# Charmed states and $SU(3)$ flavour symmetry breaking

R. Horsley, Z. Koumi, Y. Nakamura, H. Perlt, P. E. L. Rakow,
G. Schierholz, A. Schiller, H. Stüben, R. Young and J. M. Zanotti

– QCDSF-UKQCD-CSSM Collaborations –

Edinburgh – Adelaide – RIKEN (Kobe) – Leipzig – Liverpool – DESY – Hamburg

Lattice 2017, Granada, Spain

[Wednesday 21/6/17 10:40 (Andalucia III)]

## QCDSF related talks with $2+1$ flavours:

- James Zanotti — Monday 19/6/17 17:20 (Andalucia III)
  Isospin breaking effects on nucleon structure from fully dynamical QCD+QED
- Ross Young — Tuesday 20/6/17 17:50 (Andalucia III)
  Partonic structure from the Compton amplitude
- Holger Perlt — Tuesday 20/6/17 19:30 (Palacio de Congresos) [Poster]
  The IMOM renormalization scheme on the lattice
- Hinnerk Stüben — Tuesday 20/6/17 19:30 (Palacio de Congresos) [Poster]
  An update on the BQCD Hybrid Monte Carlo program
- Gerrit Schierholz — Friday 23/6/17 15:00 (Seminarios 6+7)
  Dynamical QCD+Axion simulation: First results
- Paul Rakow — Friday 23/6/17 17:50 (Seminarios 1+2)
  Charge creation, finite size effects, and infra-red photons in simulations of QCD plus QED

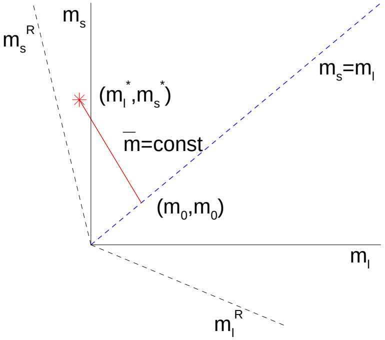
## Introduction

- Background:
  - Given $2+1$ simulations (at quark masses larger than physical quark masses), how can we usefully approach the physical point?
  - Possibility: $SU(3)$ flavour expansion about flavour symmetric line
- Extend expansion to PQ quark masses (ie valence quarks $\neq$ sea quarks)
- Determine (quenched) charm quark
- Open charm masses
- Conclusions

# QCDSF strategy:

[arXiv:1102.5300 ]

$2 + 1$ simulations: many paths to approach the physical point [$m_u = m_d \equiv m_l$ case]




QCDSF: extrapolate from a point on the $SU(3)_F$ flavour symmetry line to the physical point

$$(m_0, m_0) \longrightarrow (m_l^*, m_s^*)$$

Choice here: keep the singlet quark mass $\overline{m}$ constant

$$\overline{m} = m_0 = \frac{1}{3}\,(2m_l + m_s)$$

## QCDSF strategy

[arXiv:1102.5300 ]

- develop $SU(3)$ flavour symmetry breaking expansion for hadron masses
- expansion in:

$SU(3)$ flavour symmetric point $\delta m_q = 0$

$$\delta m_q = m_q - \overline{m}\,, \quad \overline{m} = \tfrac{1}{3}(m_u + m_d + m_s) = m_0$$

  - expansion coefficients are functions of $\overline{m}$
  - trivial constraint

$$\delta m_u + \delta m_d + \delta m_s = 0$$

- path called 'unitary line' as expand in both sea and valence quarks

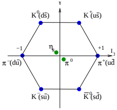
# $SU(3)$ flavour symmetry breaking expansions

[known to $O(3)$ in $\delta m$, ie NNLO]

