

Nucleon structure from 2+1 flavor lattice QCD near the physical point

N. Tsukamoto K.-I. Ishikawa^{A,B}
Y. Kuramashi^{B,C} S. Sasaki and T. Yamazaki^{B,C}
for PACS Collaboration

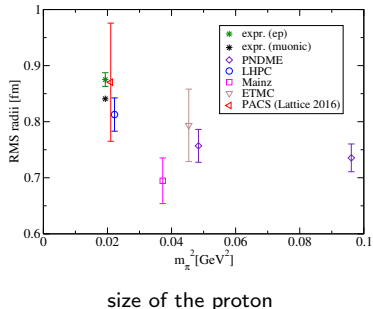
Tohoku Univ., Hiroshima Univ.^A, RIKEN AICS^B, Univ. of Tsukuba^C

June 19, 2017

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Introduction

We access the information on the **nucleon structure** through nucleon **form factors (FFs)** that can be computed from first principles with lattice QCD.



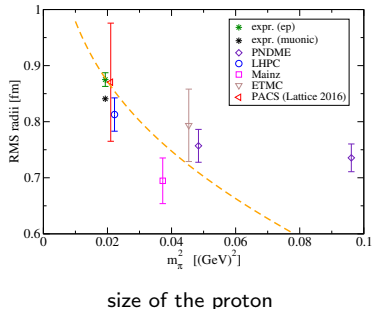
- Lattice results of the electric **charge radius (RMS)** have been often **underestimated**.
- Note that chiral effective field theory predicts that the charge radius has the logarithmic divergence in the **chiral limit**.

Our preliminary result (Lattice 2016)* computed with almost physical pion mass on very large volume shows agreement with experimental results.

*Y. Kuramashi *et al.*, PoS Lattice2016 158 (2017).

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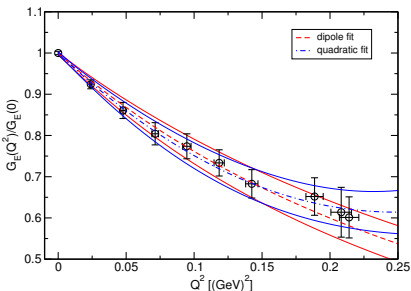
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How can we determine the proton radius?

$$G(t = Q^2) = 1 - \frac{1}{6} \langle r^2 \rangle t + \mathcal{O}(t^2) \quad (1)$$

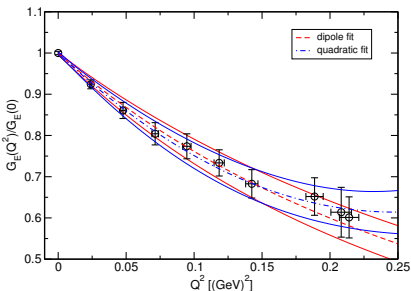


- We can estimate the **root mean squared (RMS) radius** from the slope at zero momentum transfer.
 - **Low Q^2 data point** is crucial for the determination of the **RMS radius**.
- **Large spacial size** is required.
- $$Q^2 = \left(\frac{2\pi}{aL}\right)^2 \times \text{integer}$$

The slope determination is highly sensitive to how we fit the Q^2 -dependence of the form factor.

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polynomial? $G(t) = \sum a_k t^k$ or dipole? $G(t) = \frac{1}{(1+\frac{t}{m})^2}$

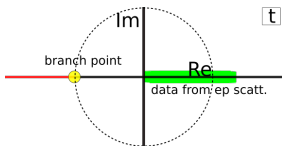
model dependence

Method

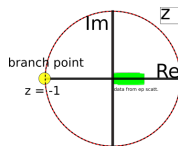
z -expansion method[†] is a model-independent approach.

$$G(t = Q^2) = \sum c_k z^k \quad z = \frac{\sqrt{t_{\text{cut}} + t} - \sqrt{t_{\text{cut}}}}{\sqrt{t_{\text{cut}} + t} + \sqrt{t_{\text{cut}}}} \quad t_{\text{cut}} = 4m_\pi^2 \quad (2)$$

We can avoid **non-analytic** region due to thresholds of two or more particles.



