

# ChPT loops for the lattice: pion mass and decay constant; HVP at finite volume and $n\bar{n}$ -oscillations

ChPT loops  
for the lattice

Johan Bijnens

Introduction

Two-point

Pion mass and  
decay constant

$n\bar{n}$  oscillations

Conclusions



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## 1 Introduction

## 2 Vector two-point functions for $a_\mu$ LO-HVP

- Connected and disconnected in infinite volume
- Finite volume
- Twisting
- Results

## 3 Pion mass and decay constant

## 4 $n\bar{n}$ oscillations

## 5 Conclusions

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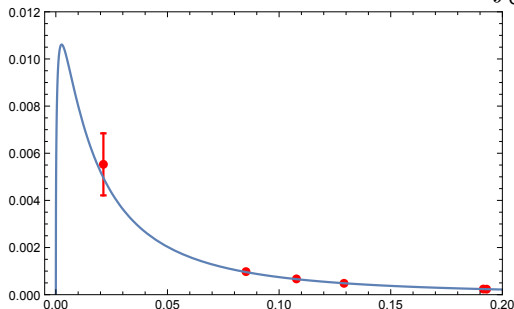
Conclusions

- ChPT = Effective field theory describing the lowest order pseudo-scalar representation
- or the (pseudo) Goldstone bosons from spontaneous breaking of chiral symmetry.
- The number of degrees of freedom depend on the case we look at
- Treat  $\pi, \eta, K$  as light and pointlike with a derivative and quark-mass expansion
- Recent review of LECs:

JB, Ecker, *Ann.Rev.Nucl.Part.Sci.* 64 (2014) 149 [arXiv:1405.6488]

# Why

Muon:  $a_\mu = (g - 2)/2$  and  $a_\mu^{\text{LO,HVP}} = \int_0^\infty dQ^2 f(Q^2) \hat{\Pi}(Q^2)$



plot:  $f(Q^2) \hat{\Pi}(Q^2)$  with  $Q^2 = -q^2$  in GeV<sup>2</sup>

Figure and data: Aubin, Blum, Chau, Golterman, Peris, Tu,  
Phys. Rev. D93 (2016) 054508 [arXiv:1512.07555]

Low energy quantity so ChPT should be useful

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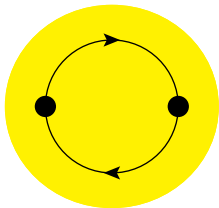
(Dis)connected  
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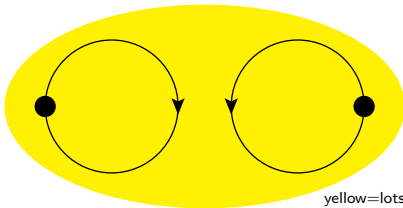
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# Two-point: Connected versus disconnected



Connected



Disconnected

yellow=lots of quarks/gluons

- $\Pi_{ab}^{\mu\nu}(q) \equiv i \int d^4x e^{iq \cdot x} \langle T(j_a^\mu(x) j_a^{\nu\dagger}(0)) \rangle$
- $j_{\pi^+}^\mu = \bar{d} \gamma^\mu u$
- $j_u^\mu = \bar{u} \gamma^\mu u$ ,  $j_d^\mu = \bar{d} \gamma^\mu d$ ,  $j_s^\mu = \bar{s} \gamma^\mu s$
- $j_e^\mu = \frac{2}{3} \bar{u} \gamma^\mu u - \frac{1}{3} \bar{d} \gamma^\mu d - \frac{1}{3} \bar{s} \gamma^\mu s$
- ChPT  $p^4$ : Della Morte, Jüttner, JHEP 1011(2010)154 [arXiv:1009.3783]
- ChPT  $p^6$ : JB, Relefors, JHEP 1611(2016)086 [arXiv:1609.01573]