- octet pseudoscalar mesons:

$$
\begin{aligned}
M^2(a\bar{b}) \quad = \quad & M_{0\pi}^2 + \alpha(\delta m_a + \delta m_b) \\
& + \beta_0 \frac{1}{6}(\delta m_u^2 + \delta m_d^2 + \delta m_s^2) \\
& + \beta_1(\delta m_a^2 + \delta m_b^2) + \beta_2(\delta m_a - \delta m_b)^2 \\
& + \ldots
\end{aligned}
$$

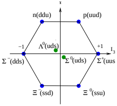
$[a, b = u, d, s$ (outer ring)$]$




- octet baryons:

$$
\begin{aligned}
M^{N\,2}(aab) \quad = \quad & M_{0N}^2 + A_1(2\delta m_a + \delta m_b) + A_2(\delta m_b - \delta m_a) \\
& + B_0 \frac{1}{6}(\delta m_u^2 + \delta m_d^2 + \delta m_s^2) \\
& + B_1(2\delta m_a^2 + \delta m_b^2) + B_2(\delta m_b^2 - \delta m_a^2) + B_3(\delta m_b - \delta m_a)^2 \\
& + \ldots
\end{aligned}
$$

$[a, b = u, d, s$ (outer ring)$]$




stable under strong ints.

$$
\begin{aligned}
M^{\Lambda\,2}(aab) \quad = \quad & M_{0\Lambda}^2 + A_1(2\delta m_a + \delta m_b) - A_2(\delta m_b - \delta m_a) \\
& + B_0 \frac{1}{6}(\delta m_u^2 + \delta m_d^2 + \delta m_s^2) \\
& + B_1(2\delta m_a^2 + \delta m_b^2) - B_2(\delta m_b^2 - \delta m_a^2) + B_4(\delta m_b - \delta m_a)^2 \\
& + \ldots
\end{aligned}
$$

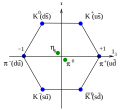
$[a, b = l, s$ (when no $\Lambda^0 - \Sigma^0$ mixing)$]$

# Singlet quantities

- Pseudoscalar mesons (centre of mass):

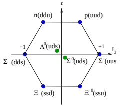
[no QED, $m_u = m_d$, FLAG3]

$$X_\pi^2 = \frac{1}{6}(M_{K^+}^2 + M_{K^0}^2 + M_{\pi^+}^2 + M_{\pi^-}^2 + M_{\overline{K}^0}^2 + M_{K^-}^2) = (0.4116\ \text{GeV})^2$$
$$= M_{0\pi}^2 + \left(\frac{1}{6}\beta_0 + \frac{2}{3}\beta_1 + \beta_2\right)(\delta m_u^2 + \delta m_d^2 + \delta m_s^2) = M_{0\pi}^2 + O(\delta m_q^2)$$




- Octet baryons (centre of mass):

$$X_N^2 = \frac{1}{6}(M_p^2 + M_n^2 + M_{\Sigma^+}^2 + M_{\Sigma^-}^2 + M_{\Xi^0}^2 + M_{\Xi^-}^2) = (1.160\ \text{GeV})^2$$
$$= M_{0N}^2 + \frac{1}{6}(B_0 + B_1 + B_3)(\delta m_u^2 + \delta m_d^2 + \delta m_s^2) = M_{0N}^2 + O(\delta m_q^2)$$




stable under strong ints.

- Many other possibilities

$$X_S^2 = \begin{cases} \frac{1}{2}(M_\Sigma^2 + M_\Lambda^2) & S = \Lambda \quad \text{baryon octet} \\ M_{\Sigma^*}^2,\ \frac{1}{2}(M_\Delta^2 + M_{\Xi^*}^2) & S = \Sigma^*, \Delta \quad \text{baryon decuplet, unstable under QCD} \\ \frac{1}{6}(M_{K^{*+}}^2 + M_{K^{*0}}^2 + M_{\rho^+}^2 + M_{\rho^-}^2 + M_{\overline{K}^{*0}}^2 + M_{K^{*-}}^2) & S = \rho \quad \text{vector octet} \\ 1/r_0^2,\ 1/t_0,\ 1/w_0^2 & S = r_0, t_0, w_0 \quad \text{force, Wilson flow scales} \\ \frac{1}{6}(f_{K^+} + f_{K^0} + f_{\pi^+} + f_{\pi^-} + f_{\overline{K}^0} + f_{K^-}) & S = f \quad \text{decay constants} \end{cases}$$

