Toward pion-pion scattering amplitudes with controlled systematic errors

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Scattering amplitudes in lattice QCD

- In imaginary time, $\langle 0|T[\hat{\mathcal{O}}'(x')\hat{\mathcal{O}}^\dagger(x)]|0\rangle$ generally contains no info about on-shell amplitudes. L. Maiani, M. Testa, *Phys. Lett.* **B245** (1990) 585

- Below $n \geq 3$ hadron thresholds (identical particles),

$$E_{\rm cm} = \sqrt{E_{\rm lat}^2 - \boldsymbol{P}^2}, \qquad \boldsymbol{q}_{\rm cm}^2 = \frac{1}{4}E_{\rm cm}^2 - m^2$$

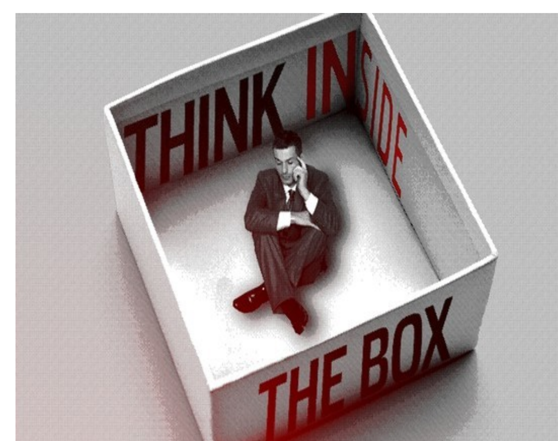
$$\det[1 - K(E_{\rm cm})B(L\mathbf{q}_{\rm cm})] = \mathrm{O}(\mathrm{e}^{-ML})$$

M. Lüscher, *Nucl. Phys.* **B354** (1991) 531

- Determinant over partial wave and channel indices

- Progress in all-to-all (distillation, stochastic LapH) => more precise $E_{\rm cm}$

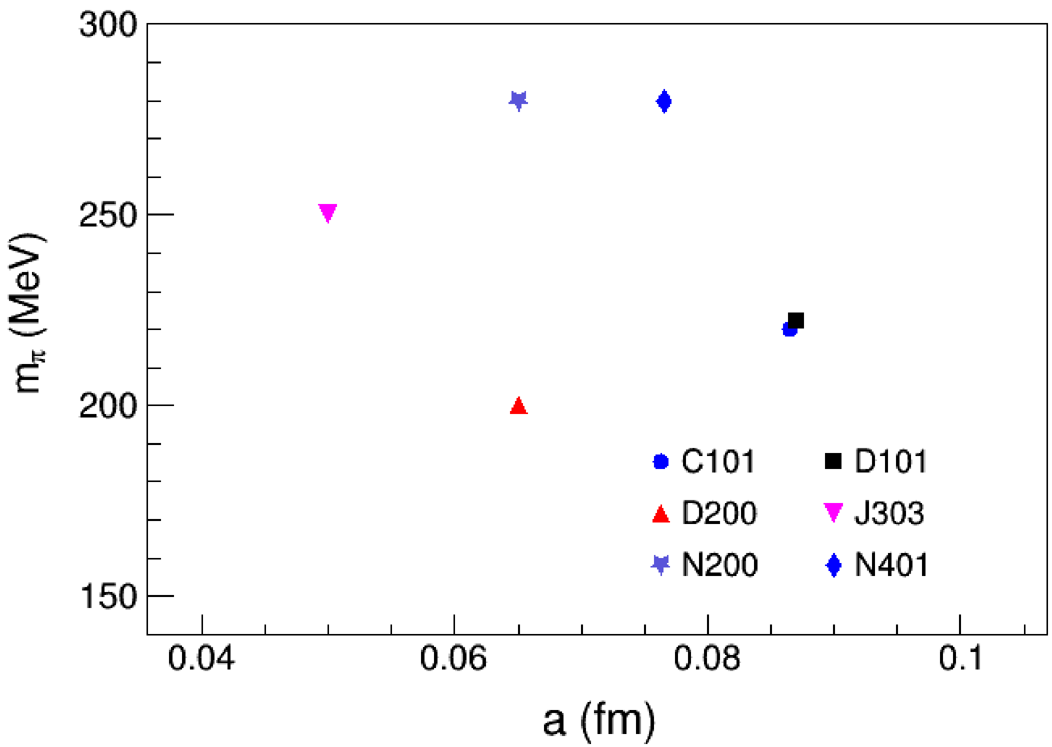
C. Morningstar, JB, J. Foley, K. Juge, D. Lenkner, M. Peardon, C. H. Wong, *Phys. Rev.* **D83** (2011) 114505, (Many talks at this conference)



