

Numerical investigation of QED finite-volume effects for meson mass and HVP

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Introduction

- We have made an exploratory calculation of isospin-breaking corrections to the HVP

[V. Guelpers, Thursday, 17:30; arXiv:1706.05293].

- QED finite volume effects (FVE) must be taken into account for a physical calculation of the HVP.
- 2-loop analytical calculation of HVP FVE has not yet been carried out.
- We use lattice scalar QED, as a quicker numerical calculation of FVE and as a cross-check for the analytical result.
- Cross-checked method against known results for meson mass FVE.
- Preliminary results for FV behaviour of HVP.

Isospin breaking

- Isospin-breaking effects enter at $\mathcal{O}\left(\alpha, \frac{m_u - m_d}{\Lambda_{QCD}}\right) \sim 1\%$
- Precision in QCD now approaching 1% for several quantities.
 - e.g. hadronic contribution to muon $g - 2$: 1% precision or better required to compete with determination from $e^+ e^- \rightarrow$ hadrons, and to prepare for upcoming experiments at Fermilab and J-PARC.
- Isospin is significant systematic uncertainty
- Electromagnetism and light quark mass difference need to be included.

QED in finite volume

- For sufficiently large volume, QCD FVE are exponentially suppressed.
- QED is long-range due to massless photon, so QED FVE are much larger than for QCD.
- Typically scale with inverse powers of L , not exponential.
- QED FVE can be comparable in magnitude to QED corrections [[Borsanyi et al., arXiv:1406.4088](#)].
- FVE must be accounted for in any lattice calculation involving QED.

QED finite volume effects

- FVE are not very sensitive to higher modes (discretisation, hadron structure).
- Can study these analytically using an effective theory, with point-like hadrons
 - Scalar QED (pseudoscalar mesons)
 - Spinor QED (spin-1/2 baryons)
- In momentum space, integrals become discrete sums in finite volume.
- Derive relations between finite-volume quantities and their infinite-volume counterparts.
- FVE have been calculated for hadron masses [Davoudi & Savage, arXiv:1402.6741; Borsanyi et al., arXiv:1406.4088] and leptonic decay amplitudes [Lubicz et al., arXiv:1611.08497].

QED finite volume effects

Alternative: lattice scalar QED.

- Generate $U(1)$ gauge configurations
- Calculate scalar propagators and expectation values
- Repeat for several volumes
- Fit polynomial in $1/L$ to extract coefficients

Offers a quick numerical method for obtaining FVE, which is generally applicable to a wide range of observables.

Scalar QED on the lattice

Discretised scalar QED action:

$$\begin{aligned}S(\phi, A_\mu) &= \frac{a^4}{2} \sum_x \phi^*(x) \Delta \phi(x) + S_\gamma(A_\mu) \\ \Delta &= - \sum_\mu D_\mu^* D_\mu + m^2 \\ D_\mu f(x) &= a^{-1} \left[e^{ieaA_\mu(x)} f(x + a\hat{\mu}) - f(x) \right]\end{aligned}$$

Quenched theory: set scalar determinant = 1 in path integral.

Sampling gauge field configurations

Non-compact photon action:

$$\mathcal{S}_\gamma(A_\mu) = \frac{a^4}{4} \sum_x \sum_{\mu,\nu} (\partial_\mu A_\nu - \partial_\nu A_\mu)^2$$

$$\partial_\mu f(x) = a^{-1} [f(x + a\hat{\mu}) - f(x)]$$

- Feynman gauge.
- In momentum space, $\tilde{A}_\mu(k)$ is Gaussian - cheap to sample gauge configurations [Duncan, Eichten & Thacker, arXiv:hep-lat/9602005].
- Subtract zero mode using QED_L scheme [Uno & Hayakawa, arXiv:0804.2044].

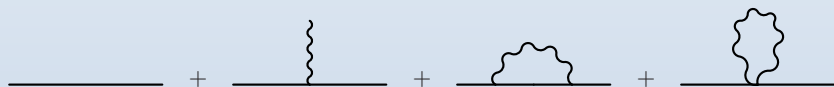
Calculating the scalar propagator

Alternative to CG, making use of FFT.