- Form dimensionless ratios (within a multiplet):

$$\tilde{M}^2 \equiv \frac{M^2}{X_S^2}, \quad S = \pi, N, \ldots, \qquad \tilde{A}_i \equiv \frac{A_i}{M_{0N}^2}, \ldots \quad \text{in expansions}$$

# Lattice

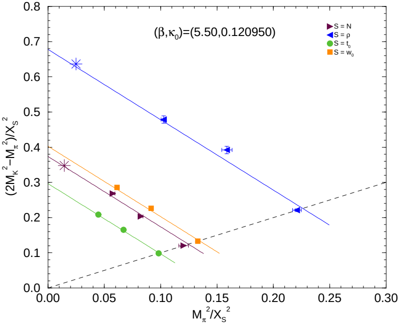
- $O(a)$ NP improved clover action
  - tree level Symanzik glue
  - mildly stout smeared $2+1$ clover fermion
  - $\beta = [5.40], 5.50, 5.65, 5.80, 5.95$ $[24^3 \times 48,\ 32^3 \times 64,\ 48^3 \times 96]$, unitary $M_\pi \sim 220 - 410\,\mathrm{MeV}$ $[M_\pi L \geq 4]$

- $$m_q = \frac{1}{2}\left(\frac{1}{\kappa_q} - \frac{1}{\kappa_{0c}}\right)$$

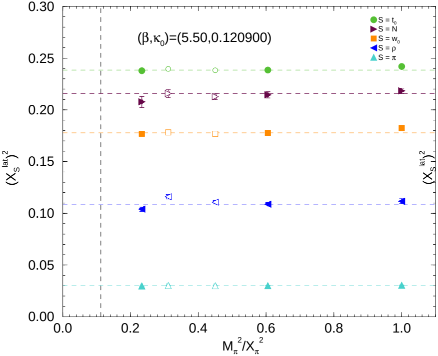
  $\kappa_{0c}$ is chiral limit along symmetric line

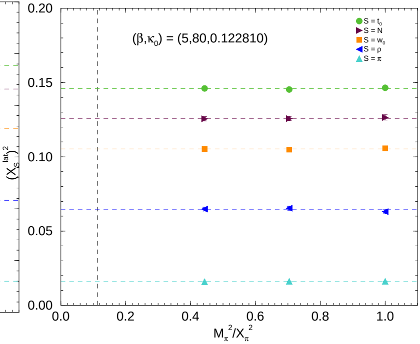
- $$\delta m_q = m_q - \overline{m} = \frac{1}{2}\left(\frac{1}{\kappa_q} - \frac{1}{\kappa_0}\right)$$

- Path in quark mass plane



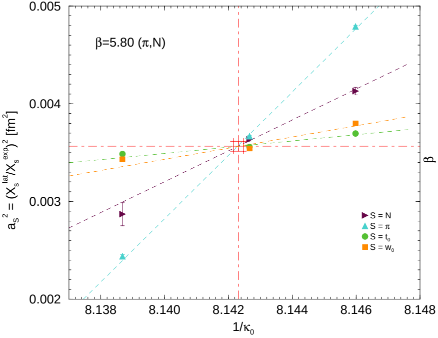
# Constancy of $X_S^2$

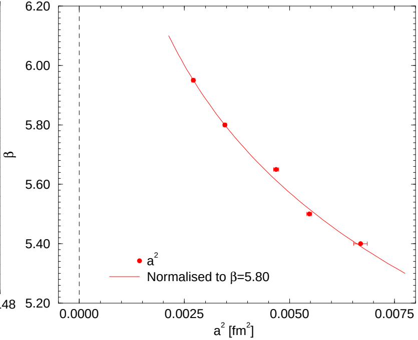



Use this constancy to determine the scale

# Scale




- Crossing of

$$a_S^2 = \frac{X_S^{LAT\ 2}}{X_S^{\exp\ 2}}$$

for $(\pi, N)$, ... determines the $SU(3)$ flavour symmetric starting point $\kappa_0$ and the scale $a^2$

