The η' meson at the physical point with $N_f = 2$ Wilson twisted mass fermions

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European Twisted Mass Collaboration

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Motivation

- $U(1)$ axial anomaly thought to be responsible for large η' mass

$$\partial^\mu J^q_{\mu 5} = 2m_q(\bar{q} i\gamma_5 q) + \frac{\alpha_s}{4\pi} F\tilde{F}$$

- tightly connected to topology
- results at physical pion mass missing so far
- how do errors scale towards the physical point?
- $N_f = 2$ technically easier: no mixing of η–η'
- test bed for e.g. $\eta' \to \gamma^\star\gamma^\star$

$\Rightarrow$ relevant for HLbL contribution to $g-2$ of the muon

Status η and η' Mesons

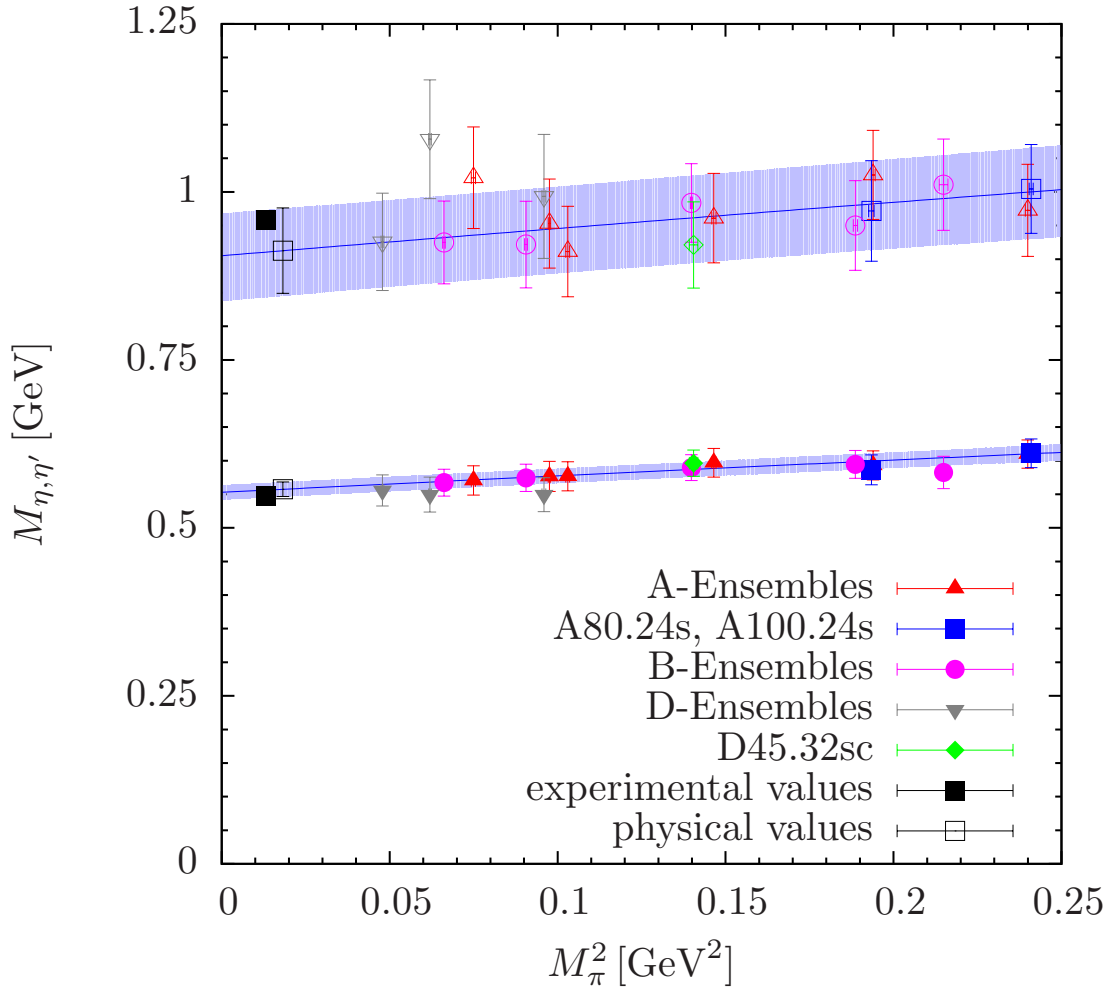
- $N_f = 2 + 1 + 1$ computation with three values of the lattice spacing

  [Michael, Ottnad, CU, PRL (2013)]

- here: updated results

  [Ottnad, Urbach, to be published]

- extrapolations including lattice artefacts and quark mass effects (ChPT reliable?)



- long extrapolation from $M_\pi \sim 250$ MeV

- decay constants available as well

  [Ottnad, Michael, CU, (LATTICE2013); Ottnad, CU, to be published]

Ensembles and Analysis

- Iwasaki gauge action at $\beta = 2.10$
- $N_f = 2$ Wilson twisted-mass clover ensembles