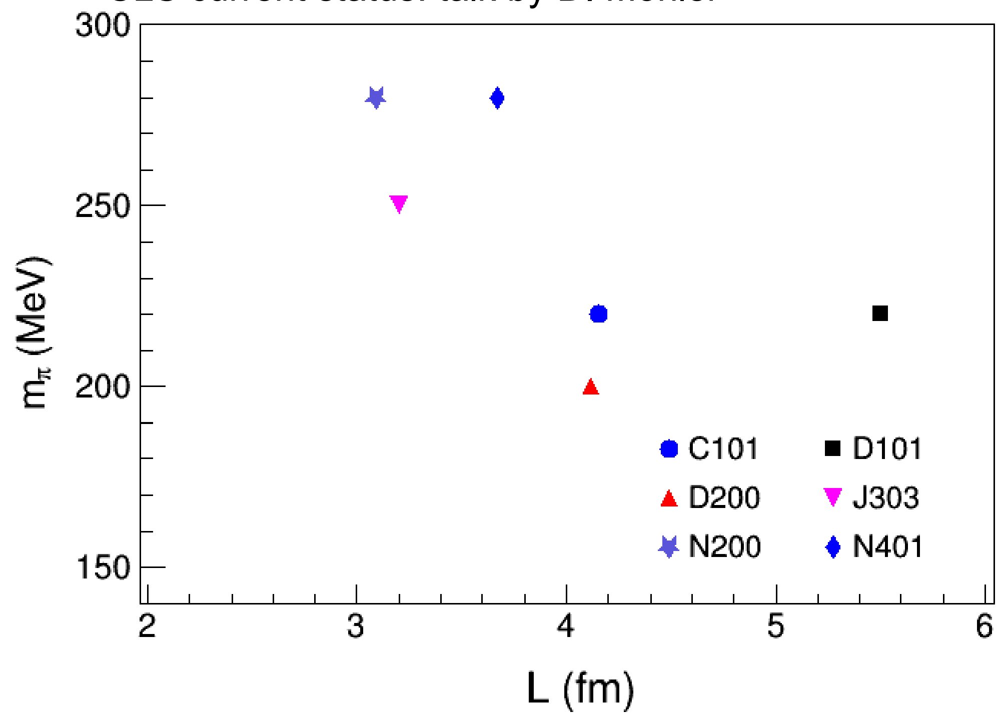
CLS ensembles

Together with : C. Andersen, B. Hoerz, C. Morningstar

- Systematic erorrs: higher partial waves, finite $a,\ m_\pi,\ L$
- 4 lattice spacings $a \geq 0.05\mathrm{fm}$, pion masses $m_\pi \gtrsim 190\mathrm{MeV}$
- Two $N_\mathrm{f} = 2+1$ chiral limits: $m_s = const.$ $\mathrm{Tr}M = const.$



CLS current status: talk by D. Mohler



Correlator Measurements

- Stochastic LapH dilution Scheme:

Performance details + volume scaling study: See B. Hoerz's talk

N_{ev}	line type	N_r	scheme	N_{t_0}	N_{inv}
192 (3.2fm)-	fixed	5	(TF, SF, LI8)	2	320
928 (5.5fm)	relative	2	(TI16, SF, LI8)	-	1024

- Elastic, isovector, *p*-wave: 3-5 two-pion operators and 1 (local) rho-like operator in each irrep.

Operator Construction: C. Morningstar, JB, B. Fahy, J. Foley, Y. C. Jhang, K. J. Juge, D. Lenkner, C. H. Wong, Phys. Rev. D88 (2013) 014511

- All possible irreps with total momenta:

$$\boldsymbol{d}^2 = 0, 1, 2, 3, 4$$

Energy determination

- Form a matrix of correlation functions:

$$C_{mn}(t) = \langle \mathcal{O}_m(t) \mathcal{O}_n^\dagger(0) \rangle$$

- Solve the GEVP once for a single $\{t_0, t_d\}$ and rotate

$$C(t_d) v_n = \lambda_n C(t_0) v_n \qquad \hat{C}_n(t) = v_n^\dagger C(t) v_n$$

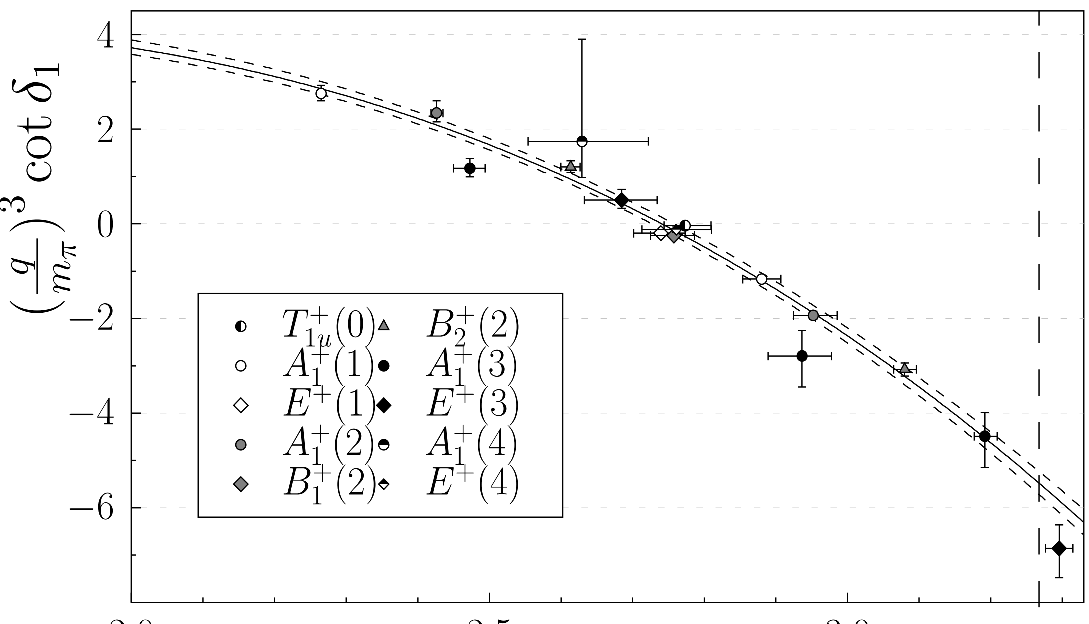
C. Micheal, I. Teasdale, *Nucl. Phys.* **B215** (1983) 433

