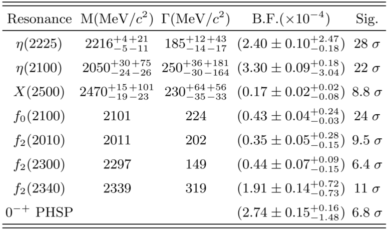
$$J/\psi \longrightarrow \gamma X \longrightarrow \gamma\eta\eta \text{ and } J/\psi \longrightarrow \gamma X \longrightarrow \gamma\phi\phi.$$

[M.Ablikim et al, *Phys.Rev.*D.87, 092009 (2013)]



Resonance	Mass(MeV/c^2)	Width(MeV/c^2)	$\mathcal{B}(J/\psi \to \gamma X \to \gamma\eta\eta)$	Significance
$f_0(1500)$	1468^{+14+23}_{-15-74}	$136^{+41+28}_{-26-100}$	$(1.65^{+0.26+0.51}_{-0.31-1.40}) \times 10^{-5}$	8.2 σ
$f_0(1710)$	$1759\pm6^{+14}_{-25}$	$172\pm10^{+32}_{-16}$	$(2.35^{+0.13+1.24}_{-0.11-0.74}) \times 10^{-4}$	25.0 σ
$f_0(2100)$	$2081\pm13^{+24}_{-36}$	273^{+27+70}_{-24-23}	$(1.13^{+0.09+0.64}_{-0.10-0.28}) \times 10^{-4}$	13.9 σ
$f_2'(1525)$	$1513\pm5^{+4}_{-10}$	75^{+12+16}_{-10-8}	$(3.42^{+0.43+1.37}_{-0.51-1.30}) \times 10^{-5}$	11.0 σ
$f_2(1810)$	1822^{+29+66}_{-24-57}	$229^{+52+88}_{-42-155}$	$(5.40^{+0.60+3.42}_{-0.67-2.35}) \times 10^{-5}$	6.4 σ
$f_2(2340)$	$2362^{+31+140}_{-30-63}$	$334^{+62+165}_{-54-100}$	$(5.60^{+0.62+2.37}_{-0.65-2.07}) \times 10^{-5}$	7.6 σ

[M.Ablikim et al, *Phys.Rev.*D.93, 112011 (2016)]



Resonance	M(MeV/c^2)	Γ(MeV/c^2)	B.F.($\times10^{-4}$)	Sig.
$\eta(2225)$	2216^{+4+21}_{-5-11}	185^{+12+43}_{-14-17}	$(2.40 \pm 0.10^{+2.47}_{-0.18})$	28 σ
$\eta(2100)$	2050^{+30+75}_{-24-26}	$250^{+36+181}_{-30-164}$	$(3.30 \pm 0.09^{+0.18}_{-3.04})$	22 σ
$X(2500)$	$2470^{+15+101}_{-19-23}$	230^{+64+56}_{-35-33}	$(0.17 \pm 0.02^{+0.02}_{-0.08})$	8.8 σ
$f_0(2100)$	2101	224	$(0.43 \pm 0.04^{+0.24}_{-0.03})$	24 σ
$f_2(2010)$	2011	202	$(0.35 \pm 0.05^{+0.28}_{-0.15})$	9.5 σ
$f_2(2300)$	2297	149	$(0.44 \pm 0.07^{+0.09}_{-0.15})$	6.4 σ
$f_2(2340)$	2339	319	$(1.91 \pm 0.14^{+0.72}_{-0.73})$	11 σ
0^{-+} PHSP			$(2.74 \pm 0.15^{+0.16}_{-1.48})$	6.8 σ

Lattice study on J/ψ radiatively decay into glueballs

- From experimental $J/\psi \longrightarrow \gamma X \longrightarrow \gamma\pi\pi$, $X \longrightarrow \pi\pi$ we can estimated

$$Br(J/\psi \longrightarrow \gamma f_0(1710)) = 2.5 \times 10^{-3}$$
$$Br(J/\psi \longrightarrow \gamma f_0(1500)) = 3.1 \times 10^{-4}$$

- Quenched LQCD predicted the J/ψ radiatively decay into scalar glueball with branching ratio :

  [L.C Gui et al, CLQCD Collaboration, *Phys.Rev.Lett.* **110** (2013) no.2, 021601]

$$Br(J/\psi \longrightarrow \gamma G_{0^{++}}) = 3.8(9) \times 10^{-3},$$

  which suggested the $f_0(1710)$ as scalar glueball candidate.

