Auxiliary field approach to extended operators for quasi-PDFs

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Outline

1. Quasi-PDFs
2. Auxiliary field formalism: continuum
3. Auxiliary field formalism: lattice
4. Relation to static quark theory
5. Non-perturbative renormalization
6. Effect on quasi-PDF data

Parton distribution functions and quasi-PDFs

Parton distribution functions (PDFs):

- $q(x, \mu)$, $g(x, \mu)$ describe probability of finding a quark or gluon with momentum fraction x of a proton's total momentum.

Quasi-PDFs: X. Ji, Phys. Rev. Lett. **110**, 262002 [1305.1539]; many follow-up papers

- Idea: define a quasi-PDF $\tilde{q}(x, \mu, p_z)$ with $p_z = p \cdot n$, using nucleon matrix elements of a nonlocal operator where ψ and $\bar{\psi}$ have spacelike separation in direction n. At large p_z:

$$\tilde{q}(x, \mu, p_z) = \int_x^1 \frac{dy}{y} Z\left(\frac{x}{y}, \frac{\mu}{p_z}\right) q(x, \mu) + O\left(\frac{\Lambda_{\text{QCD}}^2}{p_z^2}, \frac{m_p^2}{p_z^2}\right).$$

- Boost so that n is pointing in a purely spatial direction. This makes it suitable for computing on the lattice. → plenary talk by L. Del Debbio

Operator for quark quasi-PDFs

We compute nucleon matrix elements of the operator

$$\mathcal{O}_\Gamma(x, \xi, n) \equiv \bar{\psi}(x + \xi n)\Gamma W(x + \xi n, x)\psi(x),$$

where $n^2 = 1$ is a unit vector, ξ is the separation, and W is a Wilson line:

$$W(x + \xi n, x) \equiv \mathcal{P} \exp\left(-ig \int_0^\xi d\xi' \, n \cdot A(x + \xi' n)\right).$$

On the lattice we can restrict n to point along an axis, and simply form W from a product of gauge links.

This is a non-local operator. How can we understand its renormalization?

Auxiliary field approach

(Loosely following H. Dorn, Fortsch. Phys. **34**, 11 (1986))
The Wilson line satisfies the equation of motion

$$\left[\frac{d}{d\xi} + ign\cdot A(x+\xi n)\right] W(x+\xi n, x) = \delta(\xi).$$

Introduce a scalar, color triplet field $\zeta_n(\xi)$ that is defined on the line $x+\xi n$.
(We omit the subscript n most of the time.) Give it the action

$$S_\zeta = \int d\xi\, \bar{\zeta} \left[\frac{d}{d\xi} + ign\cdot A + m\right] \zeta.$$

Then its propagator for fixed gauge background is

$$\left\langle \zeta(\xi_2)\bar{\zeta}(\xi_1)\right\rangle_\zeta = \theta(\xi_2-\xi_1) W(x_2,x_1) e^{-m(\xi_2-\xi_1)}$$

We want zero mass but there is no symmetry that forbids it. Unless we use dimensional regularization, a power-divergent counterterm is needed.

Auxiliary field approach: quark operator

Introduce the spinor-valued color singlet ζ-quark bilinear

$$\phi \equiv \bar{\zeta}\psi.$$

Then the extended operator for quasi-PDFs is given (for $m = 0$ and $\xi > 0$) by

$$\mathcal{O}_\Gamma(x, \xi, n) = \left\langle \bar{\phi}(x + \xi n)\Gamma\phi(x) \right\rangle_\zeta .$$

For $\xi < 0$, we can use the relation

$$\mathcal{O}_\Gamma(x, \xi, n) = \mathcal{O}_\Gamma(x, -\xi, -n).$$

Thus, any QCD correlator involving $\mathcal{O}_\Gamma$ can be rewritten as a correlator in QCD+ζ involving the *local operators* ϕ and $\bar{\phi}$. To renormalize this, we need:

1. Z_ϕ to renormalize the local operators.
2. The mass counterterm.

Auxiliary field on the lattice

Discretize S_ζ, restricting n to be $n = \pm\hat{\mu}$:

$$S_\zeta = a \sum_\xi \frac{1}{1 + am_0} \bar{\zeta}(x + \xi n)[\nabla_n + m_0]\zeta(x + \xi n),$$

where

$$\nabla_n = \begin{cases} n \cdot \nabla^* = \nabla^*_\mu, & \text{if } n = \hat{\mu} \\ n \cdot \nabla = -\nabla_\mu, & \text{if } n = -\hat{\mu} \end{cases}.$$