→ **conformal** →
→ **mapping** →



Analytic domain is mapped into the open unit disk ($|z|^2 < 1$).

So the z -expansion series $\sum_0^\infty c_k z^k$ should converge and reduce model-dependence.

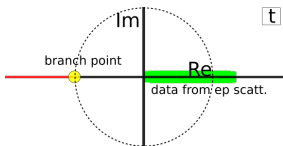
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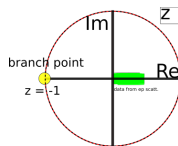
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We perform a rigorous analysis of the Q^2 -dependence of nucleon form factors including very low Q^2 data point, by using z -expansion method.

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Near Physical Point 2 + 1 flavor Lattice QCD Simulation using K computer (PACS Collaboration)[‡]



HPCI Strategic Program Field 5
"The origin of matter and the universe"

(Nos. [hp120281](#),[hp130023](#),[hp140209](#),[hp140155](#),[hp150135](#),[hp160125](#),[hp170022](#))

- Iwasaki gauge action
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- cutoff $a^{-1} \sim 2.3\text{GeV}$
- pion mass $m_\pi \sim 0.145\text{GeV}$
- 200 configurations (updated)

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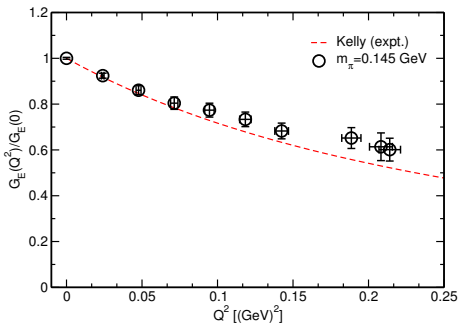
We calculated **isovector nucleon form factors** in the vector (V), axial-vector (A) and also pseudo-scalar (P) channels.

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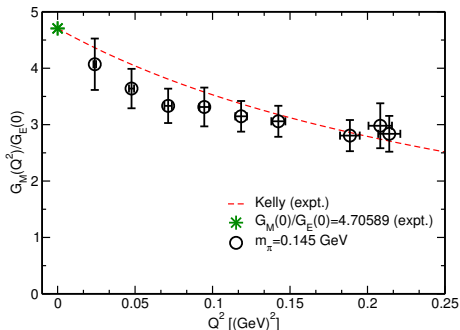
Numerical Results (Electric & Magnetic FFs)

Updated from LATTICE 2016

146 \rightarrow 200 configurations



isovector G_E



isovector G_M

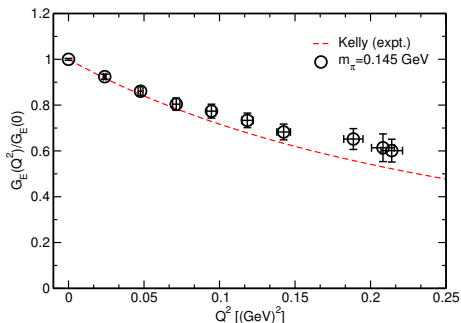
expt. : Kelly J.J., Phys. Rev. C 70, 068202 (2004).

Both G_E and G_M are consistent with experiments

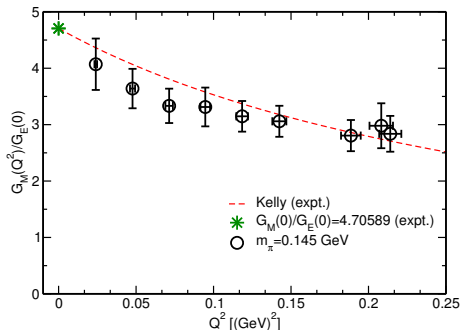
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Both G_E and G_M are consistent with experiments

We analyze electric & magnetic form factors using z -expansion method.