- $\beta_0 = 5.80$

$$\frac{a^2(\beta)}{a^2(\beta_0)} = \left(\frac{\beta_0}{\beta}\right)^{-\frac{b_1}{b_0^2}} \exp\left(-\frac{1}{10 b_0}(\beta - \beta_0)\right)$$

- $a \sim 0.052, 0.059, 0.068, 0.074,$ $[0.082]$ fm

## Reaching the charm quark mass range

- Unitary range rather small so introduce PQ partially quenching (ie valence quark masses $\neq$ sea quark masses) and NNLO
- Furthermore can generalise to different valence quark masses, $\mu_q$ to sea quark masses $m_q$ without increasing number of expansion coefficients

$$\delta\mu_q = \mu_q - \overline{m}$$

- Meson octet

Useful: $\tilde{M}^2 = M^2/X_\pi^2$; $\tilde{\alpha}(\overline{m}) = \alpha/M_{0\pi}^2, \ldots$

$$\begin{aligned}\tilde{M}^2(a\bar{b}) &= 1 + \tilde{\alpha}(\delta\mu_a + \delta\mu_b) \\ &\quad -(\tfrac{2}{3}\tilde{\beta}_1 + \tilde{\beta}_2)(\delta m_u^2 + \delta m_d^2 + \delta m_s^2) + \tilde{\beta}_1(\delta\mu_a^2 + \delta\mu_b^2) + \tilde{\beta}_2(\delta\mu_a - \delta\mu_b)^2 + \ldots\end{aligned}$$

- Baryon octet

Useful: $\tilde{M}^2 = M^2/X_N^2$; $\tilde{A}_i = A_i/M_{0N}^2, \ldots$

$$\begin{aligned}\tilde{M}_\Sigma^2(aab) &= 1 + \tilde{A}_1(2\delta\mu_a + \delta\mu_b) + \tilde{A}_2(\delta\mu_b - \delta\mu_a) \\ &\quad -(\tilde{B}_1 + \tilde{B}_3)(\delta m_u^2 + \delta m_d^2 + \delta m_s^2) + \tilde{B}_1(2\delta\mu_a^2 + \delta\mu_b^2) + \tilde{B}_2(\delta\mu_b^2 - \delta\mu_a^2) + \tilde{B}_3(\delta\mu_b - \delta\mu_a)^2 + \ldots \\ \tilde{M}_\Lambda^2(aa'b) &= 1 + \tilde{A}_1(2\delta\mu_a + \delta\mu_b) - \tilde{A}_2(\delta\mu_b - \delta\mu_a) \\ &\quad -(\tilde{B}_1 + \tilde{B}_3)(\delta m_u^2 + \delta m_d^2 + \delta m_s^2) + \tilde{B}_1(2\delta\mu_a^2 + \delta\mu_b^2) - \tilde{B}_2(\delta\mu_b^2 - \delta\mu_a^2) + \tilde{B}_4(\delta\mu_b - \delta\mu_a)^2 + \ldots\end{aligned}$$