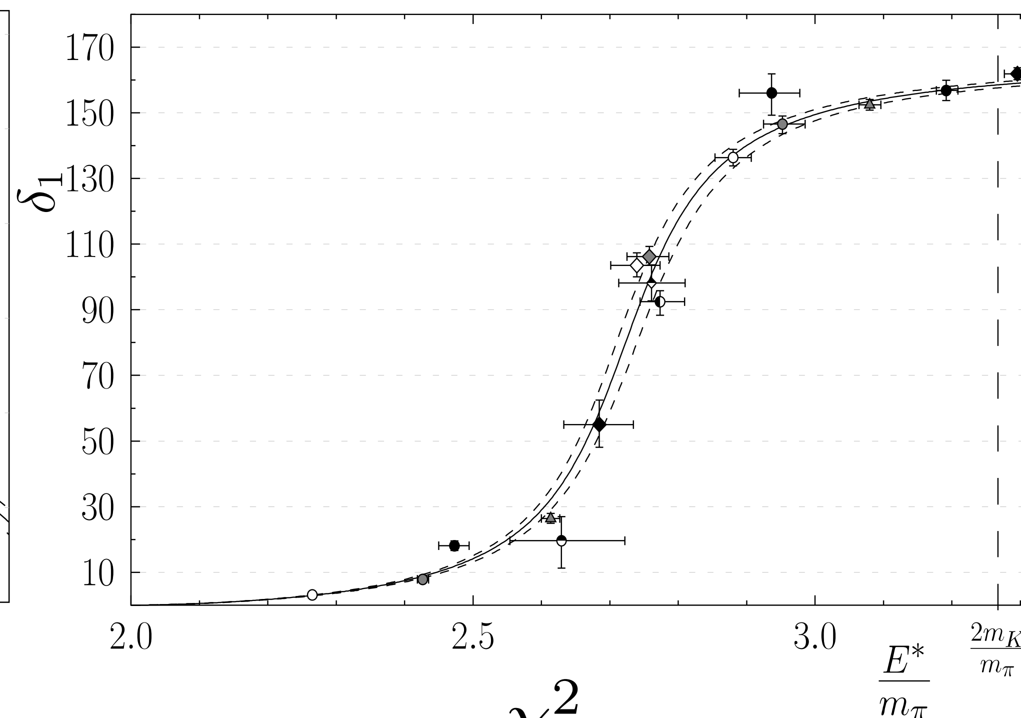
- Perform single-exponential fits to ratios and obtain energy shifts.

$$R(t) = \frac{\hat{C}_n(t)}{C_{\pi,1}(t) C_{\pi,2}(t)} \xrightarrow[t \to \infty]{} A \mathrm{e}^{-\Delta E t}$$

Isovector p-wave results: N200

$$(L = 3.12\text{fm}, a = 0.065\text{fm},\ m_\pi = 280\text{MeV})$$





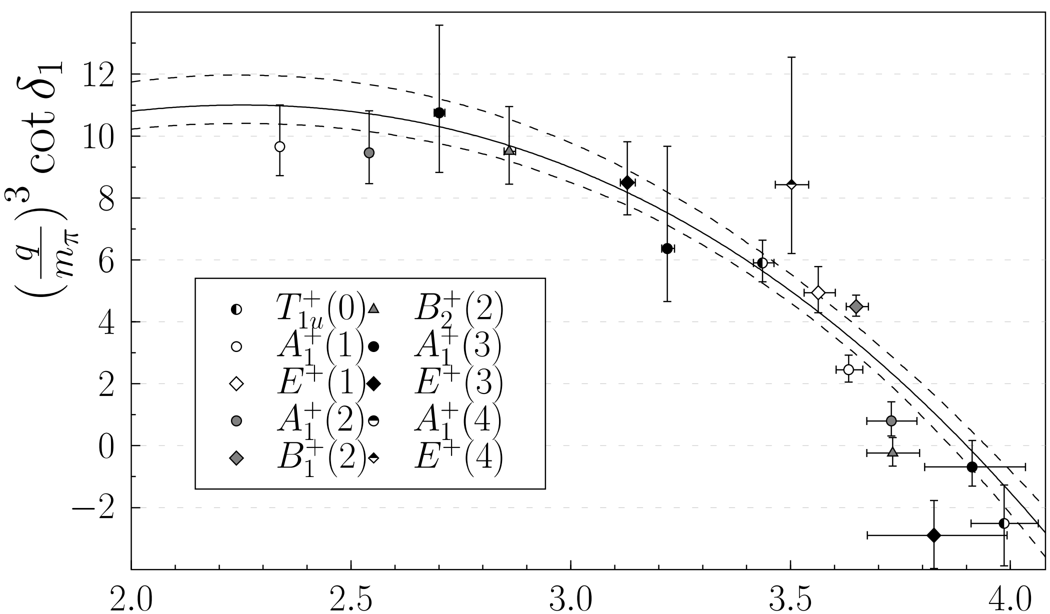
$$\frac{m_\rho}{m_\pi} = 2.741(16), \qquad g_{\rho\pi\pi} = 5.97(10), \qquad \frac{\chi^2}{d.o.f} = 0.75$$