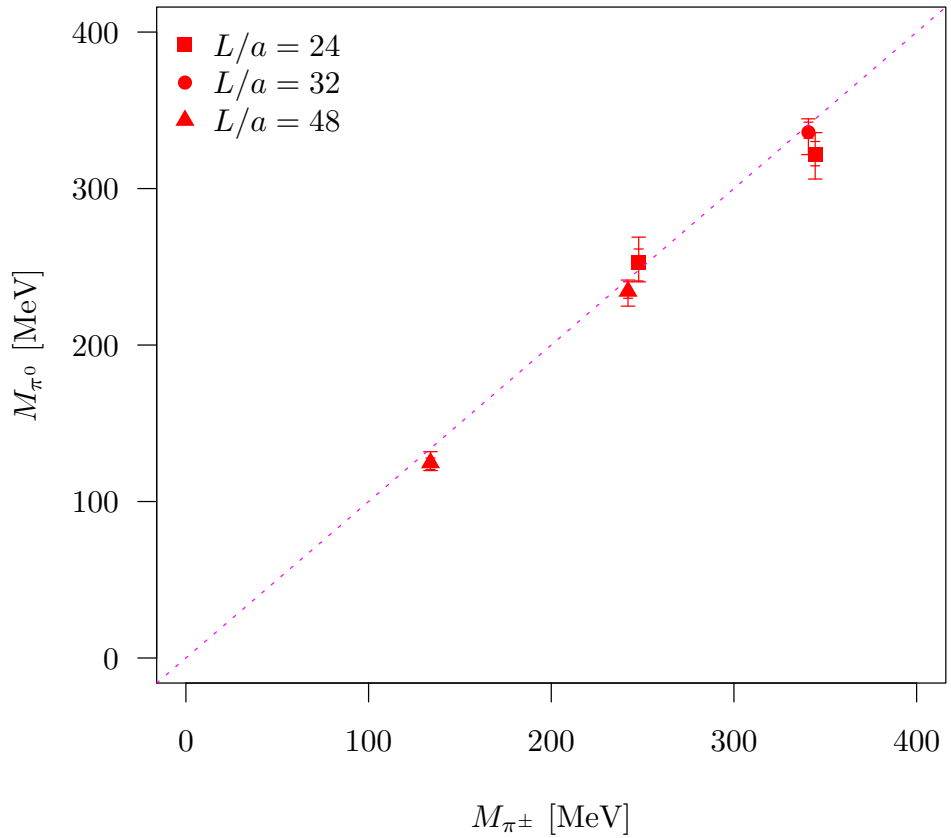
  [ETMC, LAT15 (2015), ETMC PRD (2017)]
- maximal twist guarantees $\mathcal{O}(a)$ improvement

  [Frezzotti, Rossi, (2004)]
- clover term introduced to reduce pion mass splitting
- single lattice spacing $a \sim 0.9$ fm, $r_0/a = 5.317$
- three pion mass values from 130 MeV to 350 MeV
- two volumes per pion mass value from $L/a = 24$ to $L/a = 64$
- stochastic Laplacian Heaviside for correlators

  [HSC, Peardon et al., PRD (2009), Morningstar et al., PRD (2011)]
- blocked bootstrap statistics

Pion Mass Splitting

- previously: large pion mass splitting in tmLQCD
- simulations @$M_\pi^{\rm phys}$ only at tiny lattice spacing possible



- added clover term to action
- clover coefficient set to its non-perturbative value

$$c_{\rm sw} \;=\; 1.57551$$

[CP-PACS, JLQCD, (2006)]

$\Rightarrow$ mass splitting zero within errors

- $c_{\rm sw}$–value also compatible with boosted PT

$$c_{\rm sw} \;=\; 1 + \frac{g_0^2}{2\langle P\rangle}$$

The η' in $N_f = 2$ flavour QCD

- $N_f = 2$ light pseudo scalar mesons: a pion triplet and the η_2
- pions: (pseudo) GOLDSTONE bosons
- η_2: related to the $U(1)$ axial anomaly, i.e. the η' for $N_f > 2$
- correlation function

$$C_{\eta_2}(t - t') \;=\; \langle \mathcal{O}^s(t)\,(\mathcal{O}^s(t'))^\dagger \rangle\,, \quad \mathcal{O}^s(t) \;=\; \frac{1}{\sqrt{2}} \sum_{\mathbf{x}} \bar{\psi}\, i\gamma_5\, \mathbb{1}_f\, \psi(\mathbf{x}, t)\,.$$

$\Rightarrow$ fermionic connected and disconnected contributions

- delicate cancellation and difficult statistical analysis

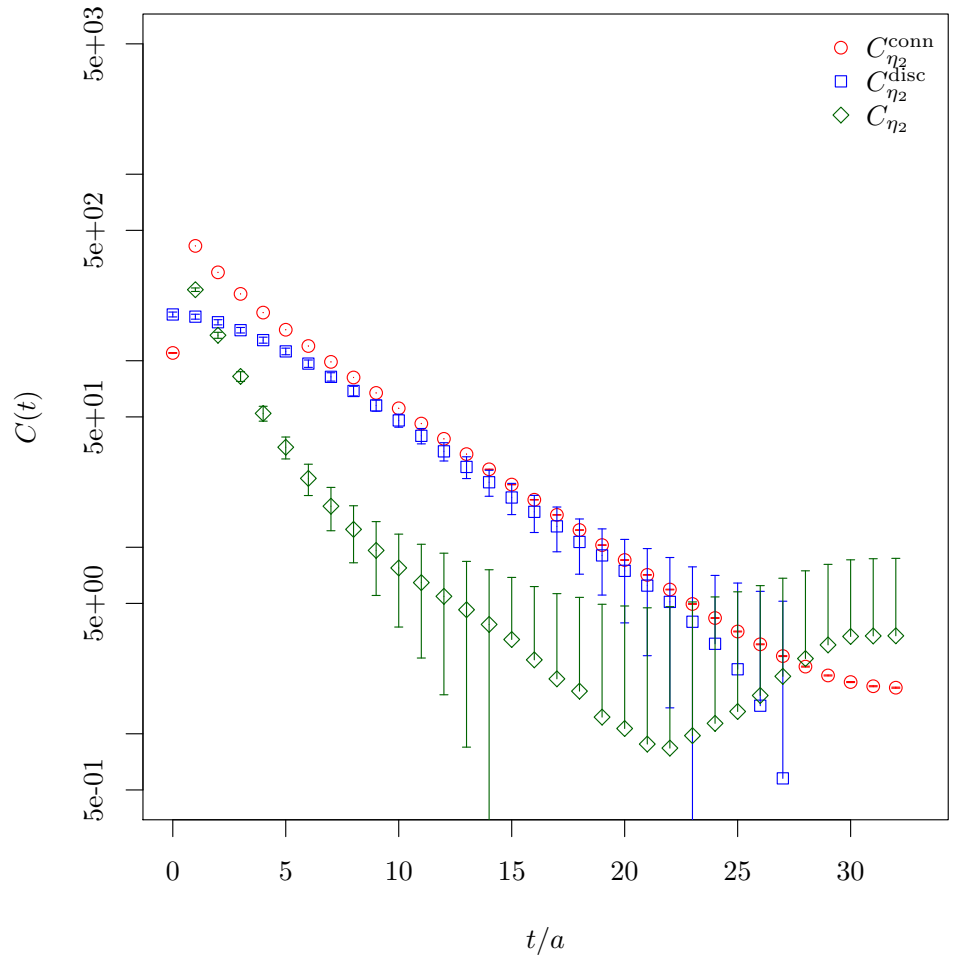
$$C_{\eta_2} \;=\; C_{\eta_2}^{\rm conn} \;-\; C_{\eta_2}^{\rm disc}$$