- mixed sea/valence mass terms
- unitary limit: $\delta\mu_q \to \delta m_q$

## Method

- Use PQ data to determine expansion coefficients
  - $\tilde{\alpha}$, $\tilde{\beta}$, $\tilde{\gamma}$ – meson octet
  - $\tilde{A}$, $\tilde{B}$, $\tilde{C}$ – baryon octet
- Determine 'physical' quark masses

$$\underbrace{\delta m_u^*, \delta m_d^*}_{\delta m_l^*}, \quad \delta m_s^*, \quad \delta \mu_c^*$$

by fitting to (eg) [FLAG3 where possible, minimise isospin effects]

$$M_\pi^{\exp}, \quad M_K^{\exp}, \quad M_{\eta_c}^{\exp}(c\bar{c})$$

- Can then determine open charmed states

## $\kappa_0$ (fine) tuning

- If miss (slightly) starting point on $SU(3)$ flavour symmetric line
- Tune using PQ results so that get physical values of (say)

$$M_\pi^{\exp} \qquad X_N^{\exp} \qquad M_K^{\exp}$$

  correct
- Gives

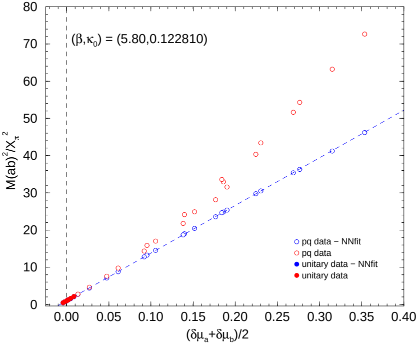
$$\delta\mu_l^* \qquad \delta\mu_s^*$$

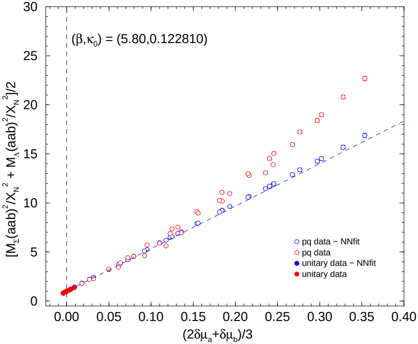
- Philosophy: most change is due to change in valence quark mass, rather than sea quark mass $\delta\overline{\mu} \equiv (2\delta\mu_l^* + \delta\mu_s^*)/3 \neq 0$ necessarily
- Modifying dimensionless appropriately gives

$$\tilde{M}^2(a\overline{b}) \to \tilde{M}^2(a\overline{b})/c \qquad c = \frac{X_\pi^{LAT\,2}}{X_N^{LAT\,2}} \Big/ \frac{X_\pi^{\exp\,2}}{X_N^{\exp\,2}}$$

- $c = 1 + O(1\%)$, ie typically a $\sim 1\%$ effect

# Determining the expansion coefficients



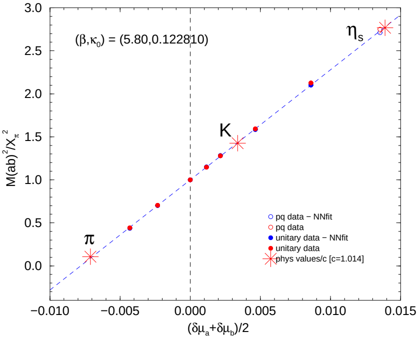



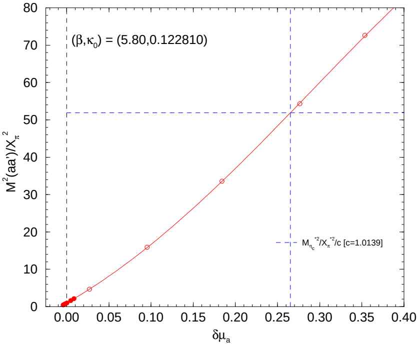

- PQ+unitary data
- At LO, $\tilde{M}(a\bar{b})^2$ only function of $\delta\mu_a + \delta\mu_b$

- PQ+unitary data
- At LO, $(\tilde{M}_\Sigma(aab)^2 + \tilde{M}_\Lambda(aab)^2)/2$ only function of $2\delta\mu_a + \delta\mu_b$

- To compare to fit, subtract NN terms, LO remains
- Very different $x$-regions (for unitary/PQ data)