- J/ψ radiative decay into tensor glueball gives

  [Yi-Bo Yang et al, CLQCD Collaboration, *Phys. Rev. Lett.* **111**, 091601 (2013)]

$$Br(J/\psi \longrightarrow \gamma G_{2^{++}}) = 1.1(2) \times 10^{-2}$$

  relatively large branching ratio.

Gauge configuration details

- $N_f = 2$ anisotropic gauge configuration.
- Tadpole-improved gauge action and Clover-improved Wilson fermion action.
- Ground state 0^{++}, 2^{++}, 0^{-+} was investigated in our study. [W. Sun et al, CLQCD Collaboration, arXiv:1702.08174]

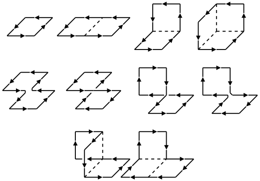
Table: Parameters of configurations

β	$L^3 \times T$	ξ	a_s	m_π	N_{conf}
2.5	$12^3 \times 128$	5	$0.114fm$	~ 650 MeV	4800
2.5	$12^3 \times 128$	5	$0.118fm$	~ 938 MeV	10400

Glueball operators construction

- Particle states denoted by J^{PC} in continuum
- $SU(2)(\text{continuum}) \xrightarrow{reduction} {}^2O(\text{lattice})$
- Glueballs are bosons $({}^2O \to O)$
- Octahedral group O has five IRs, A_1, A_2, E, T_1, T_2, denoted by R
- Subduced representation of $SU(2)$ with respect to group O is generally reducible($J \geq 2$)
- $R \not\leftrightarrow J$ $(A_1 \longrightarrow J = 0, J = 4, ...)$
- Assuming that the ground state on the lattice corresponds to the lowest spin state in continuum
- Using different spatial oriented Wilson loops to construct glueball operators with quantum number denoted by R^{PC}

J R	0	1	2	3	4	5
A_1	1	0	0	0	1	0
A_2	0	0	0	1	0	0
E	0	0	1	0	1	1
T_1	0	1	0	1	1	2
T_2	0	0	1	1	1	1



[C.J. Morningstar and M.J. Peardon, *Phys. Rev.* D 60, 034509(1999)]

Effective mass plateaus

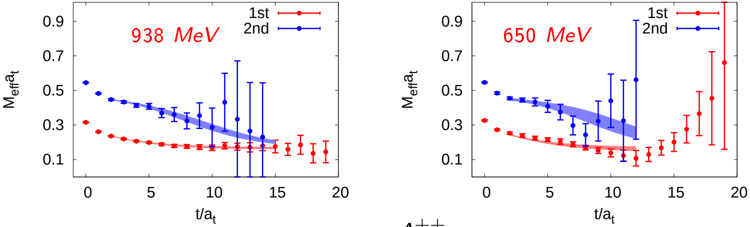
- 24 operators $\phi_\alpha^{(R^{PC})}$ for each R^{PC}
- Use the variational method to get a optimal operator $\Phi_i^{R^{PC}}$
- The optimal correlation function is

$$\tilde{C}_i^{(R^{PC})}(t) = \sum_\tau \langle 0|\Phi_i^{(R^{PC})}(t+\tau)\Phi_i^{(R^{PC})}(\tau)|0\rangle,$$

- Use two state union fit

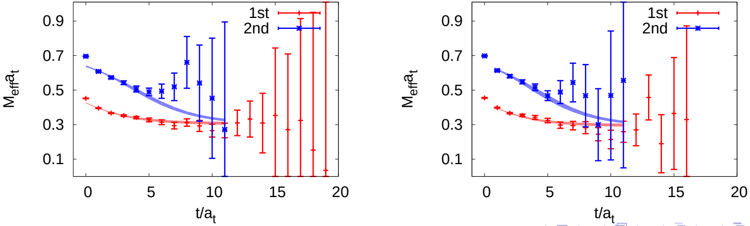
$$\begin{aligned}\tilde{C}_1^{(R^{PC})}(t) &= W_{11}^{(R^{PC})}e^{-m_1 t} + W_{12}^{(R^{PC})}e^{-m_2 t},\\ \tilde{C}_2^{(R^{PC})}(t) &= W_{21}^{(R^{PC})}e^{-m_1 t} + W_{22}^{(R^{PC})}e^{-m_2 t},\end{aligned}$$

Effective mass plateaus (A_1^{++} & A_1^{-+})