The η' in $N_f = 2$ flavour QCD

- delicate cancellation
- $C(t) \geq 0$ required by theory

$\Rightarrow$ $C^{\mathrm{disc}}_{\eta 2} \leq C^{\mathrm{conn}}_{\eta 2}$

- signal lost early in time
- $M_\pi \approx 340$ MeV
- $L/a = 32$

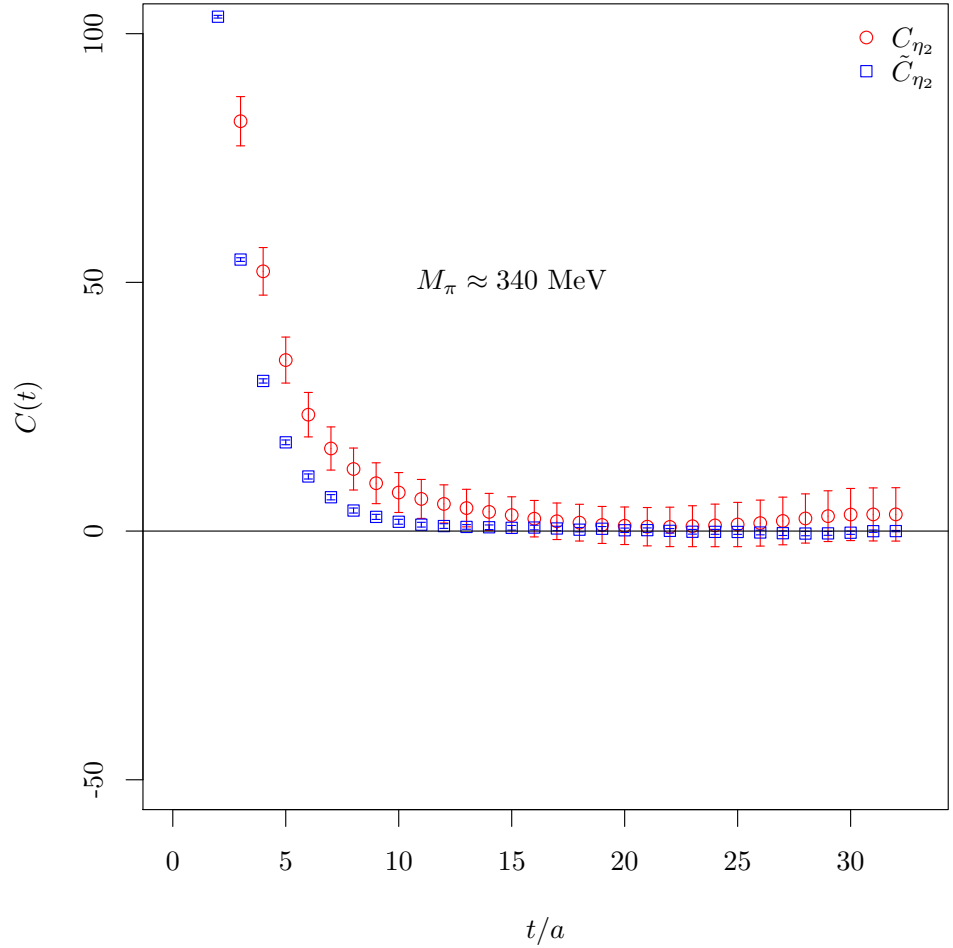


Correlation Functions

- disconnected contribution introduces large correlation
- increasing with $M_\pi \to 0$
- use instead

$$\tilde{C}_{\eta_2}(t) \;=\; C_{\eta_2}(t) - C_{\eta_2}(t+1)$$

- drastically reduces correlation



- also removes possible constant FSE [Aoki et al., PRD (2014), Bali et al., PRD (2014)]