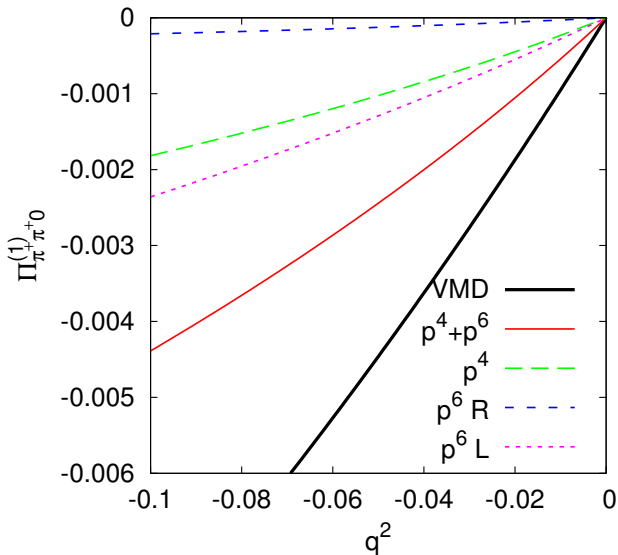
# Two-point: Connected versus disconnected

- Include also singlet part of the vector current
- There are new terms in the Lagrangian
- $p^4$  only one more:  $\langle L_{\mu\nu} \rangle \langle L^{\mu\nu} \rangle + \langle R_{\mu\nu} \rangle \langle R^{\mu\nu} \rangle$   
(drops out when subtracting  $\Pi(0)$ )
- $\implies$  The pure singlet vector current does not couple to mesons until  $p^6$
- $\implies$  Loop diagrams involving the pure singlet vector current only appear at  $p^8$  (implies relations)
- $p^6$  (no full classification, just some examples)  
 $\langle D_\rho L_{\mu\nu} \rangle \langle D^\rho L^{\mu\nu} \rangle + \langle D_\rho R_{\mu\nu} \rangle \langle D^\rho R^{\mu\nu} \rangle,$   
 $\langle L_{\mu\nu} \rangle \langle L^{\mu\nu} \chi^\dagger U \rangle + \langle R_{\mu\nu} \rangle \langle R^{\mu\nu} \chi U^\dagger \rangle, \dots$
- Results at two-loop order, unquenched isospin limit

# Two-point: Connected versus disconnected

- $\Pi_{\pi^+\pi^+}^{\mu\nu}$ : only connected
- $\Pi_{ud}^{\mu\nu}$ : only disconnected
- $\Pi_{uu}^{\mu\nu} = \Pi_{\pi^+\pi^+}^{\mu\nu} + \Pi_{ud}^{\mu\nu}$
- $\Pi_{ee}^{\mu\nu} = \frac{5}{9}\Pi_{\pi^+\pi^+}^{\mu\nu} + \frac{1}{9}\Pi_{ud}^{\mu\nu}$
- Infinite volume (and the  $ab$  considered here):  
$$\Pi_{ab}^{\mu\nu} = (q^\mu q^\nu - q^2 g^{\mu\nu}) \Pi_{ab}^{(1)}$$
- Large  $N_c$  + VMD estimate:  $\Pi_{\pi^+\pi^+}^{(1)} = \frac{4F_\pi^2}{M_V^2 - q^2}$
- Plots on next pages are for  $\Pi_{ab0}^{(1)}(q^2) = \Pi_{ab}^{(1)}(q^2) - \Pi_{ab}^{(1)}(0)$
- At  $p^4$  the extra LEC cancels, at  $p^6$  there are new LEC contributions, but no new ones in the loop parts

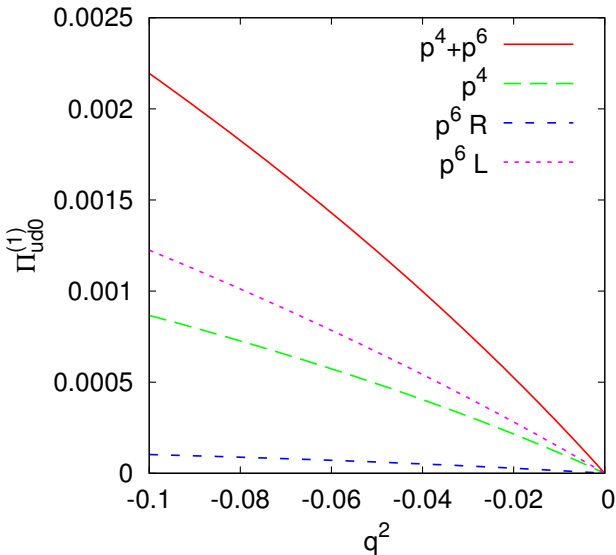
# Two-point: Connected versus disconnected