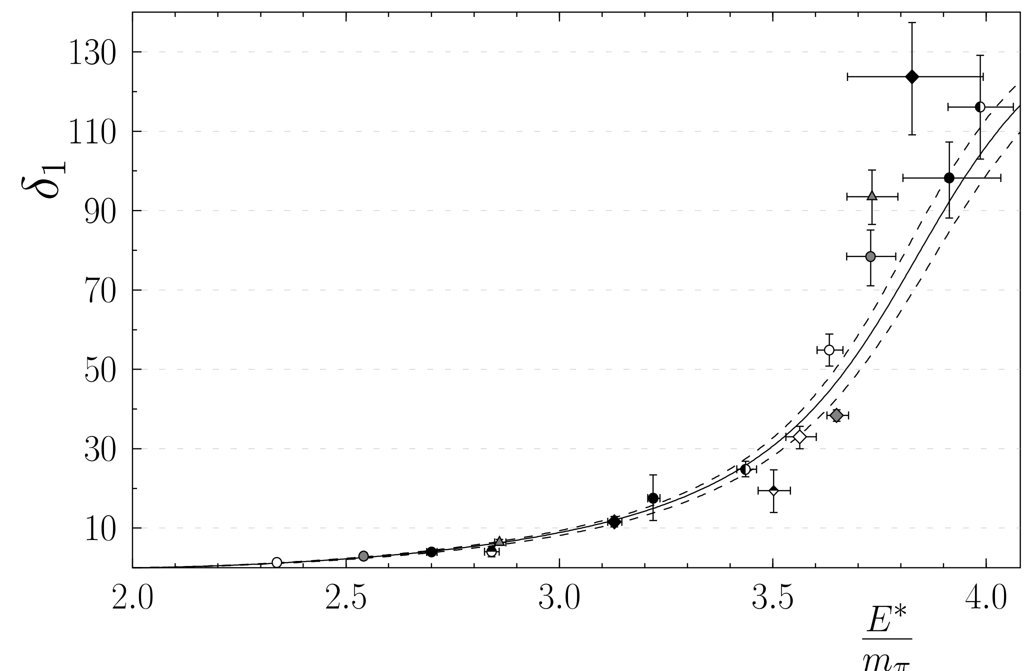
$$m_\rho = 775(4)(8)\text{MeV}$$

Scale determination/uncertainties from M. Bruno, T. Korzec, S. Schaefer, arXiv:1608.08900 [hep-lat]

Isovector *p*-wave results: D200

$$(L = 4.16\text{fm}, a = 0.065\text{fm},\ m_\pi = 200\text{MeV})$$



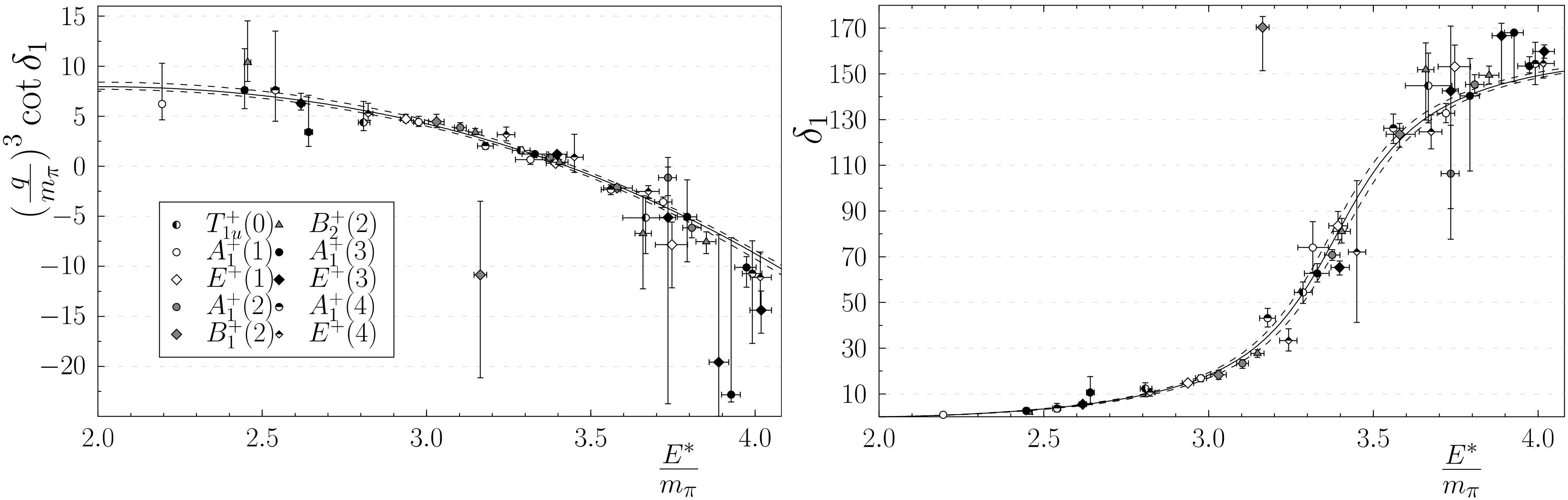


$$\frac{m_\rho}{m_\pi} = 3.901(41), \qquad g_{\rho\pi\pi} = 6.25(25), \qquad \frac{\chi^2}{d.o.f} = 1.34$$