$$\propto \frac{a^5}{T}\left(\chi_t + \frac{Q^2}{V}\right)$$

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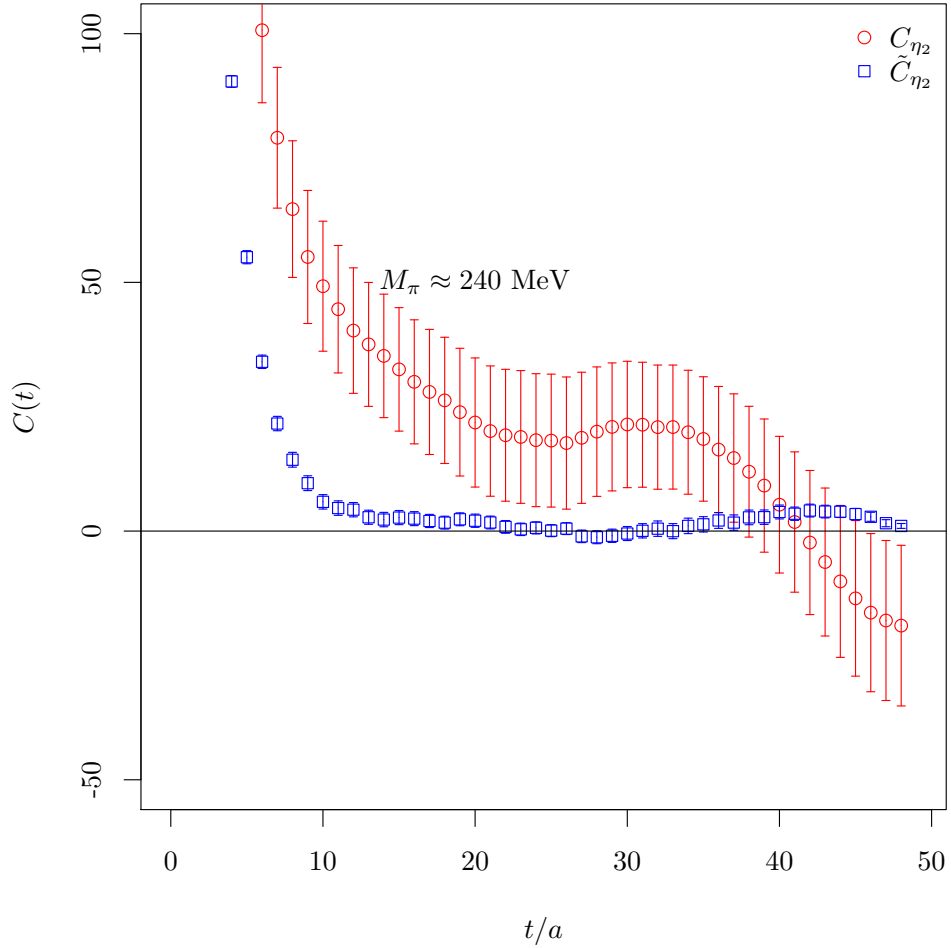

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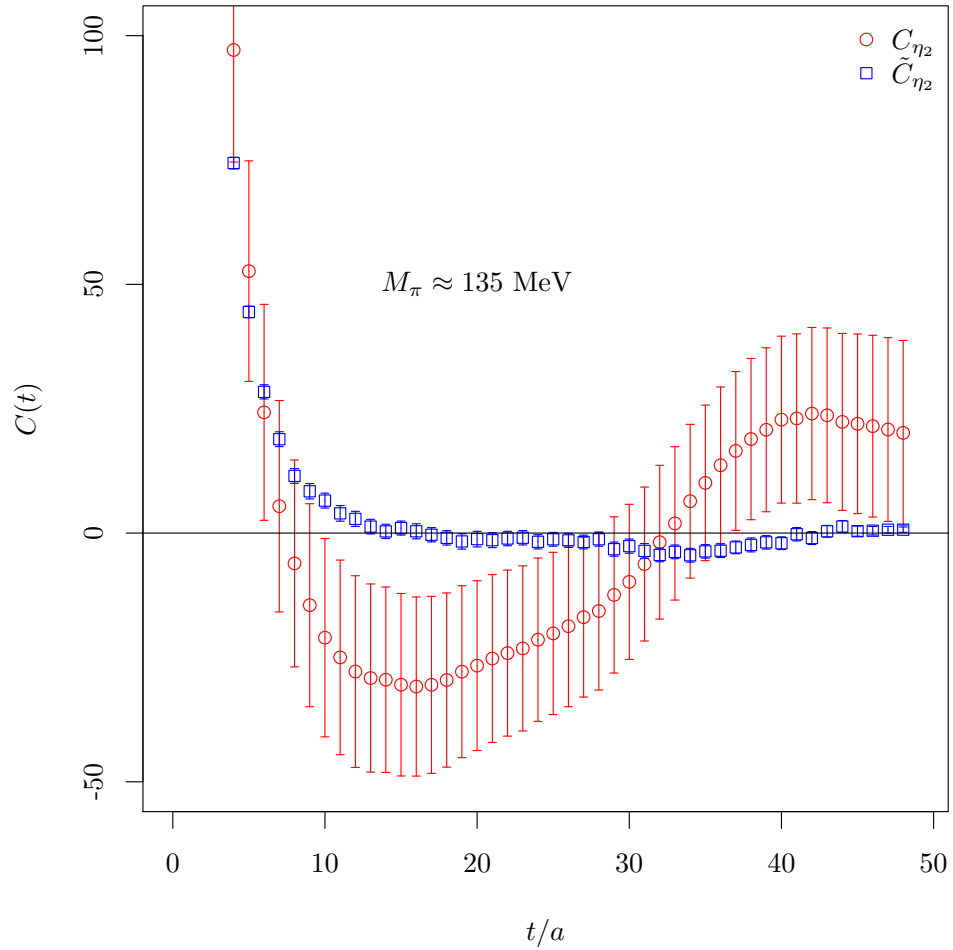