- **Connected**
- $p^6$  is large
- Due to the  $L_i^r$  loops



# Two-point: Connected versus disconnected

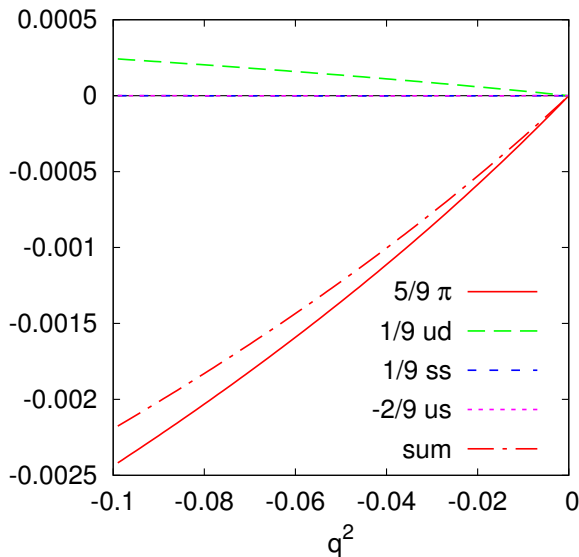


**Disconnected**

- $p^6$  is large
- Due to the  $L_i^r$  loops
- about  $-\frac{1}{2}$  connected
- $-\frac{1}{10}$  is from

$$\Pi_{ee}^{(1)} = \frac{5}{9} \Pi_{\pi^+\pi^+}^{(1)} + \frac{1}{9} \Pi_{ud}^{(1)}$$

# Two-point: with strange, electromagnetic current



- $\pi$  connected u,d
  - $ud$  disconnected u,d
  - $ss$  strange current
  - $us$  mixed s-u,d
  - new  $p^6$  LEC cancels
  - Disconnected strange  $\approx -15\%$  of total strange
- JB, Relefors,  
LUTP 16-51 to appear

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- One-loop calculation in finite volume done by  
Aubin et al, Phys.Rev. D88 (2013) 7, 074505 [arXiv:1307.4701]  
Aubin et al. Phys. Rev. D93 (2016) 054508 [arXiv:1512.07555]  
and found to fit lattice data well
- two-loop in partially quenched  
JB, Relefors, LU TP 16-51 to appear
- I will stay with ChPT and the  $p$  regime ( $M_\pi L \gg 1$ )
- $1/m_\pi = 1.4$  fm  
may need to (and I will) go beyond leading  $e^{-m_\pi L}$  terms  
“around the world as often as you like”
- Convergence of ChPT is given by  $1/m_\rho \approx 0.25$  fm

# Finite volume and Twisted boundary conditions

- On a lattice at finite volume  $p^i = 2\pi n^i/L$ : very few momenta directly accessible
- Put a constraint on certain quark fields in some directions:  
 $q(x^i + L) = e^{i\theta^i} q(x^i)$
- Then momenta are  $p^i = \theta^i/L + 2\pi n^i/L$ . Allows to map out momentum space on the lattice much better

Bedaque,...

- Small note:
  - Beware what people call momentum: is  $\theta^i/L$  included or not?
  - Reason: a colour singlet gauge transformation  $G_\mu^S \rightarrow G_\mu^S - \partial_\mu \epsilon(x)$ ,  $q(x) \rightarrow e^{i\epsilon(x)} q(x)$ ,  $\epsilon(x) = -\theta^i x^i/L$
  - Boundary condition  
Twisted  $\Leftrightarrow$  constant background field+periodic

## Drawbacks:

- Box: Rotation invariance  $\rightarrow$  cubic invariance
- Twisting: reduces symmetry further

## Consequences:

- $m^2(\vec{p}^2) = E^2 - \vec{p}^2$  is not constant
- There are typically more form-factors
- In general: quantities depend on more (all) components of the momenta
- Charge conjugation involves a change in momentum

# Two-point function: twisted boundary conditions

JB, Relfors, JHEP 05 (201)4 015 [arXiv:1402.1385]