Numerical Results (Electric & Magnetic FFs)

$$G(t = Q^2) = \sum_{k=0}^{k_{\max}} c_k z^k \quad z = \frac{\sqrt{t_{\text{cut}} + t} - \sqrt{t_{\text{cut}}}}{\sqrt{t_{\text{cut}} + t} + \sqrt{t_{\text{cut}}}} \quad t_{\text{cut}} = 4m_\pi^2 \quad (3)$$

To obtain coefficient c_k , we have to solve “minimize” problem :

$$\min |\mathbf{A}\mathbf{c} - \mathbf{b}| \quad A_{ij} = \frac{z(Q_i^2)^j}{dG(Q_i^2)} \quad \mathbf{b}_i = \frac{G(Q_i^2)}{dG(Q_i^2)} \quad (4)$$

We can solve this problem using **SVD** and a pseudoinverse matrix.

$$\mathbf{A} = \mathbf{U} \times \text{diag}(\mathbf{s}_1, \dots) \times \mathbf{V}^\dagger \quad (5)$$

$$\mathbf{A}^+ = \mathbf{V} \times \text{diag}(1/\mathbf{s}_1, \dots) \times \mathbf{U}^\dagger \quad (6)$$

$$\mathbf{c} = \mathbf{A}^+ \mathbf{b} = \sum_i^N \frac{1}{s_i} \mathbf{v}_{(i)} \mathbf{u}_{(i)}^\dagger \mathbf{b} \quad (7)$$

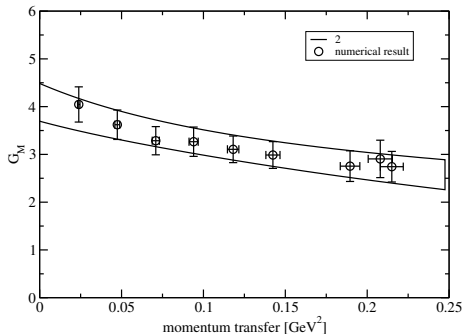
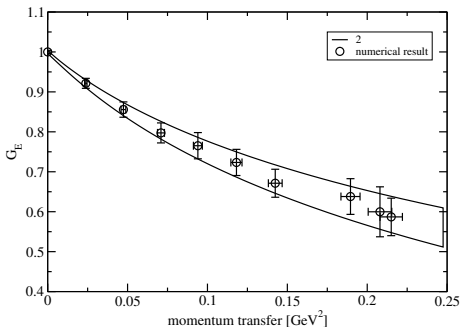
We may truncate the summation of Eq.(7) up to N before the **singular values** s_i become too small in order to prevent over-fitting.

Numerical Results (Electric & Magnetic FFs)

We show N -dependence.

$$\mathbf{c} = \mathbf{A}^+ \mathbf{b} = \sum_i^N \frac{1}{s_i} \mathbf{v}_{(i)} \mathbf{u}_{(i)}^\dagger \mathbf{b}$$

$$G(z) = c_k z^k$$

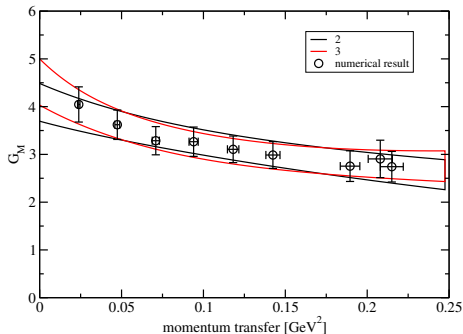
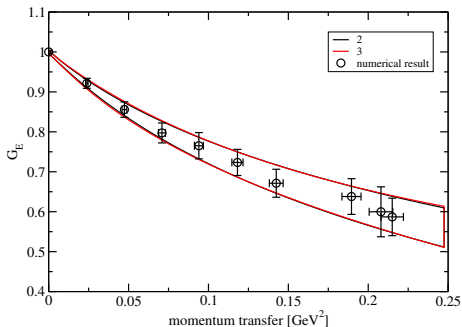