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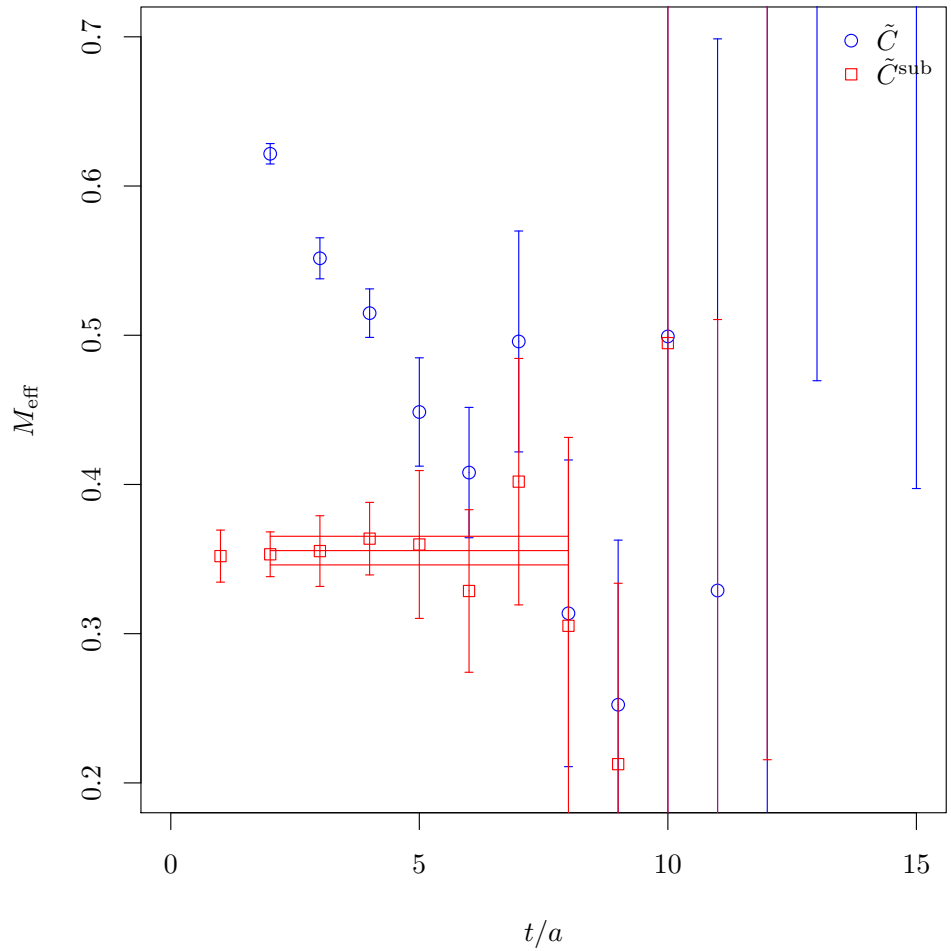
Excited State Subtraction

- signal lost early in time
- assume disconnected contribute only to ground state
- subtract excited states from connected part

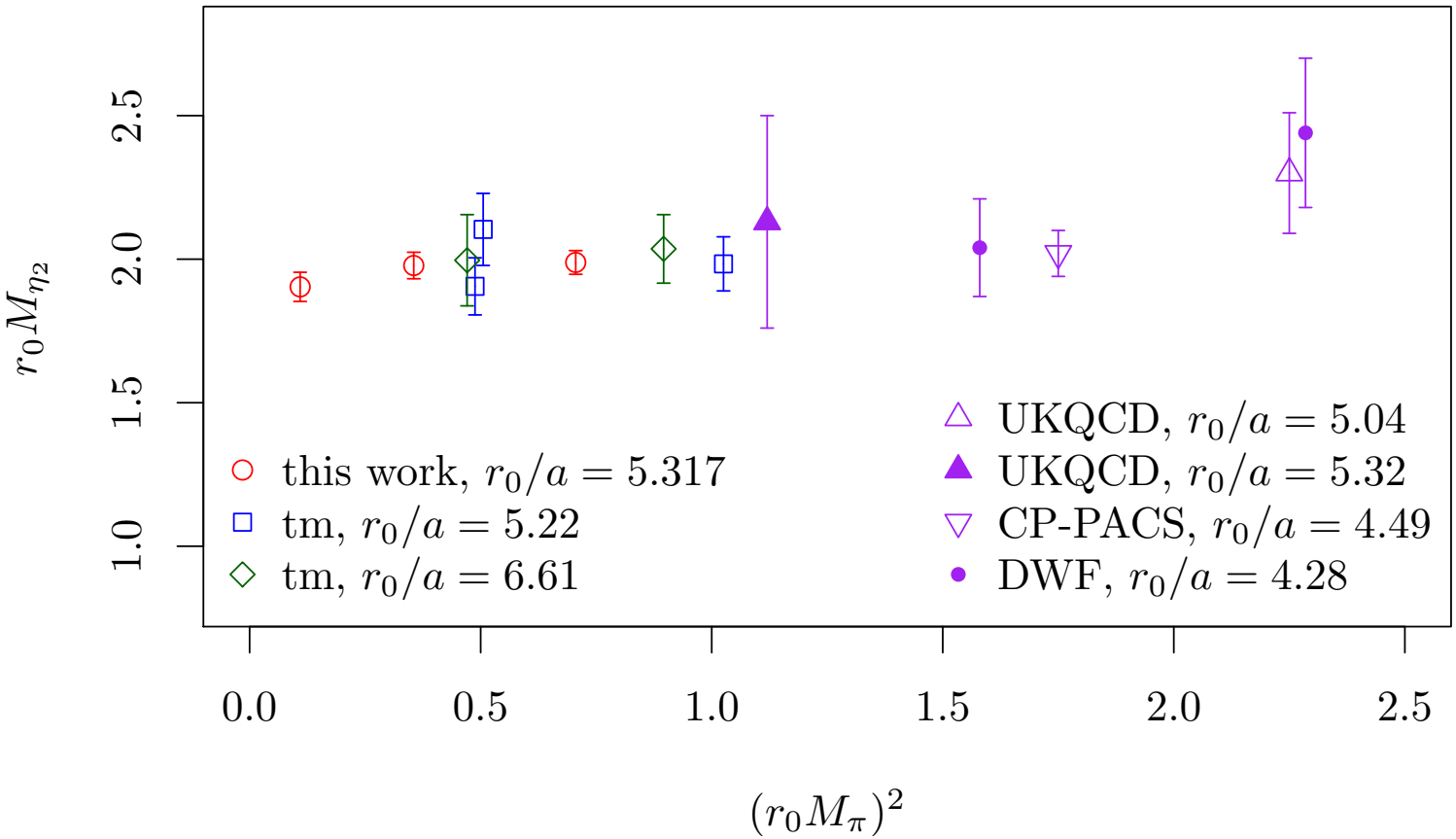
  [Neff et al. (2001), K.Jansen, C.Michael, C.U. (2008)]