- $\int_V \frac{d^d k}{(2\pi)^d} \frac{k_\mu}{k^2 - m^2} \neq 0$
- $\langle \bar{u} \gamma^\mu u \rangle \neq 0$
- $j_{\pi^+}^\mu = \bar{d} \gamma^\mu u$   
satisfies  $\partial_\mu \langle T(j_{\pi^+}^\mu(x) j_{\pi^+}^{\nu\dagger}(0)) \rangle = \delta^{(4)}(x) \langle \bar{d} \gamma^\nu d - \bar{u} \gamma^\nu u \rangle$
- $\Pi_a^{\mu\nu}(q) \equiv i \int d^4 x e^{iq \cdot x} \langle T(j_a^\mu(x) j_a^{\nu\dagger}(0)) \rangle$   
Satisfies WT identity.  $q_\mu \Pi_{\pi^+}^{\mu\nu} = \langle \bar{u} \gamma^\mu u - \bar{d} \gamma^\mu d \rangle$
- ChPT at one-loop satisfies this  
see also Aubin et al, Phys.Rev. D88 (2013) 7, 074505 [arXiv:1307.4701]
- two-loop in partially quenched  
JB, Relfors, LU TP 16-51, to appear  
satisfies the WT identity (as it should)

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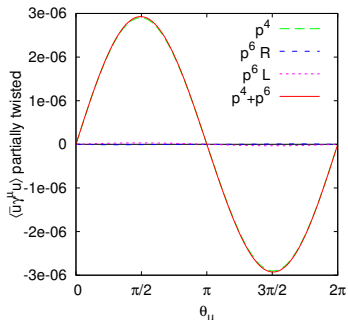
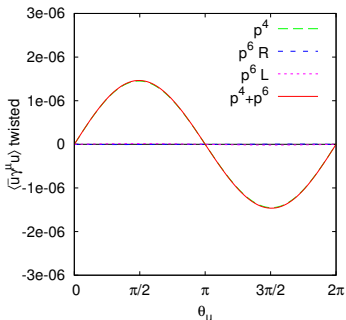
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$$\langle \bar{u} \gamma^\mu u \rangle$$



Fully twisted

$\theta_u = (0, \theta_u, 0, 0)$ , all others untwisted

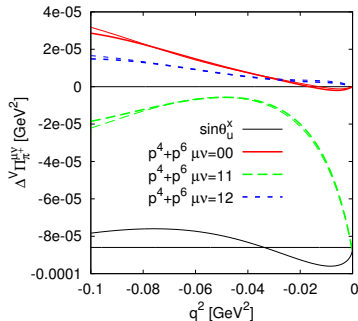
$m_\pi L = 4$

For comparison:  $\langle \bar{u} u \rangle^V \approx -2.4 \cdot 10^{-5} \text{ GeV}^3$   
 $\langle \bar{u} u \rangle \approx -1.2 \cdot 10^{-2} \text{ GeV}^3$

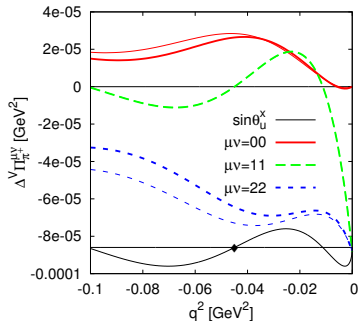
Partially twisted

(ratio at  $p^4 \equiv 2$  up to kaon loops)

# Two-point partially twisted: components



Twisting spatially symmetric

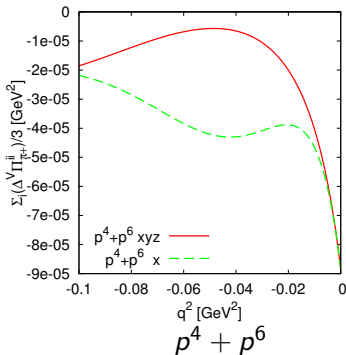
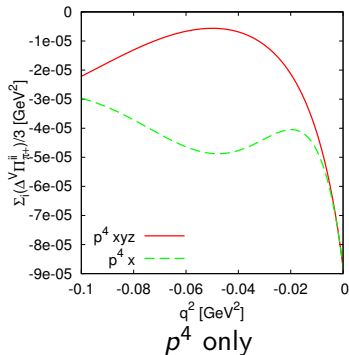


Twisting in x-direction

- Small  $p^6$  corrections (thin lines:  $p^4$  only)
- $m_{\pi 0} L = 4 \quad m_{\pi 0} = 0.135 \text{ GeV}$
- $-q^2 \Pi_{\text{VMD}}^{(1)} = \frac{-4q^2 F_\pi^2}{M_V^2 - q^2} \approx 5e-3 \cdot \frac{q^2}{0.1} \implies$  **Correction at % level**
- Can use the difference between different twists with same  $q^2$  to check the finite volume corrections