For $n = \hat{\mu}$, this yields the propagator

$$\left\langle \zeta(x + \xi\hat{\mu})\bar{\zeta}(x) \right\rangle_\zeta = \theta(\xi) e^{-m\xi} U^\dagger_\mu\Big(x + (\xi - a)\hat{\mu}\Big) U^\dagger_\mu\Big(x + (\xi - 2a)\hat{\mu}\Big) \ldots U^\dagger_\mu(x),$$

where $m = a^{-1}\log(1 + am_0)$.
(We could use smeared links U in defining the covariant derivative.)

Auxiliary field on the lattice: renormalization and mixing

Mixing on the lattice first noted at one loop in

M. Constantinou and H. Panagopoulos, 1705.11193 → talk at 17:10.

In our approach, this appears as mixing between ϕ and $\not{n}\phi$ when chiral symmetry is broken. The ζ-quark bilinear $\phi = \bar{\zeta}\psi$ renormalizes as

$$\phi_R = Z_\phi\Big(\phi + r_{\text{mix}}\not{n}\phi\Big), \qquad \bar{\phi}_R = Z_\phi\left(\bar{\phi} + r_{\text{mix}}\bar{\phi}\not{n}\right).$$

We can use $P_\pm \equiv \frac{1}{2}(1 \pm \not{n})$ to define operators that don't mix:

$$\phi^\pm \equiv P_\pm\phi \implies \phi_R^\pm = Z_\phi^\pm\phi^\pm, \quad \text{where } Z_\phi^\pm = Z_\phi(1 \pm r_{\text{mix}}).$$

The renormalized extended quark bilinear has the form

$$O_\Gamma^R(x,\xi,n) = Z_\phi^2 e^{-m|\xi|}\bar{\psi}(x+\xi n)\Gamma' W(x+\xi n, x)\psi(x),$$
$$\text{where } \Gamma' = \Gamma + \operatorname{sgn}(\xi) r_{\text{mix}}\{\not{n},\Gamma\} + r_{\text{mix}}^2\not{n}\Gamma\not{n}.$$

Relation to static quark theory

The Lagrangian for a static quark on the lattice is

$$\mathcal{L}(x) = \frac{1}{1+am_0}\bar{Q}(x)\left[\nabla_0^* + m_0\right]Q(x),$$

where Q is a color triplet spinor satisfying $\frac{1}{2}(1+\gamma_0)Q = Q$.
Other than the spin degres of freedom (which don't couple in the action) this is the same as for ζ with $n = \hat{0}$. The propagators are also related:

$$\left\langle Q(x)\bar{Q}(y)\right\rangle_Q = \left\langle \zeta(x)\bar{\zeta}(y)\right\rangle_\zeta P_+.$$

In the continuum, the connection between renormalization of quasi-PDFs and the static quark theory was discussed in

X. Ji and J.-H. Zhang, Phys. Rev. D **92**, 034006 [1505.07699].

With broken chiral symmetry, there are two renormalization factors for static-light bilinears:

$$Z_V^{\text{stat}} \text{ for } \bar{\psi}\gamma_0 Q \quad \text{and} \quad Z_A^{\text{stat}} \text{ for } \bar{\psi}\gamma_0\gamma_5 Q.$$

Inserting P_+, we identify $Z_V^{\text{stat}} = Z_\phi^+$ and $Z_A^{\text{stat}} = Z_\phi^-$.

Further implications

1. Lattice artifacts are $O(a)$.
   Even with chiral symmetry, the static-light currents need improvement at $O(a)$: e.g.

$$A_0^{\mathrm{stat},I} = \bar{\psi}\gamma_0\gamma_5 Q + a c_A^{\mathrm{stat}} \bar{\psi}\gamma_j\gamma_5 \frac{1}{2}\left(\overleftarrow{\nabla}_j + \overleftarrow{\nabla}^*_j\right) Q.$$

2. No mixing with gluons.
   - The local bilinear $\phi = \bar{\zeta}\psi$ is in the flavor fundamental irrep.
     The corresponding gluon operator is flavor singlet.
   - Mixing between quark and gluon PDFs must occur in:
     a. the matching from quasi-PDF to PDF,
     b. the dependence of quasi-PDFs on p_z.