- analyse subtracted connected plus disconnected correlation function $\tilde{C}^{\mathrm{sub}}$

$\Rightarrow$ plateau very early in time!



Results M_{η_2}



- with $r_0 = 0.4907(5)$ fm we arrive at

$$M_{\eta_2}^{N_f=2} \;=\; 768(24)\ \text{MeV}$$

Results Topological Susceptibility

- finite η_2 mass

$\Rightarrow$ χ_t should vanish in chiral limit

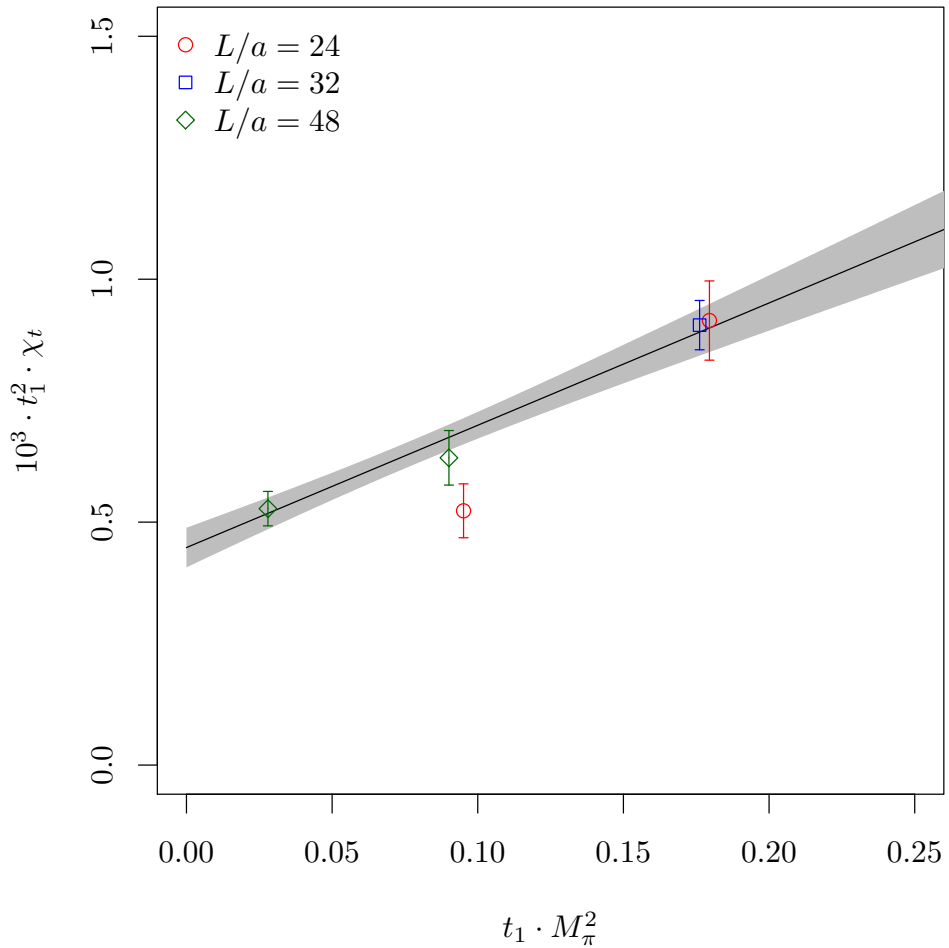
- but: lattice artefacts

$$t_1^2 \chi_t = c_2 t_1 M_\pi^2 + a^2 c_2 / t_1 + \ldots$$

[ALPHA, JHEP (2014)]

- determine χ_t from gradient flow at flowtime t_1

[Lüscher, JHEP (2010)]