$$m_\rho = 780(8)(8)\text{MeV}$$

Isovector *p*-wave results: D101

$$(L = 5.53\text{fm}, a = 0.086\text{fm},\ m_\pi = 220\text{MeV})$$

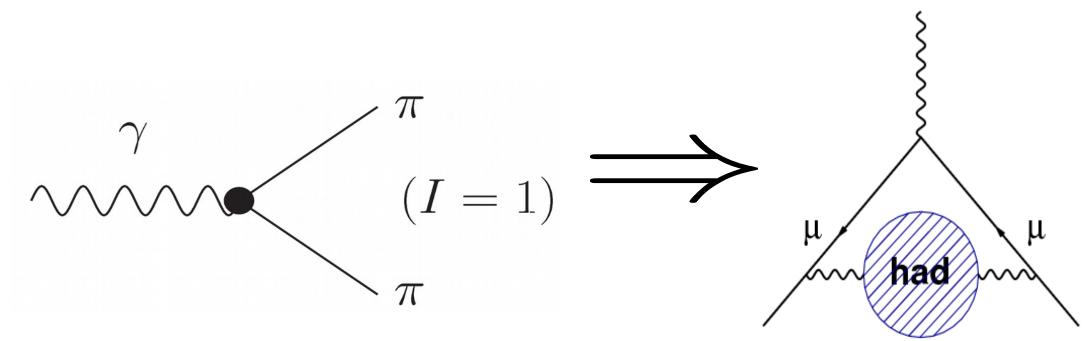


$$\frac{m_\rho}{m_\pi} = 3.42(2), \qquad g_{\rho\pi\pi} = 6.05(11), \qquad \frac{\chi^2}{d.o.f} = 0.9$$

$$m_\rho = 763(9)\text{MeV}$$

Timelike pion form factor

- Low-energy hadron vacuum polarization $\Pi(q^2)$: important theoretical uncertainty in $(g-2)_\mu$



- Optical Theorem:

$$\mathrm{Im}\,\Pi(s) = \frac{\alpha(s)}{3} R(s)$$

$$R(s) = \sigma_{\mathrm{tot}}(\mathrm{e}^+\mathrm{e}^- \to \text{ hadrons}) \left(\frac{4\pi\alpha(s)^2}{3s}\right)^{-1}$$

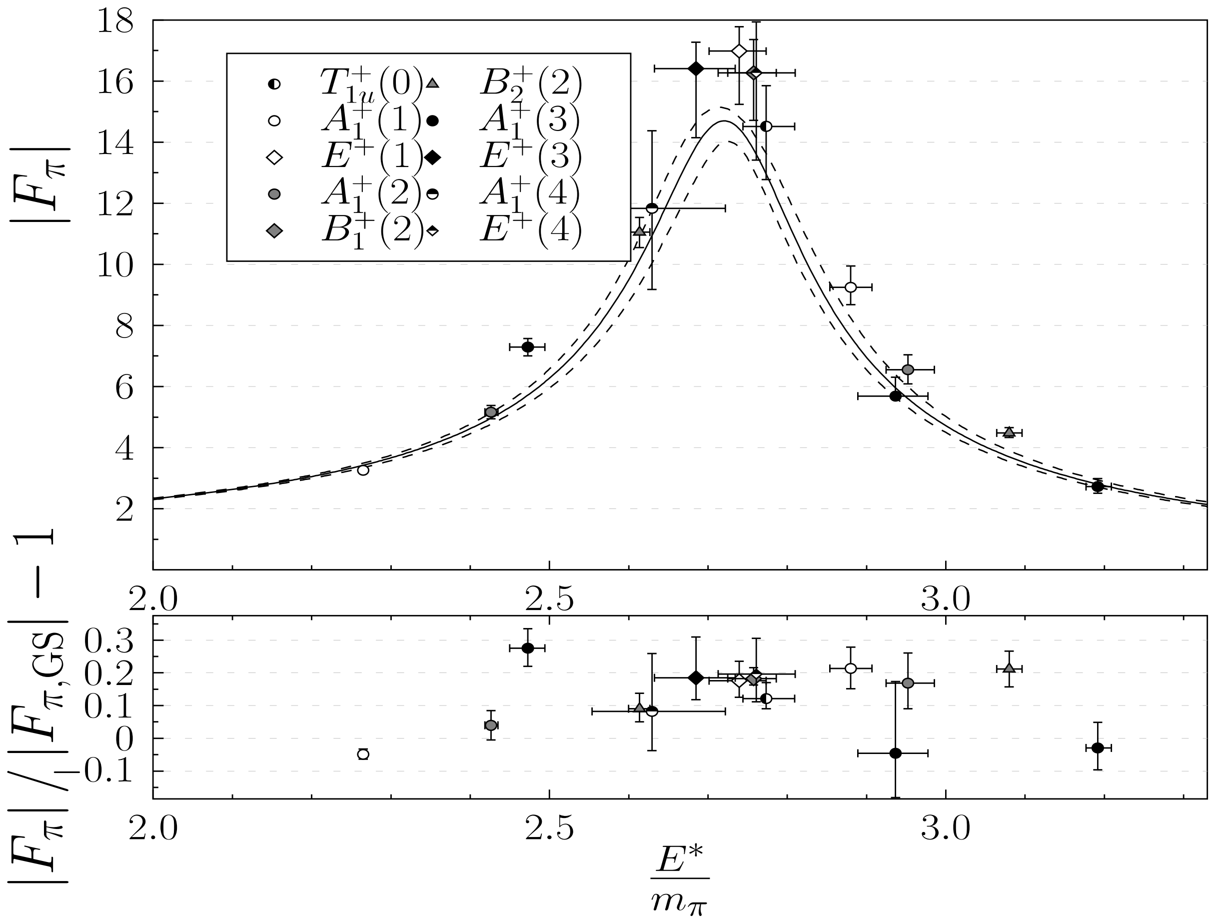
- At low energies, given by the time-like pion form-factor

$$R(s) = \frac{1}{4}\left(1-\frac{4m_\pi^2}{s}\right)^{\frac{3}{2}} |F_\pi(s)|^2,\ 4m_\pi^2 < s < 9m_\pi^2$$