# Determining ‘physical’ quark masses




- blow-up of previous graph
- no visible curvature
- $\pi$, $K$ to determine $\delta\mu_l^*$, $\delta\mu_s^*$
- Test: compare determined $\delta\mu_s^*$ with fictional $M_{\eta_s}(s\bar{s}) = 0.6885\,\mathrm{GeV}$ [HPQCD] – little/no $O(a^2)$ effects

- Diagonal terms, with fit
- $\tilde{M}^2_{\eta_c} \Longrightarrow \delta\mu_c^*$

# Open Charm masses

Can describe states with same wavefunction (and hence expansion) as previously used

- pseudoscalar mesons

$$\underbrace{D^0(c\overline{u})\,,D^+(c\overline{d})}_{D(c\bar{l})}\,,\quad D_s^+(c\overline{s})$$

which all have the wavefunction $\mathcal{M} = \overline{q}\gamma_5 c \quad q = u, d, s$

- baryons
  - single open charm ($C = 1$) states

$$\underbrace{\Sigma_c^{++}(uuc)\,,\Sigma_c^0(ddc)}_{\Sigma_c(llc)}\,,\quad \Omega_c^0(ssc)$$

which all have the wavefunction $\mathcal{B} = \epsilon(q^T C\gamma_5 c)q \quad q = u, d, s$

  - double open charm ($C = 2$) states

$$\underbrace{\Xi_{cc}^{++}(ccu)\,,\Xi_{cc}^+(ccd)}_{\Xi_{cc}(ccl)}\quad \Omega_{cc}^+(ccs)$$

which all have the wavefunction $\mathcal{B} = \epsilon(c^T C\gamma_5 q)c \quad q = u, d, s$

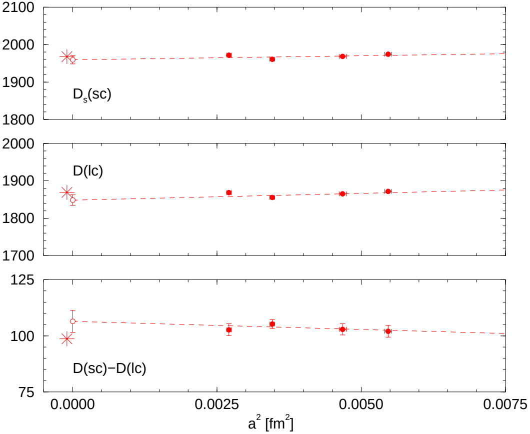
## $SU(4)$ 20-plet

Present status of results

[we presently only have $m_u = m_d = m_l$ results]