$\Rightarrow$ fits with expected LO ChPT prediction

- largish lattice artefact

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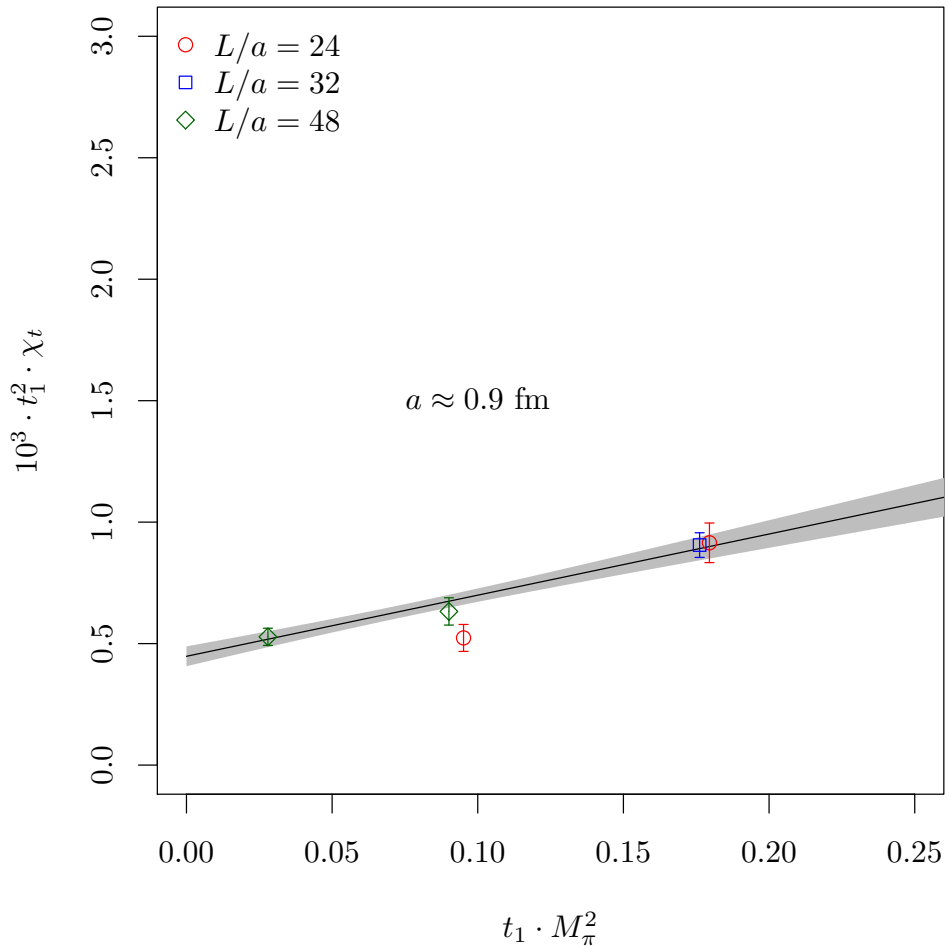

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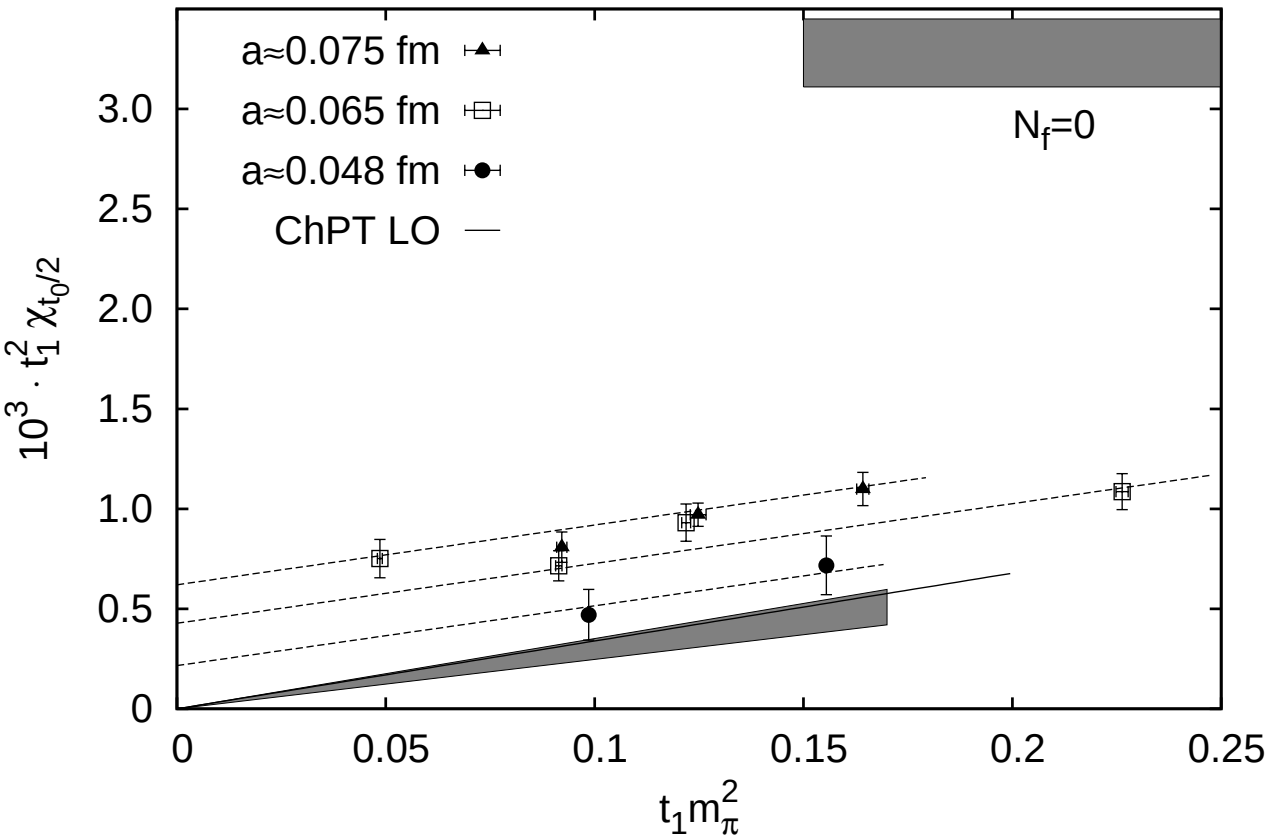
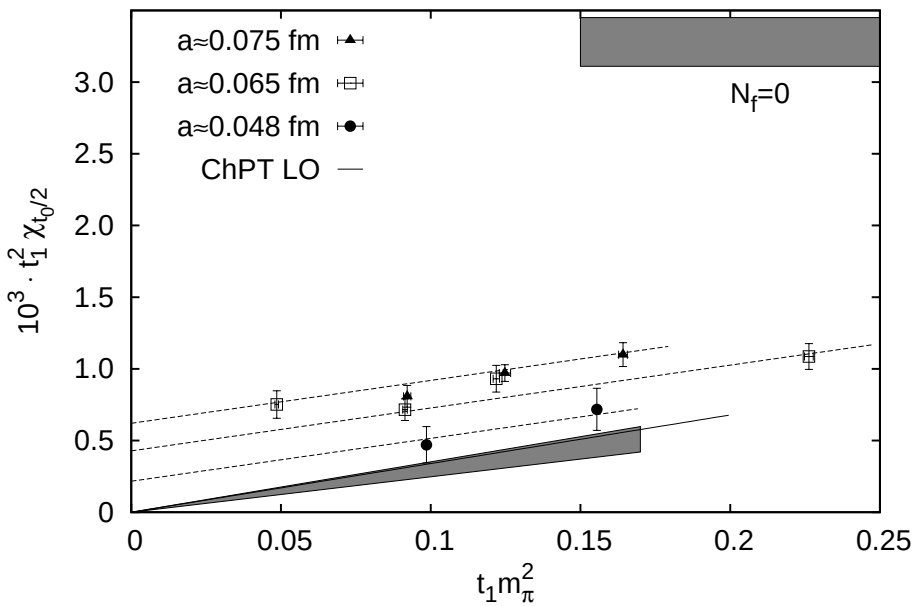
Comparison $N_f = 2$ Topological Susceptibility