# Two-point partially twisted: spatial average



- Plotted: volume correction to  $\bar{\Pi} = \frac{1}{3} \sum_{i=x,y,z} \Pi^{ii}$
- Small  $p^6$  corrections: compare left and right
- Can use the difference between different twists with same  $q^2$  to check the finite volume corrections

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# Two flavour ChPT: mass and decay constant

- First step towards finding out why hard-pion ChPT does not work at three-loops
- Lowest order: Gell-Mann, Oakes, Renner (1968)
- Chiral logarithm Langacker, Pagels (1973)
- Full NLO (and properly starting ChPT) Gasser-Leutwyler (1984)
- NNLO Buergi (1996), JB, Colangelo, Ecker, Gasser, Sainio (1996)
- NNNLO JB, Hermansson-Truedsson (2017)

- LO and Chiral logs: current algebra
- NLO: Feynman diagrams (by hand) and direct expansion of functional integral (with REDUCE)
- NNLO: Feynman diagrams (with a little help from FORM)
- NNLO: Feynman diagrams purely with FORM
- Main stumbling block: integrals
  - Reduction to master integrals with REDUZE Studerus (2009)
  - Master Integrals known  
Laporta-Remiddi (1996); Melnikov, van Ritbergen (2001)
- Lots of book-keeping: FORM
- Checks:
  - All nonlocal divergences must cancel
  - Use different parametrizations of the Lagrangian
  - Agree with known leading log result JB, Carloni (2009)

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# Diagrams



LUND  
UNIVERSITY

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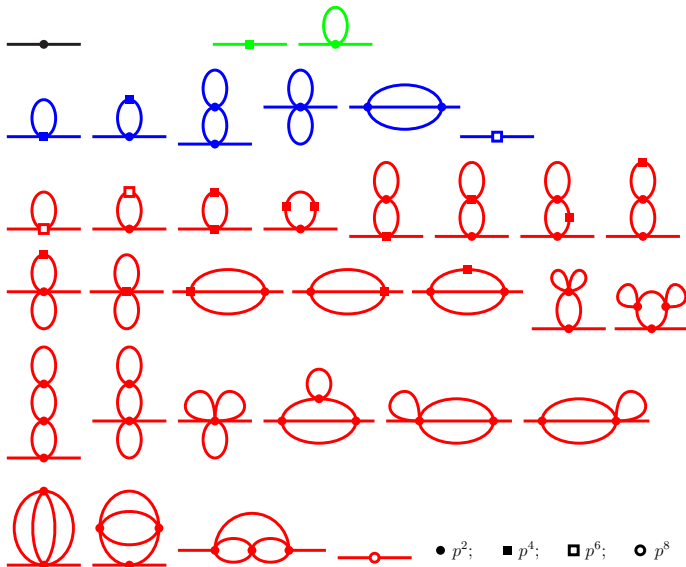
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# Results: LO or $x$ -expansion / physical or $\xi$ -expansion

- $x = \frac{M^2}{16\pi^2 F^2}, \quad L_x = \log \frac{M^2}{\mu^2}, \quad M^2 = 2B\hat{m}$

$$\frac{M_\pi^2}{M^2} = 1 + x \left( a_{11}^M L_x + a_{10}^M \right) + x^2 \left( a_{22}^M L_x^2 + a_{21}^M L_x + a_{20}^M \right) + x^3 \left( a_{33}^M L_x^3 + a_{32}^M L_x^2 + a_{31}^M L_x + a_{30}^M \right) + \dots$$

$$\frac{F_\pi}{F} = 1 + x \left( a_{11}^F L_x + a_{10}^F \right) + x^2 \left( a_{22}^F L_x^2 + a_{21}^F L_x + a_{20}^F \right) + x^3 \left( a_{33}^F L_x^3 + a_{32}^F L_x^2 + a_{31}^F L_x + a_{30}^F \right) + \dots$$

- $\xi = \frac{M_\pi^2}{16\pi^2 F_\pi^2}, \quad L_\pi = \log \frac{M_\pi^2}{\mu^2}$

- $\frac{M^2}{M_\pi^2} = 1 + \xi \left( b_{11}^M L_\pi + b_{10}^M \right) + \xi^2 \left( b_{22}^M L_\pi^2 + b_{21}^M L_\pi + b_{20}^M \right) + \xi^3 \left( b_{33}^M L_\pi^3 + b_{32}^M L_\pi^2 + b_{31}^M L_\pi + b_{30}^M \right) + \dots$