Expand scalar propagator around $e = 0$:

$$\Delta = - \sum_{\mu} D_{\mu}^* D_{\mu} + m^2 = \Delta_0 + e\Delta_1 + e^2\Delta_2 + \mathcal{O}(e^3)$$

$$\begin{aligned} \Delta^{-1} = & \Delta_0^{-1} - e\Delta_0^{-1}\Delta_1\Delta_0^{-1} \\ & + e^2 \left[\Delta_0^{-1}\Delta_1\Delta_0^{-1}\Delta_1\Delta_0^{-1} - \Delta_0^{-1}\Delta_2\Delta_0^{-1} \right] + \mathcal{O}(e^3) \end{aligned}$$



Calculating the scalar propagator

$$\Delta_0^{-1} = \mathcal{F}^{-1} \frac{1}{\hat{k}^2 + m^2} \mathcal{F}$$

$$\Delta_0^{-1} \Delta_1 \Delta_0^{-1} = -ia^{-1} \sum_{\mu} \mathcal{F}^{-1} \left[\frac{1}{\hat{k}^2 + m^2} \mathcal{F} A_{\mu} \mathcal{F}^{-1} \frac{e^{iak_{\mu}}}{\hat{k}^2 + m^2} - \frac{e^{-iak_{\mu}}}{\hat{k}^2 + m^2} \mathcal{F} A_{\mu} \mathcal{F}^{-1} \frac{1}{\hat{k}^2 + m^2} \right] \mathcal{F}$$

$$\Delta_0^{-1} \Delta_2 \Delta_0^{-1} = \frac{1}{2} \sum_{\mu} \mathcal{F}^{-1} \left[\frac{1}{\hat{k}^2 + m^2} \mathcal{F} A_{\mu}^2 \mathcal{F}^{-1} \frac{e^{iak_{\mu}}}{\hat{k}^2 + m^2} - \frac{e^{-iak_{\mu}}}{\hat{k}^2 + m^2} \mathcal{F} A_{\mu}^2 \mathcal{F}^{-1} \frac{1}{\hat{k}^2 + m^2} \right] \mathcal{F}$$

where \mathcal{F} represents the Fourier transform.

Finite volume corrections - meson mass

Scalar mass FV corrections are known:

$$m^2(L) \sim m_\infty^2 - \alpha \frac{\kappa}{L} \left(m_0 + \frac{2}{L} \right)$$

$$\kappa = 2.837297(1)$$

in QED_L, up to terms exponentially suppressed in $m_0 L$
[Borsanyi et al., arXiv:1406.4088]. m_0 is bare mass, m_∞ is infinite-volume mass.

We can use this as a validity check of our method.

Finite volume corrections - meson mass

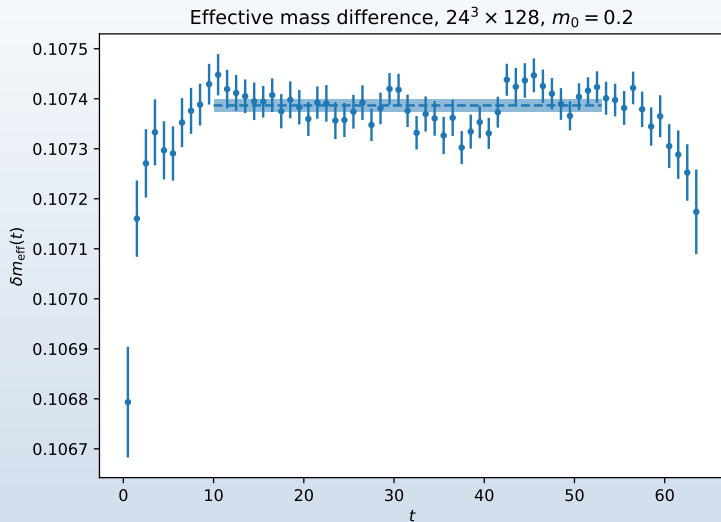
We define an effective mass difference from ratio of charged and free propagators (neglect backward propagating states for simplicity of presentation):

$$\delta m_{\text{eff}}(t) = \frac{\Delta^{-1}(t, \vec{0})}{\Delta_0^{-1}(t, \vec{0})} - \frac{\Delta^{-1}(t+1, \vec{0})}{\Delta_0^{-1}(t+1, \vec{0})}$$

Can compare to perturbation theory:

$$\frac{\Delta^{-1}(t, \vec{0})}{\Delta_0^{-1}(t, \vec{0})} = c + te^2 \left(\frac{T + \Sigma}{2m_0} \right)$$