- but not unseen large [ALPHA, JHEP (2014)]

Comparison $N_f = 2$ Topological Susceptibility



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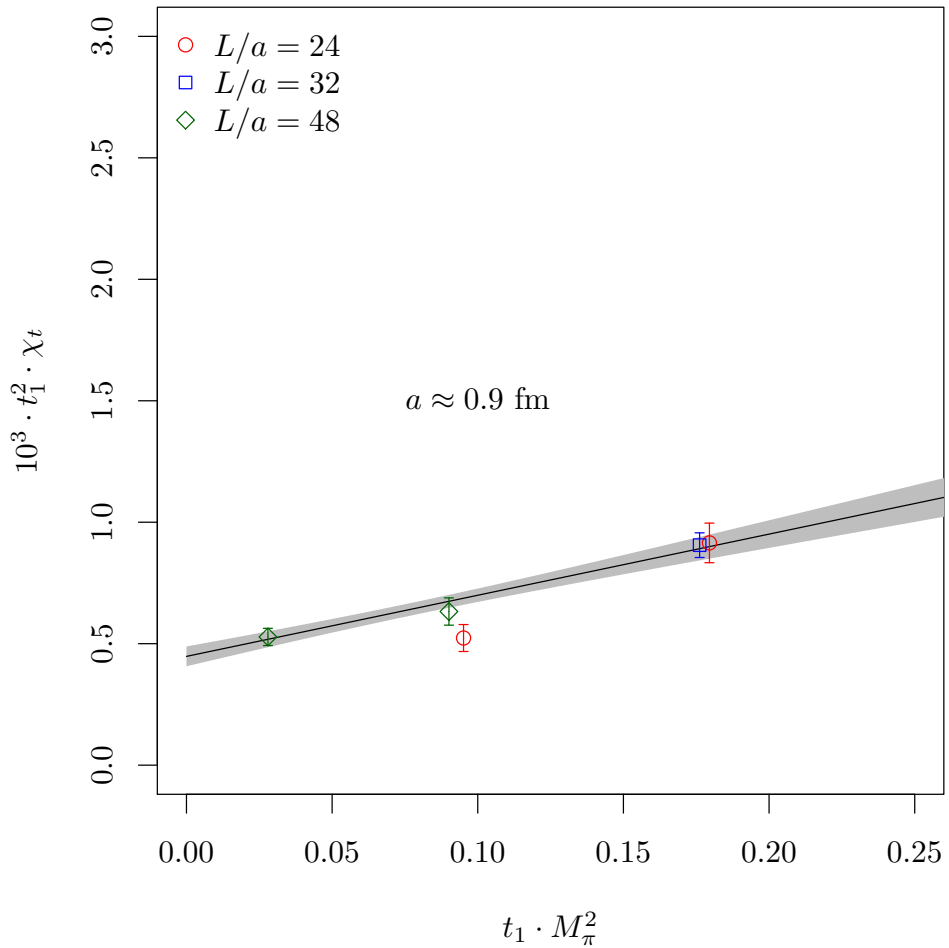

- note the lattice spacing values!

Summary

- investigated $N_f = 2$ η' meson @$M_\pi^{\rm phys}$
- previous extrapolations are confirmed
- use of shifted correlation functions important
- determined topological susceptibility @$M_\pi^{\rm phys}$
- expected pion mass scaling plus $\mathcal{O}(a^2)$

Thanks to ...

- the lattice QCD group in Bonn:
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- the ETM collaboration, in particular P. Dimopoulos, R. Frezzotti

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- ... and for your attention!