- $\frac{F}{F_\pi} = 1 + \xi \left( b_{11}^F L_\pi + b_{10}^F \right) + \xi^2 \left( b_{22}^F L_\pi^2 + b_{21}^F L_\pi + b_{20}^F \right) + \xi^3 \left( b_{33}^F L_\pi^3 + b_{32}^F L_\pi^2 + b_{31}^F L_\pi + b_{30}^F \right) + \dots$

$$\tilde{l}_i = 16\pi^2 l_i^r, \quad \tilde{c}_i = (16\pi^2)^2 c_i^r$$

$a_{11}^M$	$\frac{1}{2}$
$a_{10}^M$	$2\tilde{l}_3$
$a_{22}^M$	$\frac{17}{8}$
$a_{21}^M$	$-3\tilde{l}_3 - 8\tilde{l}_2 - 14\tilde{l}_1 - \frac{49}{12}$
$a_{20}^M$	$64\tilde{c}_{18} + 32\tilde{c}_{17} + 96\tilde{c}_{11} + 48\tilde{c}_{10} - 16\tilde{c}_9 - 32\tilde{c}_8 - 16\tilde{c}_7$ $- 32\tilde{c}_6 + \tilde{l}_3 + 2\tilde{l}_2 + \tilde{l}_1 + \frac{193}{96}$
$a_{33}^M$	$\frac{103}{24}$
$a_{32}^M$	$\frac{23}{2}\tilde{l}_3 - 11\tilde{l}_2 - 38\tilde{l}_1 - \frac{91}{24}$
$a_{31}^M$	$-416\tilde{c}_{18} - 208\tilde{c}_{17} - 32\tilde{c}_{16} + 96\tilde{c}_{14} + 8\tilde{c}_{13} - 48\tilde{c}_{12}$ $- 384\tilde{c}_{11} - 192\tilde{c}_{10} + 72\tilde{c}_9 + 144\tilde{c}_8 + 72\tilde{c}_7 + 64\tilde{c}_6 - 8\tilde{c}_5$ $- 56\tilde{c}_4 + 16\tilde{c}_3 + 32\tilde{c}_2 - 96\tilde{c}_1 - 8\tilde{l}_3^2 - 48\tilde{l}_3\tilde{l}_2 - 84\tilde{l}_3\tilde{l}_1$ $- \frac{88}{3}\tilde{l}_3 - \frac{231}{10}\tilde{l}_2 - \frac{69}{5}\tilde{l}_1 - \frac{74971}{8640}$
$a_{30}^M$	contains free $p^8$ LECs (and a lot more terms)

- Similar tables for  $a_i^F$ ,  $b_i^M$ ,  $b_i^F$
- Coefficients depend on scale  $\mu$ , but whole expression is  $\mu$ -independent
- Can be rewritten in terms of scales in the logarithm rather than in terms of LECs à la FLAG
- Leading log: a number
- NLL: depends on  $l_i^r$
- NNLL: depends on  $c_i^r$
- For the mass all needed  $c_i^r$  can be had from mass, decay-constant and  $\pi\pi$  parameters fitted to two-loop or  $p^6$  (i.e.  $r_M, r_F, r_1, \dots, r_6$ ).
- For decay need one more (busy checking if it can be had)



# Results: numerics preliminary

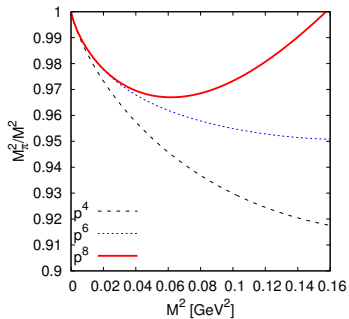
$ij$	$a_{ij}^M$	$b_{ij}^M$	$a_{ij}^F$	$b_{ij}^F$
10	+0.00282	-0.00282	+1.09436	-1.09436
11	+0.5	-0.5	-1	+1
20	+1.65296	-1.65771	-0.04734	-1.15001
21	+2.4573	-3.29038	-1.90577	+4.13885
22	+2.125	-0.625	-1.25	-0.25
30	+0.39527	-6.7854	-244.499	242.236
31	-3.75977	+4.32719	-19.0601	32.1315
32	+17.1476	+0.62039	-9.39462	-6.77511
33	+4.29167	+5.14583	-3.45833	-0.41666