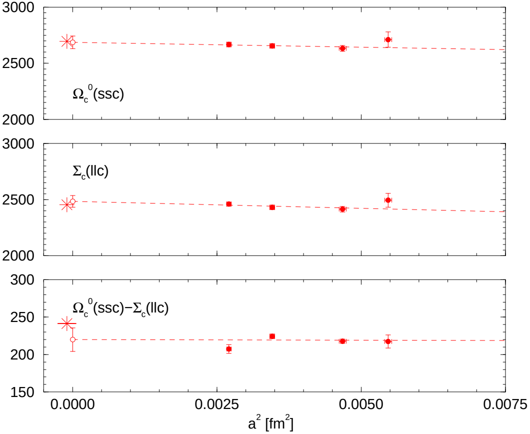
| $C$ | $S$ | $I$ | $I_3$ | baryon | wavefunction |
|---|---|---|---|---|---|
| 0 | 0 | $\frac{1}{2}$ | $+\frac{1}{2}$ | $p$ | $\epsilon(u^T C\gamma_5 d)u$ |
| 0 | 0 | $\frac{1}{2}$ | $-\frac{1}{2}$ | $n$ | $\epsilon(d^T C\gamma_5 u)d$ |
| 0 | 1 | 1 | $+1$ | $\Sigma^+$ | $\epsilon(u^T C\gamma_5 s)u$ |
| 0 | 1 | 1 | 0 | $\Sigma^0$ | $\frac{1}{\sqrt{2}}\epsilon[(u^T C\gamma_5 s)d + (d^T C\gamma_5 s)u]$ |
| 0 | 1 | 1 | $-1$ | $\Sigma^-$ | $\epsilon(d^T C\gamma_5 s)d$ |
| 0 | 2 | $\frac{1}{2}$ | $+\frac{1}{2}$ | $\Xi^0$ | $\epsilon(s^T C\gamma_5 u)s$ |
| 0 | 2 | $\frac{1}{2}$ | $-\frac{1}{2}$ | $\Xi^-$ | $\epsilon(s^T C\gamma_5 d)s$ |
| 0 | 1 | 0 | 0 | $\Lambda^0$ | $\frac{1}{\sqrt{6}}\epsilon[2(u^T C\gamma_5 d)s + (u^T C\gamma_5 s)d - (d^T C\gamma_5 s)u]$ |
| 1 | 0 | 1 | $+1$ | $\Sigma_c^{++}$ | $\epsilon(u^T C\gamma_5 c)u$ |
| 1 | 0 | 1 | 0 | $\Sigma_c^+$ | $\frac{1}{\sqrt{2}}\epsilon[(u^T C\gamma_5 c)d + (d^T C\gamma_5 c)u]$ |
| 1 | 0 | 1 | $-1$ | $\Sigma_c^0$ | $\epsilon(d^T C\gamma_5 c)d$ |
| 1 | 1 | $\frac{1}{2}$ | $+\frac{1}{2}$ | $\Xi_c'^+$ | $\frac{1}{\sqrt{2}}\epsilon[(s^T C\gamma_5 c)u + (u^T C\gamma_5 c)s]$ |
| 1 | 1 | $\frac{1}{2}$ | $-\frac{1}{2}$ | $\Xi_c'^0$ | $\frac{1}{\sqrt{2}}\epsilon[(s^T C\gamma_5 c)d + (d^T C\gamma_5 c)s]$ |
| 1 | 2 | 0 | 0 | $\Omega_c^0$ | $\epsilon(s^T C\gamma_5 c)s$ |
| 1 | 0 | 0 | 0 | $\Lambda_c^+$ | $\frac{1}{\sqrt{6}}\epsilon[2(u^T C\gamma_5 d)c + (u^T C\gamma_5 c)d - (d^T C\gamma_5 c)u]$ |
| 1 | 1 | $\frac{1}{2}$ | $+\frac{1}{2}$ | $\Xi_c^+$ | $\frac{1}{\sqrt{6}}\epsilon[2(s^T C\gamma_5 u)c + (s^T C\gamma_5 c)u - (u^T C\gamma_5 c)s]$ |
| 1 | 1 | $\frac{1}{2}$ | $-\frac{1}{2}$ | $\Xi_c^0$ | $\frac{1}{\sqrt{6}}\epsilon[2(s^T C\gamma_5 d)c + (s^T C\gamma_5 c)d - (d^T C\gamma_5 c)s]$ |
| 2 | 0 | $\frac{1}{2}$ | $+\frac{1}{2}$ | $\Xi_{cc}^{++}$ | $\epsilon(c^T C\gamma_5 u)c$ |
| 2 | 0 | $\frac{1}{2}$ | $-\frac{1}{2}$ | $\Xi_{cc}^+$ | $\epsilon(c^T C\gamma_5 d)c$ |
| 2 | 1 | 0 | 0 | $\Omega_{cc}^+$ | $\epsilon(c^T C\gamma_5 s)c$ |