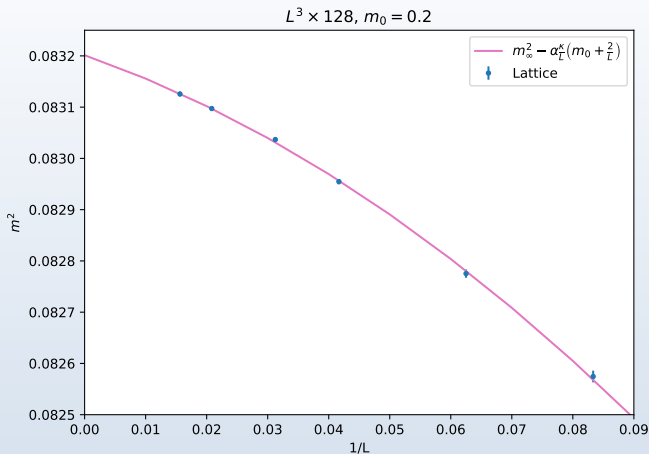
Finite volume corrections - meson mass



Finite volume corrections - meson mass

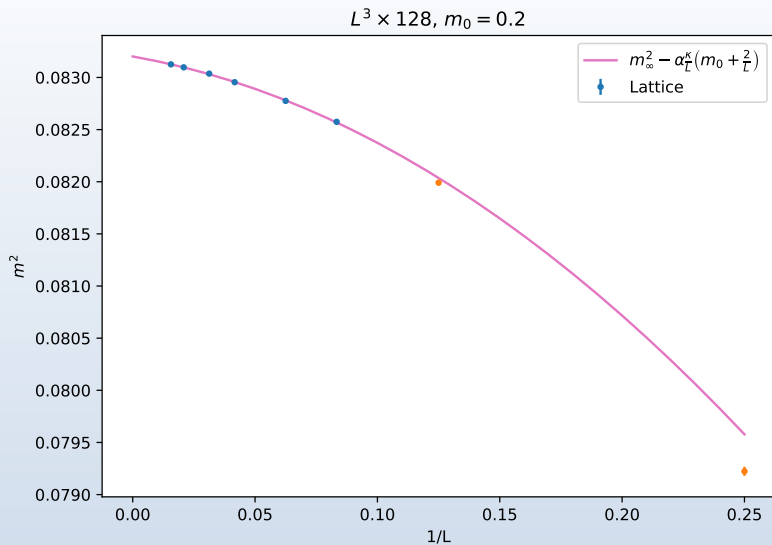
- Eight volumes, from $L = 4$ to $L = 64$
- $T = 128$ for all L
- 100 configurations per volume
- Computed on a single KNL
- Largest volume took ~ 1 day

Finite volume corrections - meson mass



Good agreement between lattice and analytical:
 $\chi^2/\text{d.o.f.} = 0.44$ (note: this is not a fit).

Finite volume corrections - meson mass



Finite volume corrections - HVP

Leading effective field theory contribution to HVP is scalar bubble diagram.



Vacuum polarisation tensor:

$$C_{\mu\nu}(x) = \langle V_\mu(x) V_\nu(0) \rangle$$

$$\Pi_{\mu\nu}(Q) = a^4 \sum_x e^{-iQ \cdot x} C_{\mu\nu}(x) - a^4 \sum_x C_{\mu\nu}(x)$$

$$\Pi_{\mu\nu}(Q) = (\delta_{\mu\nu} \hat{Q}^2 - \hat{Q}_\mu \hat{Q}_\nu) \Pi(\hat{Q}^2)$$

Finite volume corrections - HVP

Conserved vector current:

$$V_\mu(x) = a^2 \left[\phi^*(x) e^{ieaA_\mu(x)} \phi(x + a\hat{\mu}) - \phi^*(x + a\hat{\mu}) e^{-ieaA_\mu(x)} \phi(x) \right]$$