Note the large coefficients in the decay constant

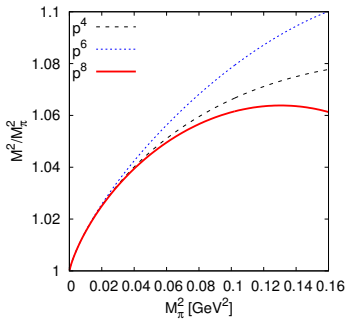
# Pion mass

$$F_\pi = 92.2 \text{ MeV}, F = F_\pi/1.037, \bar{l}_1 = -0.4, \bar{l}_2 = 4.3, \bar{l}_3 = 3.41, \bar{l}_4 = 4.51,$$

$r_i$  from [JB et al 1997](#), other  $c_i^r = 0$ ,  $\mu = 0.77 \text{ GeV}$



x-expansion ( $F$ -fixed)



$\xi$ -expansion ( $F_\pi$  fixed)

$\xi$ -expansion converges notably better

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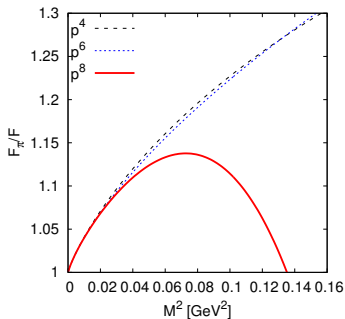
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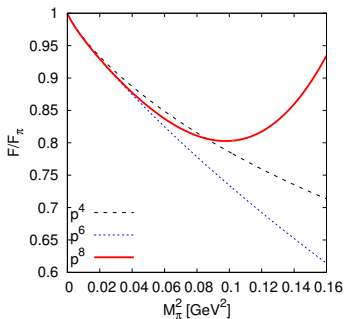
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# Pion decay constant



x-expansion ( $F$ -fixed)

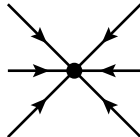


$\xi$ -expansion ( $F_\pi$  fixed)

- $\xi$ -expansion converges better
- Large  $p^6$  due to the “240” in  $a_{30}^F$  and  $b_{30}^F$

- some GUT models have neutron-anti-neutron oscillations but no proton decay
- Limits:
  - $8.6 \cdot 10^7$ s from free neutrons (ILL)
  - $2.7 \cdot 10^8$ s from oxygen nuclei (super-K)  
( $\bar{n}$  mass inside nuclei very different from  $n$ -mass)
  - Possible ESS experiment improvement by up to  $10^3$

- Effective dimension 9 operator: “ $uuduud$ ”



- Classification of quark operators and RGE to two loops:  
[Buchoff, Wagman, Phys. Rev D93\(2016\)016005 \[arXiv:1506.00647\]](#)  
and earlier papers in there
- [E. Kofoed, Master thesis LU TP 16-62; JB, Kofoed, in preparation](#)

- 14 operators
- Chiral representations under  $SU(2)_L \times SU(2)_R$ :

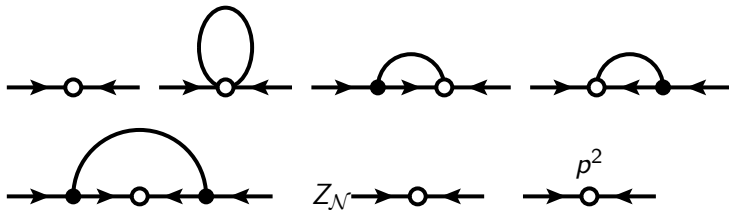
Chiral	#operators	Chiral	#operators
$(1_L, 3_R)$	3: $Q_1, Q_2, Q_3$	$(3_L, 1_R)$	3
$(5_L, 3_R)$	3: $Q_5, Q_6, Q_7$	$(3_L, 5_R)$	3
$(1_L, 7_R)$	1: $Q_4$	$(7_L, 1_R)$	1

- $n\bar{n}$  is  $\Delta I = 1$  so bottom line needs isospin breaking
- For the others the different operators are different elements within the same representation
- Use heavy baryon formalism with a baryon ( $\mathcal{N}$ ) and anti-baryon doublet ( $\mathcal{N}^c$ ) with same velocity  $v$
- These correspond two widely separated areas ( $v$  and  $-v$ ) in the relativistic field: so no double counting