Nonperturbative approach

In Landau gauge, compute the position-space ζ propagator

$$S_\zeta(\xi) \equiv \left\langle \zeta(x+\xi n)\bar\zeta(x) \right\rangle_{\mathrm{QCD}+\zeta} = \langle W(x+\xi n, x) \rangle_{\mathrm{QCD}},$$

the momentum-space quark propagator

$$S_q(p) \equiv \sum_x e^{-ip\cdot x} \left\langle \psi(x)\bar\psi(0) \right\rangle,$$

and the mixed-space Green's function for $\phi^\pm$:

$$G^\pm(\xi, p) \equiv \sum_x e^{ip\cdot x} \left\langle \zeta(\xi n)\phi^\pm(0)\bar\psi(x) \right\rangle_{\mathrm{QCD}+\zeta}.$$

These renormalize as

$$S_\zeta^R(\xi) = e^{-m\xi} Z_\zeta S_\zeta(\xi), \quad S_q^R(p) = Z_q S_q(p),$$
$$G_R^\pm(\xi, p) = e^{-m\xi} \sqrt{Z_\zeta Z_q} Z_\phi^\pm G^\pm(\xi, p).$$

Power divergence

Take the effective energy of the ζ propagator:

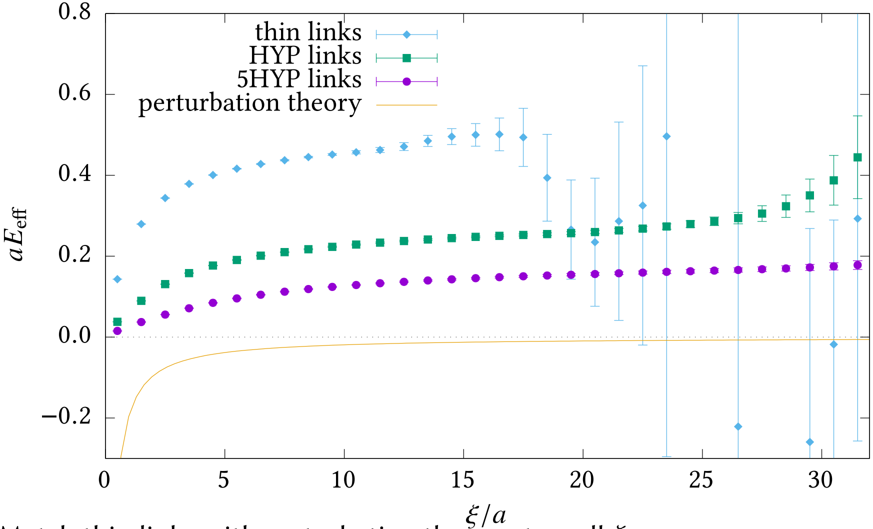
$$E_{\text{eff}}(\xi) \equiv -\frac{d}{d\xi} \log \operatorname{Tr} S_\zeta(\xi).$$

This renormalizes as $E^R_{\text{eff}}(\xi) = m + E_{\text{eff}}(\xi)$. Determine m by matching to perturbation theory at small ξ:

$$E_{\text{eff}}(\xi) = -\frac{3\alpha_s C_F}{2\pi\xi} + O(\alpha_s^2).$$

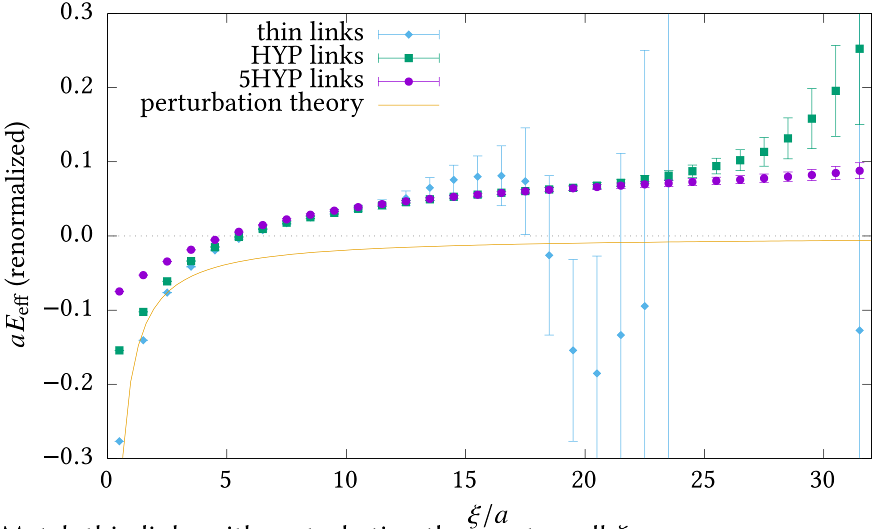
Here we use fixed $\alpha_s = 0.3$.
Preliminary results from an $N_f = 4$ twisted mass ensemble with $\beta = 2.1$, or $a = 0.064$ fm.