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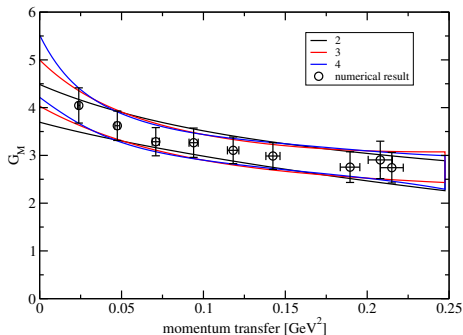
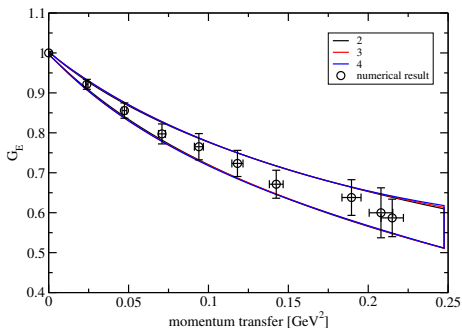


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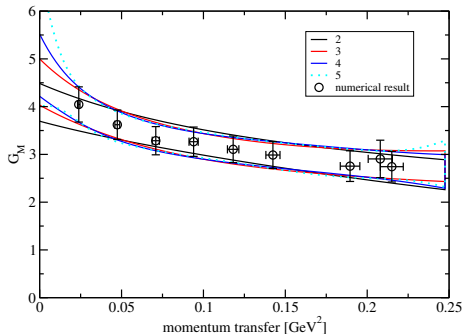
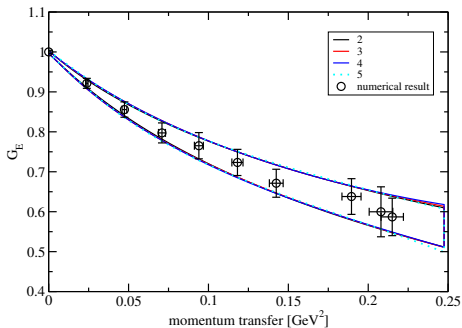


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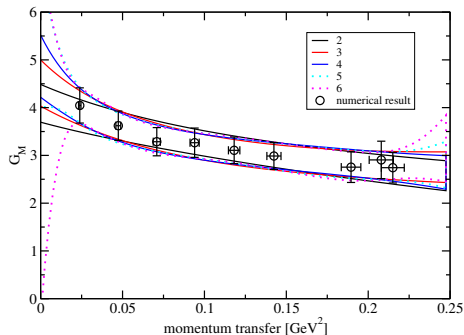
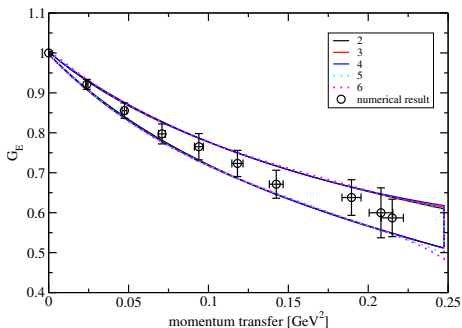


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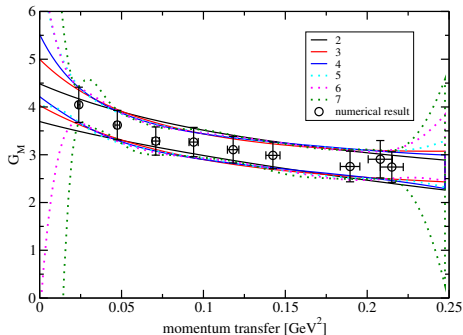
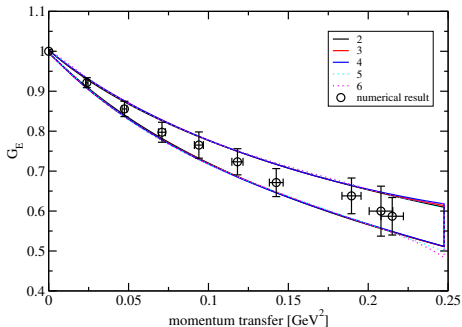