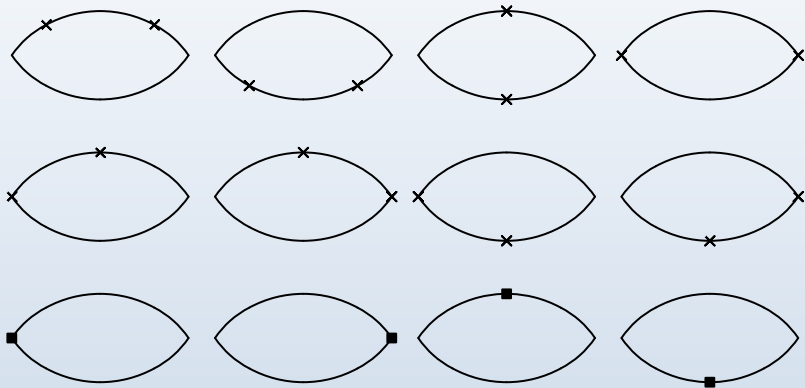
Contact term at $x = 0$ due to non-local currents:

$$T_\mu(x) = \phi^*(x) e^{ieaA_\mu(x)} \phi(x + a\hat{\mu}) + \phi^*(x + a\hat{\mu}) e^{-ieaA_\mu(x)} \phi(x)$$

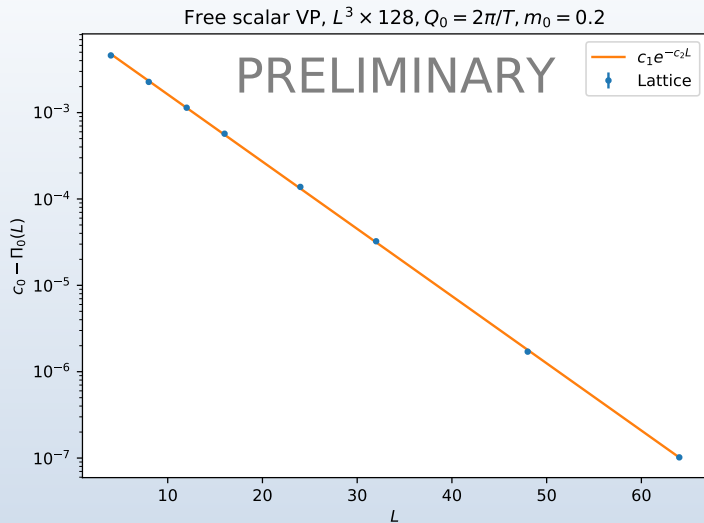
Contact term is removed by subtracting zero-mode $\tilde{\Pi}_{\mu\nu}(0)$.

Finite volume corrections - HVP

$\mathcal{O}(\alpha)$ contributions to scalar VP:

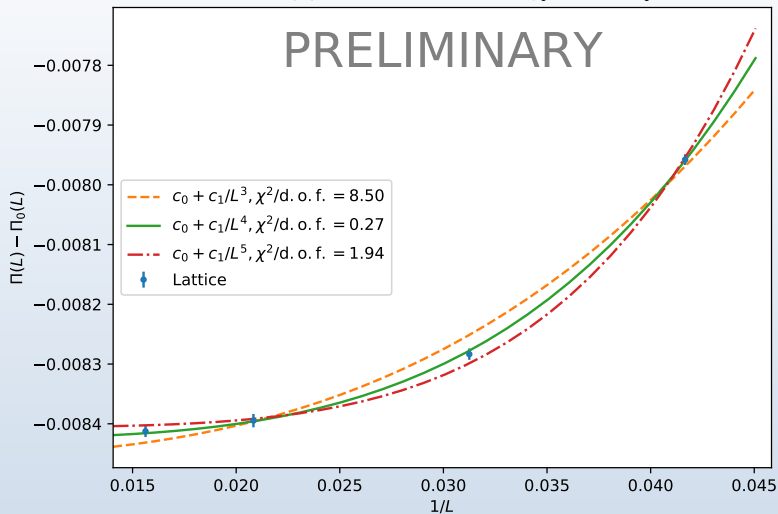


Finite volume corrections - HVP



Finite volume corrections - HVP

Scalar VP $\mathcal{O}(\alpha)$ correction, $L^3 \times 128, Q_0 = 2\pi/T, m_0 = 0.2$



Summary

- We want to calculate QED FVE for the HVP.
- As a first step before analytical calculation, we use lattice scalar QED to calculate FVE
- Simulations are very cheap (~ 1 day on a single KNL)
- We successfully reproduce known results for meson mass FVE
- Preliminary data suggest leading QED FV behaviour of HVP may be $O(1/L^4)$
- This technique is more generally applicable to a wide range of quantities

Acknowledgements

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