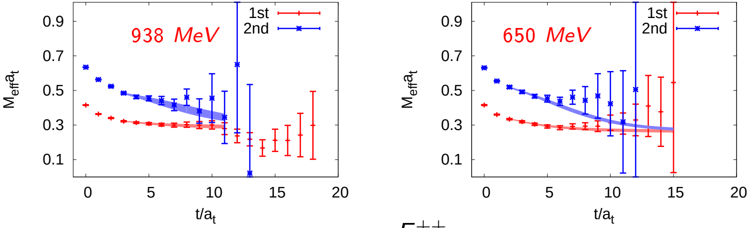


A_1^{++}

A_1^{-+}

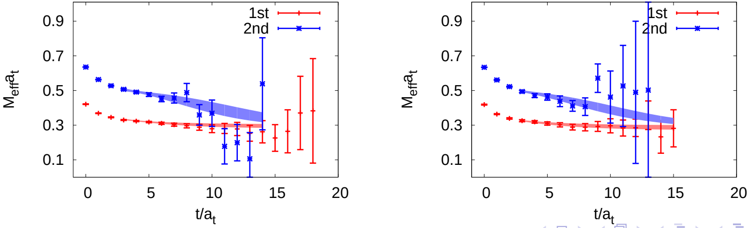


Effective mass plateaus (E^{++} & T_2^{++})

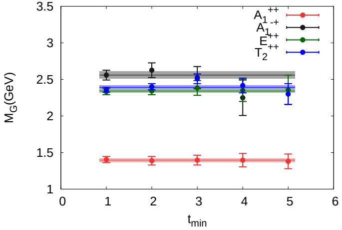


E^{++}

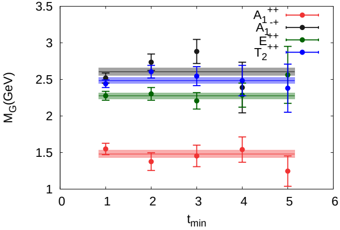
T_2^{++}



Fitted results



938 *MeV*



650 *MeV*

	m_π (MeV)	$m_{0^{++}}$ (MeV)	$m_{2^{++}}$ (MeV)	$m_{0^{-+}}$ (MeV)
$N_f = 2$	938	1397(25)	2367(35)	2559(50)
[this work]	650	1480(52)	2380(61)	2605(52)
$N_f = 2 + 1$	360	1795(60)	2620(50)	—
[E. Gregory]				
quenched	—	1710(50)(80)	2390(30)(120)	2560(35)(120)
[C. Morningstar]				
quenched	—	1730(50)(80)	2400(25)(120)	2590(40)(130)
[Y. Chen]				

Fitted results

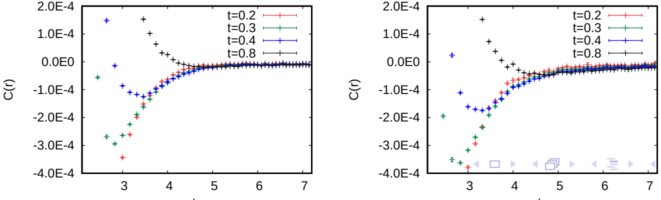
Table: Ground state meson spectrum

m_{PS}(MeV)	m_V(MeV)	m_S(MeV)	$m_{G^{0++}}$(MeV)
650(4)	993(16)	1362(53)	1480(52)
938(3)	1164(10)	1473(28)	1397(25)

- 0^{++} glueball mass is relatively lighter than previous results and close to the 0^{++} $q\bar{q}$ meson mass on our lattice (actually isovector a_0).
- Relatively large mass above 2 GeV for pseudoscalar glueball
- Physical isosinglet pseudoscalar around 1 GeV
- Further study on pseudoscalar state by topological charge density
- Heavy quark mass, sea quark effects?

η' *v.s.* pseudoscalar glueball

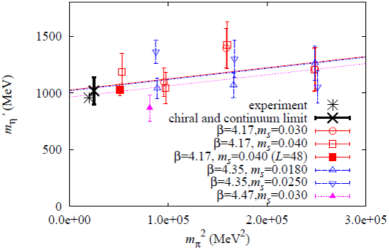
- $U(1)_A$ anomaly gives
$$\partial_\mu A^\mu(x) = 2mP(x) - \frac{N_f}{32\pi^2}\epsilon^{\mu\nu\rho\sigma} TrF_{\mu\nu}F_{\rho\sigma}$$
- topological charge density operator is defined as
$$q(x) = \frac{1}{32\pi^2}\epsilon^{\mu\nu\rho\sigma} TrF_{\mu\nu}F_{\rho\sigma}$$
- correlation of $q(x)$
$$C_q(x-y) = \langle q(x)q(y)\rangle$$