Jegerlehner and Nyffeler '09; Meyer '11;
Feng, et al. `15

Form factor results: N200

$(L = 3.12\text{fm}, a = 0.065\text{fm},\ m_\pi = 280\text{MeV})$



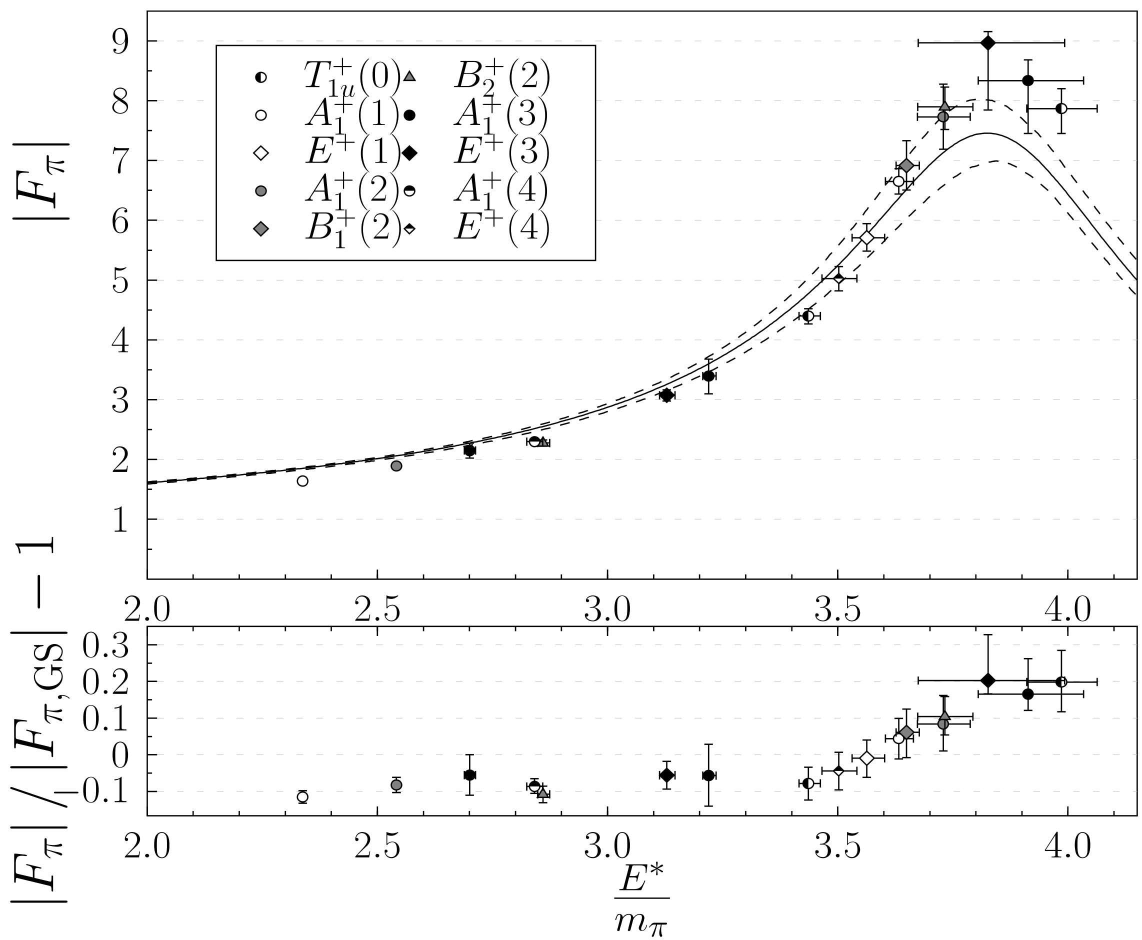
'Fit' is the Gounaris-Sakurai parametrization

Meyer, `11; Feng et al `15; See also talks by F. Erben, L. Leskovec

$$|F_\pi(E_{\mathrm{cm}})|^2 = \frac{2\pi E_{\mathrm{cm}}}{2L^3p^5} g(\gamma) \left(q\phi'(q) + p\frac{\partial \delta_1}{\partial p} \right) |\langle 0|\hat{j}_{\mathrm{em}}|\pi(\vec{p}_1)\pi(\vec{p}_2)\rangle|^2$$