# Charmed pseudoscalar mesons



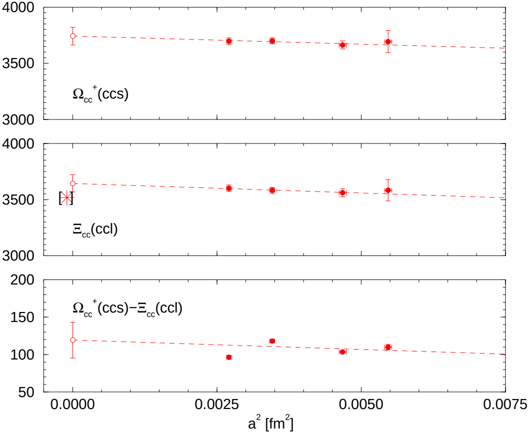

- $D_s(c\bar{s})$ [ie $D_s^+(c\bar{s})$], $D(c\bar{l})$ [ie $D^0(c\bar{u})$, $D^+(c\bar{d})$] [+Dashen to minimise QED effects]
- $D(c\bar{s}) - D(c\bar{l})$, ie $SU(3)$ flavour splitting effects
- very small lattice artifacts

# Charmed $C = 1$ baryons



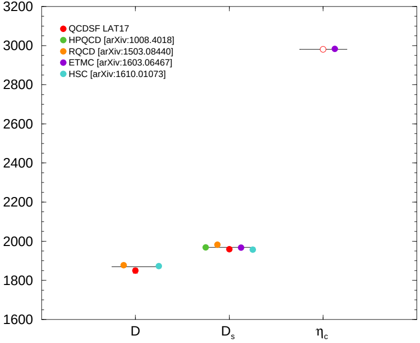

- $\Omega_c(ssc)$, [ie $\Omega_c^0(ssc)$], $\Sigma_c(llc)$ [ie $\Sigma_c^{++}(uuc)$, $\Sigma_c^0(ddc)$]
- $\Omega_c(ssc) - \Sigma_c(llc)$, ie $SU(3)$ flavour splitting effects
- very small lattice artifacts

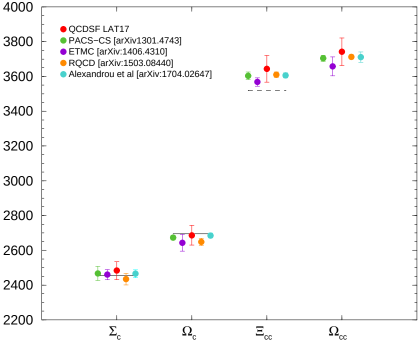
# Charmed $C = 2$ baryons




- $\Omega_{cc}(ccs)$, [ie $\Omega_{cc}^{+}(ccs)$], $\Xi_{cc}(ccl)$ [ie $\Xi_{cc}^{++}(ccu)$, $\Xi_{cc}^{+}(ccd)$]
- $\Omega_{cc}(ccs) - \Xi_{cc}(ccl)$, ie $SU(3)$ flavour splitting effects [Inflate final error bar]
- [$\star$] SELEX
- very small lattice artifacts

# Comparison with (some) other results



## Conclusions

- For $u$, $d$, $s$ quarks, have developed a method to approach the physical point
- Precise $SU(3)$ flavour symmetry breaking expansions – nothing ad-hoc
- Extend expansions – PQ (mass valence quarks $\neq$ mass sea quarks) to
  - better determine expansion coefficients
  - determine c quark mass
- Applied method to determine some open charm states.
  - $O(a^2)$ extrapolations rather delicate
- Future:
  - Immediate: $\Lambda_c$, $N$-octet
  - Extend to $m_u \neq m_d$
  - mixing:
    in a $2+1$ world no $\Sigma^0$ - $\Lambda^0$ mixing, but determined coefficients can be used to determine $\Sigma^0(uds)$ - $\Lambda^0(uds)$ mixing
    generalise to eg $\Sigma_c^+$ - $\Lambda_c^+$, $\Xi_c^0$ - $\Xi_c'^0$ mixing
  - baryon decuplet
  - QED effects