η' v.s. pseudoscalar glueball

m_π	fit range(a_s)	$m_{\eta'} a_s$	$m_{\eta'}$(MeV)	χ^2/dof
938 MeV	3.74-5.92	0.856(21)	1481(36)	1.01
650 MeV	3.87-5.48	0.514(22)	890(38)	1.43

- For quenched study, the flavor singlet pseudoscalar mass agrees with pseudoscalar glueball.
  [A. Chowdhury et al., *Phys. Rev.* **D 91** (2015)074507]
- $N_f = 2 + 1$ result agrees with physical η'.
  [H. Fukaya et al, *Phy. Rev.* **D92 (R)**, 111501 (2015)]



η' *v.s.* pseudoscalar glueball

- The continuum form of our pseudoscalar glueball operator is

$$\phi^{A_1^{-+}}(\mathbf{x}, t) \sim \epsilon_{ijk} Tr B_i(\mathbf{x}, t) D_j B_k(\mathbf{x}, t) + O(a_s^2)$$

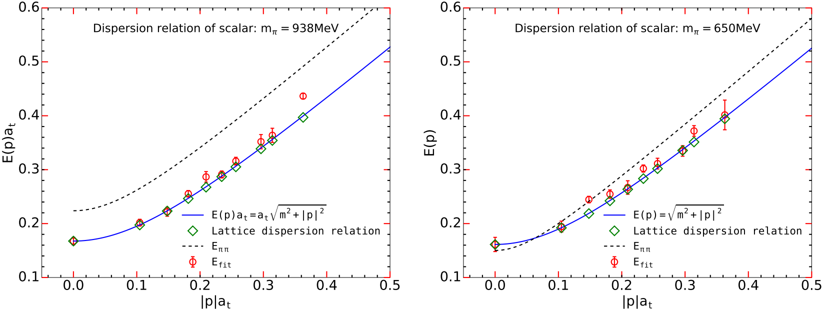
- Topological charge density operator goes like

$$q(x) \propto \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} \propto \mathbf{E}(x) \cdot \mathbf{B}(x)$$

- The large difference of our glueball and η' mass can be explained.

Preliminary results on dispersion relation

- Single particle or multi-particle state?
- Dispersion relation of one-particle and lowest free two pion state

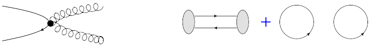


Mixing of flavor singlet $q\bar{q}$ meson and glueball in progress

- The scalar $q\bar{q}$ meson (actually isovector a_0) mass on our lattice is very close to the obtained scalar glueball mass.
- Mixing of glueball and isosinglet $q\bar{q}$ meson may need be considered.
- Disconnected quark loops can be calculated by wall source technique without gauge fixing.

(a) glueball correlator

(b) cross term



(c) cross term

(d) connected and disconnected

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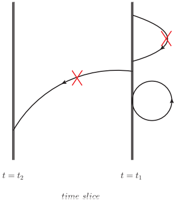

Summary

- The lowest-lying spectrum of 0^{++}, 0^{-+} and 2^{++} glueballs have been calculated in $N_f = 2$ lattice QCD with $m_\pi \sim 650$ MeV and $m_\pi \sim 938$ MeV.
- Results of 2^{++} and 0^{-+} states are consistent with quenched lattice results.
- The 0^{-+} glueball mass are much larger than ground state flavor singlet 0^{-+} meson.
- Mixing of scalar glueball and flavor singlet scalar mesons is in progress.

Thank you!

Glueball operators construction

- Operators with R^{PC} quantum number are linear combinations of Wilson loops
  - $P = \pm, \ C = +$

$$\phi_i^{R^{PC}} = \sum_{g \in O} c_R ReTr[g \circ W_i(\mathbf{x}, t) \pm \mathcal{P} g \circ W_i(\mathbf{x}, t) \mathcal{P}^{-1}]$$

  - $P = \pm, \ C = -$

$$\phi_i^{R^{PC}} = \sum_{g \in O} c_R ImTr[g \circ W_i(\mathbf{x}, t) \pm \mathcal{P} g \circ W_i(\mathbf{x}, t) \mathcal{P}^{-1}]$$

where $W_i(\mathbf{x}, t)$ denote prototype of Wilson loop, g is vector representation of group O, $\mathcal{P}$ is space inversion, c_R is combination coefficient based on group theory.