Form factor results: D200

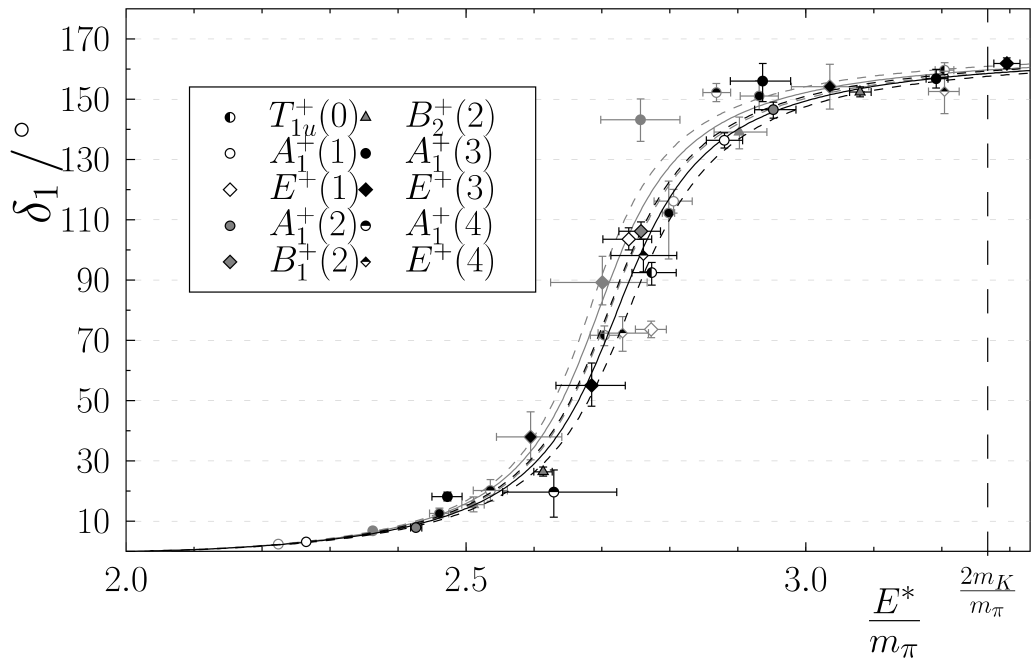
$(L = 4.16\text{fm}, a = 0.065\text{fm},\ m_\pi = 200\text{MeV})$

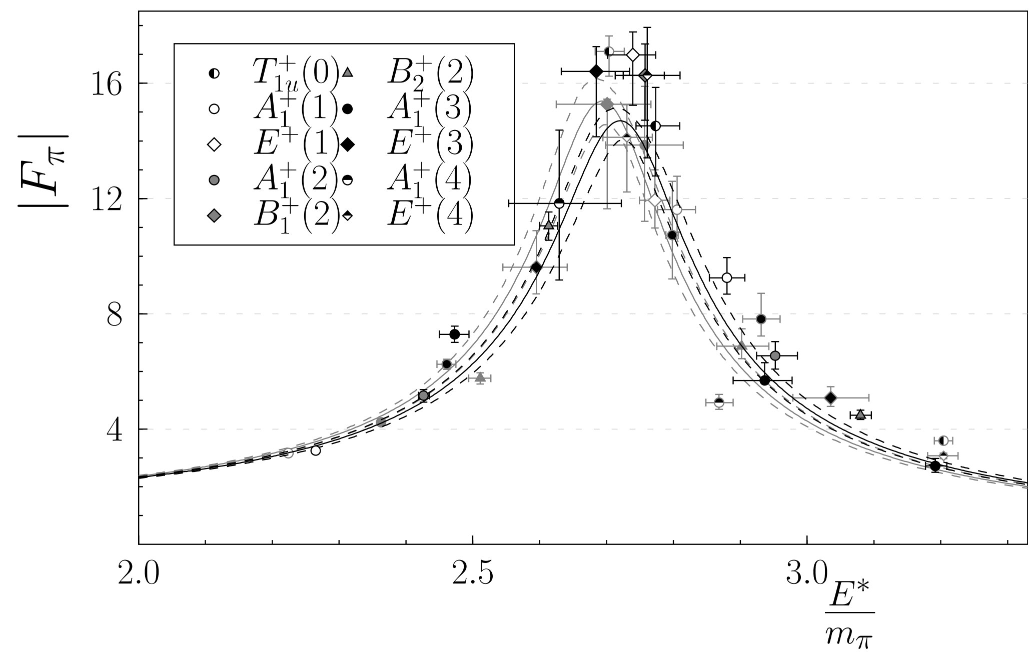


TODO:

- Investigate deviation from GS form, alternative parametrizations
- Combined determination of a_μ^{HVP}

Finite volume/lattice spacing





- Dark points: N200 $(L = 3.12\text{fm}, a = 0.065\text{fm},\ m_\pi = 280\text{MeV})$
- Gray points: N401 $(L = 3.65\text{fm}, a = 0.076\text{fm},\ m_\pi = 280\text{MeV})$

- Form factor uses non-perturbative Z_V , 1-loop c_V, b_V. Non-perturbative c_V, b_V could have significant effect.