Effective energy



Match thin links with perturbation theory at small ξ,
then match thin with smeared links at larger ξ.

Effective energy



Match thin links with perturbation theory at small ξ, then match thin with smeared links at larger ξ.

Get $am_{\text{thin}} = -0.42$, $am_{\text{5HYP}} = -0.09$.

RI-type renormalization scheme

For Z_ζ, we could use a condition

$$\frac{3\,\mathrm{Tr}\,S_\zeta^R(2\xi)}{\left(\mathrm{Tr}\,S_\zeta^R(\xi)\right)^2} = 1.$$

For $\phi^\pm$, "amputate" the Green's function:

$$\Lambda^\pm(\xi,p) \equiv S_\zeta^{-1}(\xi) G^\pm(\xi,p) S_q^{-1}(p).$$

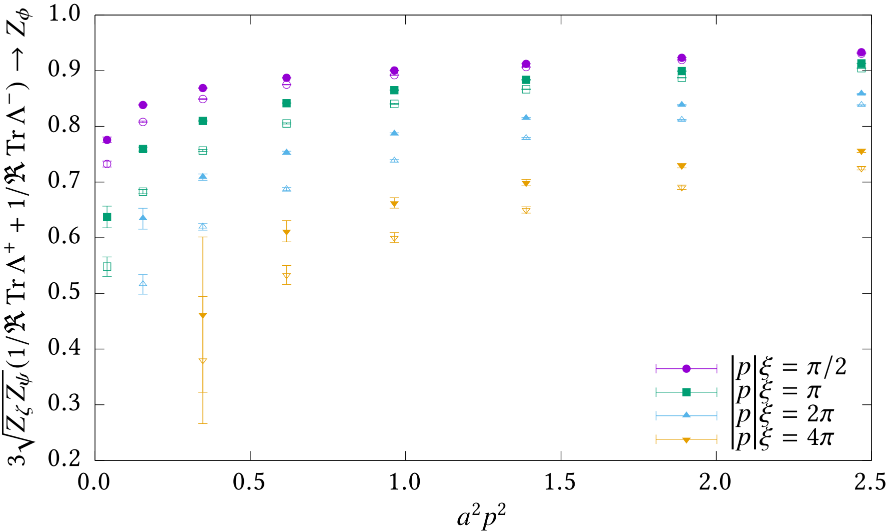
Both of these serve to eliminate the dependence on m.

Then we could impose the condition

$$\frac{1}{6}\Re\,\mathrm{Tr}\,\Lambda_R^\pm(p,\xi) = 1$$

at some scale $\mu^2 = p^2$. This is a two-parameter family of schemes, which depends on the dimensionless parameters $|p|\xi$ and $(n\cdot p)/|p|$.

Estimator for Z_ϕ



solid symbols: $p \parallel n$; open symbols: $p \perp n$.

Matching to $\overline{\mathrm{MS}}$ and evolution to fixed scale still needed.