Lattice study on J/ψ radiatively decay into glueballs

For scalar glueball candidates $f_0(1500)$ and $f_0(1710)$, the experimental result of J/ψ radiative decay are as following: [C. Patrignani et al.(Particle Data Group), Chin. Phys. C, 40, 100001 (2016)]

decay channel	branching ratio
$J/\psi \longrightarrow \gamma f_0(1500) \longrightarrow \gamma\pi\pi$	$(1.09 \pm 0.24) \times 10^{-4}$
$J/\psi \longrightarrow \gamma f_0(1500) \longrightarrow \gamma\eta\eta$	$(1.7^{+0.6}_{-1.4}) \times 10^{-5}$
$J/\psi \longrightarrow \gamma f_0(1710) \longrightarrow \gamma K\bar{K}$	$(1.00^{+0.11}_{-0.09}) \times 10^{-3}$
$J/\psi \longrightarrow \gamma f_0(1710) \longrightarrow \gamma\pi\pi$	$(3.8 \pm 0.5) \times 10^{-4}$
$J/\psi \longrightarrow \gamma f_0(1710) \longrightarrow \gamma\omega\omega$	$(3.1 \pm 1.0) \times 10^{-4}$
$J/\psi \longrightarrow \gamma f_0(1710) \longrightarrow \gamma\eta\eta$	$(2.4^{+1.2}_{-0.7}) \times 10^{-4}$

Lattice study on J/ψ radiatively decay into glueballs

- Branching ratio of $J/\psi \longrightarrow \gamma X$ (X for f_0 etc.):

$$Br(J/\psi \longrightarrow \gamma X) = \frac{Br(J/\psi \longrightarrow \gamma X \longrightarrow \gamma PP)}{Br(X \longrightarrow PP)}$$

Lattice study on J/ψ radiatively decay into glueballs

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decay channel	branching ratio	$Br(J/\psi \longrightarrow \gamma f_0(1500))$
$f_0(1500) \longrightarrow \pi\pi$	$(34.9 \pm 2.3)\%$	3.1×10^{-4}
$f_0(1500) \longrightarrow \eta\eta$	$(5.1 \pm 0.9)\%$	3.3×10^{-4}
decay channel	branching ratio	$Br(J/\psi \longrightarrow \gamma f_0(1710))$
$f_0(1710) \longrightarrow K\bar{K}$	0.36 ± 0.12	2.8×10^{-3}
$f_0(1710) \longrightarrow \eta\eta$	0.22 ± 0.12	1.1×10^{-3}
$f_0(1710) \longrightarrow \pi\pi$	0.15	2.5×10^{-3}

Lattice study on J/ψ radiatively decay into glueballs

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$f_0(1710) \longrightarrow \eta\eta$	0.22 ± 0.12	1.1×10^{-3}
$f_0(1710) \longrightarrow \pi\pi$	0.15	2.5×10^{-3}

- Quenched Lattice QCD predicted the J/ψ radiatively decay into scalar glueball with branching ratio : [Long-Cheng Gui et al, CLQCD Collaboration, Phys.Rev.Lett. 110 (2013) no.2, 021601]

$$Br(J/\psi \longrightarrow \gamma G_{0^{++}}) = 3.8(9) \times 10^{-3},$$

which suggested the $f_0(1710)$ as scalar glueball candidate.

tensor glueball & $f_2(2340)$

- Flavour blindness of glueball

$$\Gamma(G \longrightarrow \pi\pi : K\bar{K} : \eta\eta : \eta\eta' : \eta'\eta') = 3 : 4 : 1 : 0 : 1$$

$$\Gamma(G \longrightarrow \eta\eta)/\Gamma(G \longrightarrow PP) \sim 10\%$$

- Pseudo-Pseudoscalar final states in tensor glueball decay in D-wave, considering centrifugal barrier effect,

$$\Gamma(G \longrightarrow M\bar{M}) = \eta\alpha\frac{k^{2L+1}}{m_G^{2L}} = \frac{\eta\alpha}{m_G}(\frac{k}{m_G})^{2L+1}$$

$$\frac{k}{m_G} = \frac{1}{2}\sqrt{1-(\frac{2m_M}{G})^2} \sim 0.5 - 0.3$$

tensor glueball & $f_2(2340)$

- Partial width of Glueball decay into pseudoscalar-pseudoscalar final states suppressed to

$$Br(G_{2^+} \longrightarrow PP) \sim O(10\%)$$

- BESIII results give

$$Br(J/\psi \longrightarrow \gamma f_2(2340) \longrightarrow \eta\eta) = 5.6(2.3) \times 10^{-5}$$

- Large branch ratio of $f_2(2340)$ in J/ψ decay

$$Br(J/\psi \longrightarrow \gamma f_2(2340)) \sim 10^{-2}$$

- Consistent with quenched LQCD