See talks by G. De Divitiis, J. Heitger

Higher Partial Waves

Determinant conditions w/ all partial waves up to L=6, arbitrary coupled channels: Talk by Colin Morningstar

- Fit results w/o *f*-wave contribution: (N401)

$$\frac{m_\rho}{m_\pi} = 2.712(15), \qquad g_{\rho\pi\pi} = 5.92(14), \qquad \frac{\chi^2}{d.o.f} = 1.22$$

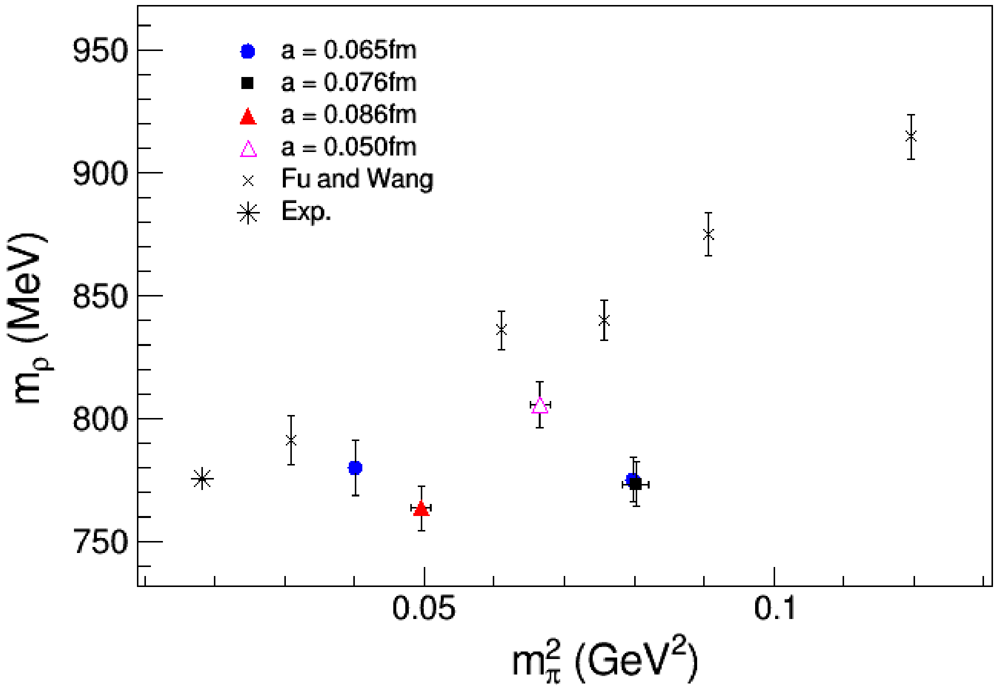
- Fit results with *f*-wave contribution:

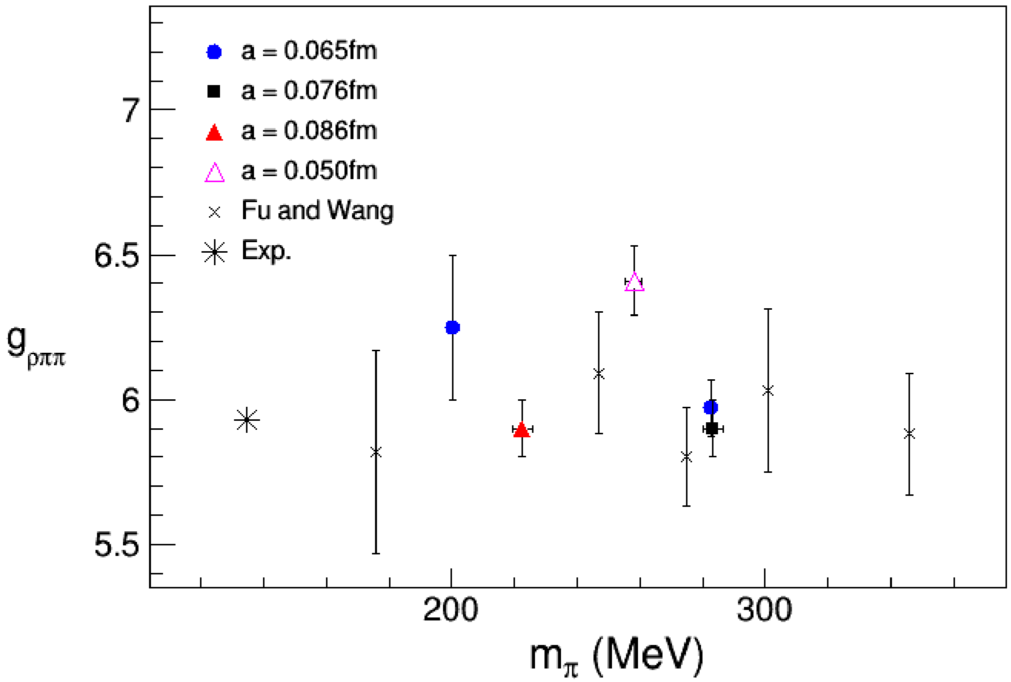
$$\frac{m_\rho}{m_\pi} = 2.728(15), \qquad g_{\rho\pi\pi} = 5.86(11),$$

$$m_\pi^7 a_3 = -0.0038(34), \qquad \frac{\chi^2}{d.o.f} = 1.8$$

- Experimentally: $m_\pi^7 a_3 = 6.3(4) \times 10^{-5}$ Pelaez, Yndurian '05

Summary





- Our 0.05fm point is preliminary (incomplete systematics)
- Gray points from: Z. Fu, L. Wang, Phys. Rev. D94 (2016) 034505
  - 3-flavor MILC ensembles, scale well-determined.
  - Different chiral trajectory: $m_s = const.$

Conclusions

- Algorithmic advances enable precise finite-volume energies.
- CLS ensembles enable exploration of continuum, chiral, and infinite volume limits
- Simple resonance photoproduction amplitude: timelike pion form factor
- Cutoff, finite volume, and higher partial wave effects appear to be small.
- Stay tuned for: I=0 pion-pion, meson-baryon, interaction with UchPT.