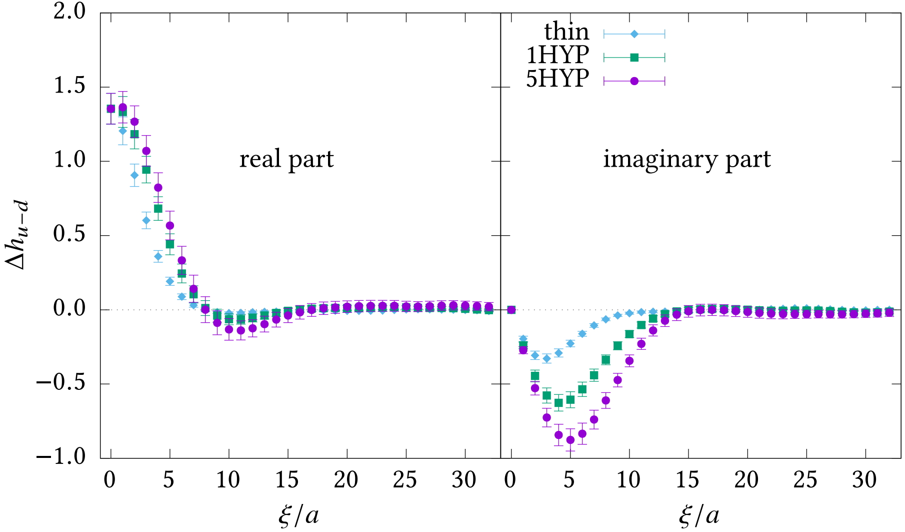
Work is in progress to understand significant $O(a)$ effects in r_{mix}.

Quasi-PDF data on fine ensemble

New calculation on fine $N_f = 2 + 1 + 1$ twisted mass ensemble:

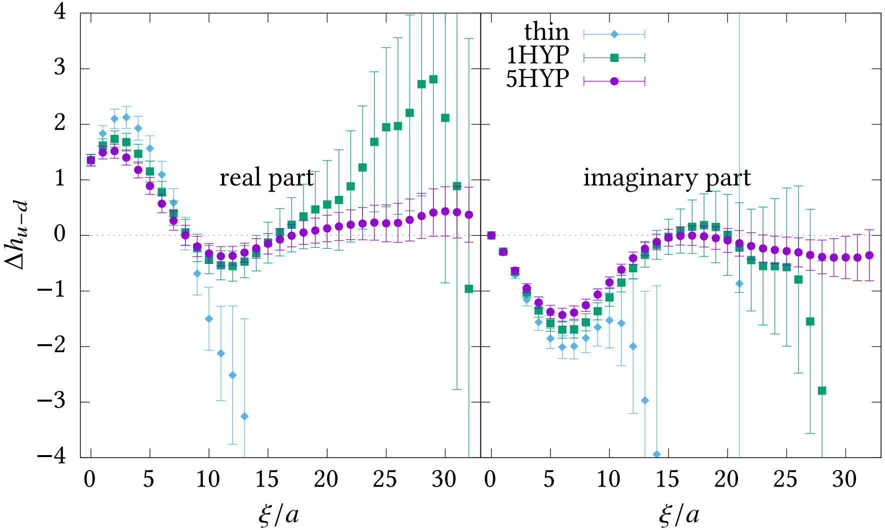
- $\beta = 2.1$, $a \approx 0.065$ fm
- $m_\pi \approx 370$ MeV
- 45 configurations $\times$ 4 source positions
- $p_z \approx 1.8$ GeV, using momentum smearing
- Various smearings applied to the links in the extended operator

Helicity matrix element, bare



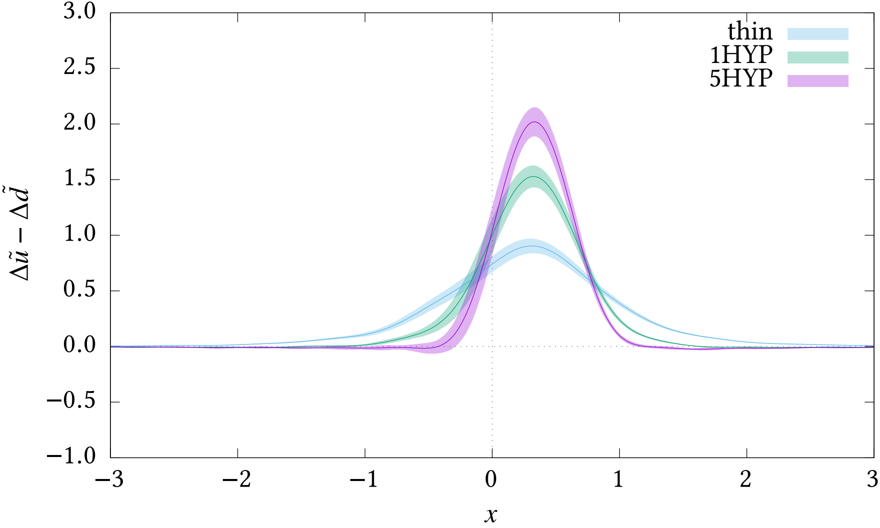
real part is even in ξ; imaginary part is odd