- $\mathcal{L} = \frac{F^2}{4} \langle u_\mu u^\mu + \chi_+ \rangle + \overline{\mathcal{N}} (i v \cdot D + g_A u \cdot S) \mathcal{N}$   
 $+ \overline{\mathcal{N}^c} \tau^2 (i v \cdot D + g_A u \cdot S) \tau^2 \mathcal{N}^c + \text{higher orders}$
- $\mathcal{N} = \begin{pmatrix} p \\ n \end{pmatrix} \rightarrow h\mathcal{N}, \quad \overline{\mathcal{N}^c} = (\overline{p^c} \ \overline{n^c}) \rightarrow \overline{\mathcal{N}^c} h^T$
- $h(g_L, g_R, u)$ :  $SU(2)_V$  compensator chiral transformation
- Spurions for each quark  $n\bar{n}$  operator:
  - $(1_L, 3_R)$ : two  $SU(2)_R$  doublet indices
  - $(5_L, 3_R)$ : four  $SU(2)_L$  and two  $SU(2)_L$  doublet indices
  - $(1_L, 7_R)$ : six  $SU(2)_R$  double indices
  - plus parity conjugates

# ChPT terms and diagrams

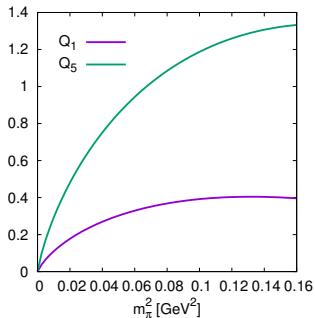
- $p^0$ :
  - $(1_L, 3_R)$ :  $(u\overline{\mathcal{N}^c})_{i_L} (u\mathcal{N})_{j_L}$
  - $(5_L, 3_R)$ :  $(u^\dagger\overline{\mathcal{N}^c})_{i_L} (u^\dagger\mathcal{N})_{j_L} (U_{T^2})_{k_R l_L} (U_{T^2})_{m_R n_L}$
  - $(7_L, 1_R)$ : none (first one at  $p^2$ )
- $p^1$ : none that directly contribute (but in loops from  $p^3$ )
- $p^2$ : many (at least 20 each for  $(3_L, 1_R)$  and  $(3_L, 5_R)$ )



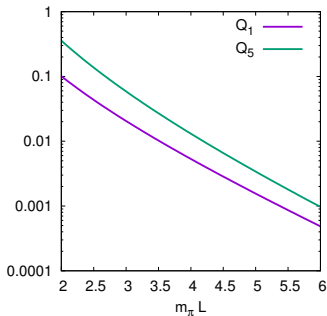
- $Q_1, Q_2, Q_3$  same factor from loops (isospin)
- $Q_5, Q_6, Q_7$  same factor  
(Conjecture: due to projection on  $I = 1$  subspace)
- $Q_1, Q_2, Q_3$ :  
$$1 + \frac{M_\pi^2}{16\pi^2 F_\pi^2} \left( \left( -\frac{3}{2} g_A^2 - 1 \right) \log \frac{M_\pi^2}{\mu^2} - 1 \right) + \text{order } p^2 \text{ LECs}$$
- $Q_5, Q_6, Q_7$ :  
$$1 + \frac{M_\pi^2}{16\pi^2 F_\pi^2} \left( \left( -\frac{3}{2} g_A^2 - 7 \right) \log \frac{M_\pi^2}{\mu^2} - 1 \right) + \text{order } p^2 \text{ LECs}$$
- Also done at finite volume



# Results (Preliminary)



Relative correction  
from loops



Relative correction  
from loops (absolute value)

Showed you results for:

- HVP: ChPT at two-loops including partially quenched
  - Connected versus disconnected at two-loops
  - Connected: twisting and finite volume at two-loops
- Two flavour ChPT correction at three loops for the pion mass and decay constant
- Two flavour ChPT correction at one-loop for  $n\bar{n}$ -oscillations
- **Be careful: ChPT must exactly correspond to your lattice calculation**
- Programs available (for published ones) via CHIRON  
Those for this talk: sometime later this year (I hope) (or ask me)

ChPT loops  
for the lattice

Johan Bijnens

Introduction

Two-point

Pion mass and  
decay constant

$n\bar{n}$  oscillations

Conclusions