Helicity matrix element, effect of power correction



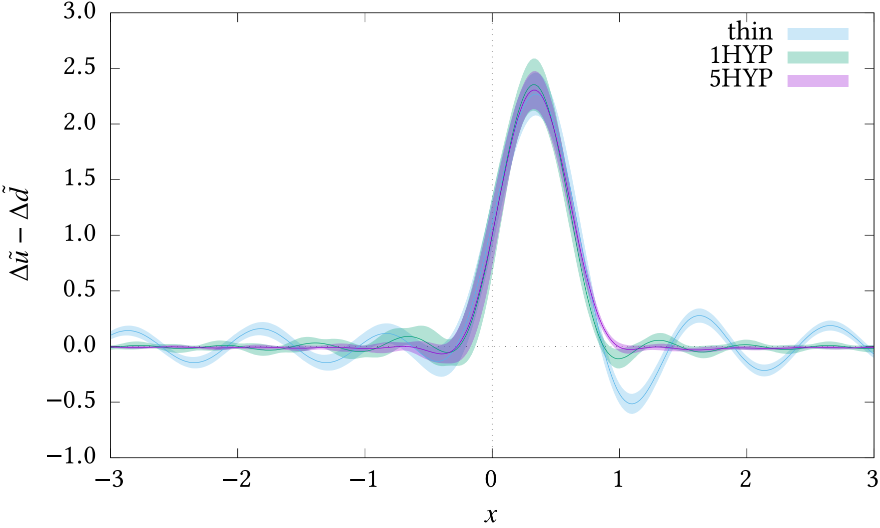
Multiplied by $e^{-m|\xi|}$. Still need ξ-independent factor $Z_\phi^2(1 - r_{\text{mix}}^2)$.

Helicity quasi-PDF, bare



$$\Delta\tilde{q}(x, p_z) = \frac{p_z}{2\pi} \int d\xi \, e^{-ixp_z\xi} \Delta h(p_z, \xi)$$

Helicity quasi-PDF, effect of power correction



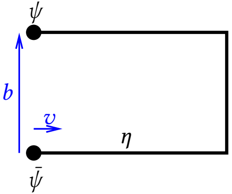
Oscillations caused by cutoff in $|\xi|$.

Summary

- The auxiliary field approach allows us to replace nonlocal operators with local operators in an extended theory.
- We can study renormalization and improvement of the local operators in the usual way.
- This is an alternative to imposing RI-MOM conditions on the extended operator $\mathcal{O}_\Gamma$ to determine $Z_\mathcal{O}(\xi)$. → H. Panagopoulos, 17:10; K. Cichy, 17:30 An advantage is that we can avoid perturbative matching to $\overline{\text{MS}}$ at large ξ.
- Can also be applied to lattice transverse momentum-dependent (TMD) PDFs, where ψ and $\bar{\psi}$ are connected by a staple-shaped gauge link.
- Renormalizing quasi-PDFs on the lattice with broken chiral symmetry requires determining three parameters: m, Z_ϕ, and r_{mix}.
- Z_ϕ and r_{mix} can be determined from Z_A^{stat} and Z_V^{stat} in the static quark theory.
- Determining m brings results with different link smearings into reasonable agreement.

Generalization: staple-shaped gauge link

Transverse momentum-dependent (TMD) PDFs are studied on the lattice using operators with staple-shaped gauge links:



$$\mathcal{O}^{\mathrm{TMD}} = \bar\psi(0)\Gamma W(0,\eta v)W(\eta v,\eta v{+}b)W(\eta v{+}b,b)\psi(b).$$

We introduce the auxiliary fields ζ_v, ζ_{-v}, and $\zeta_{-\hat b}$. Using

1. the ζ-quark bilinear $\phi_n = \bar\zeta_n\psi$,
2. the ζ-ζ "corner" bilinear $C_{n',n} = \bar\zeta_{n'}\zeta_n$,

we obtain

$$\mathcal{O}^{\mathrm{TMD}} = \left\langle \bar\phi_{-v}(0)\Gamma C_{-v,-\hat b}(\eta v) C_{-\hat b,v}(\eta v + b)\phi_v(b)\right\rangle_\zeta .$$

The corner operators also must be renormalized with a factor Z_C.
In this case mixing will occur between TMD operators with Γ and $[\not{v},\Gamma]$.