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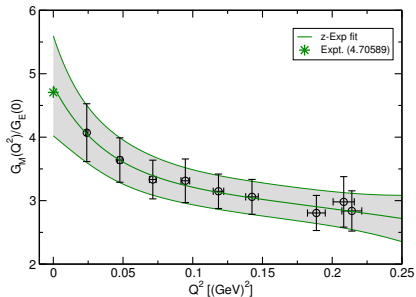
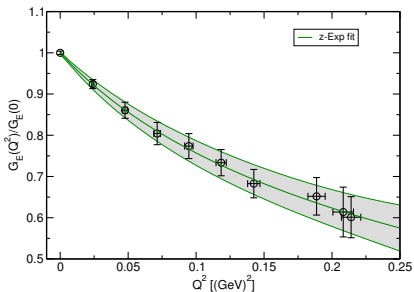
We observe that **stable fit** results are obtained up to $N = 4$.

$$s_4 \gg s_i$$

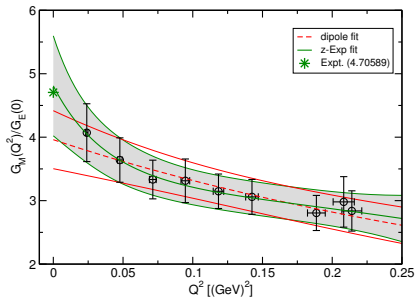
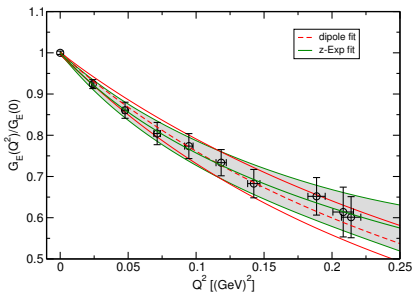
for $i > 4$

then set $\mathbf{u}_{(i)} = 0$.

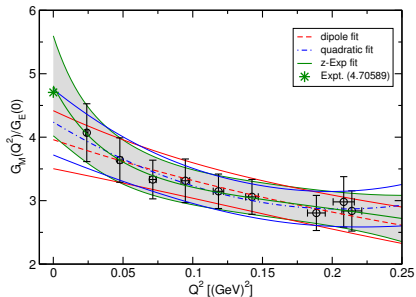
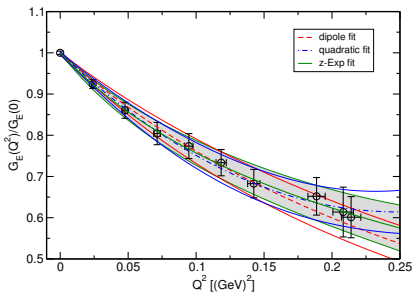
Numerical Results (Electric & Magnetic FFs)



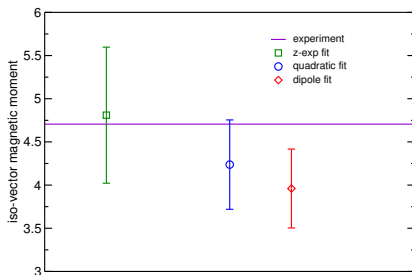
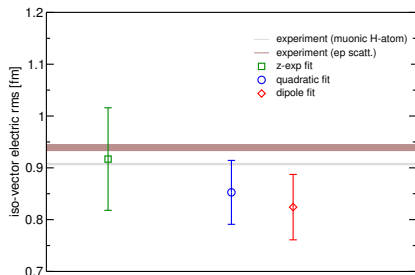
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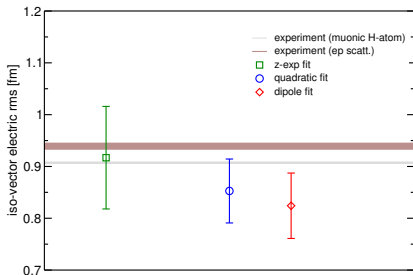


We extract the isovector (\mathcal{V}) radius and magnetic moment, which are defined as

$$\langle r_E^2 \rangle_{\mathcal{V}} = \langle r_E^2 \rangle_p - \langle r_E^2 \rangle_n, \quad (8)$$

$$\mu_{\mathcal{V}} = \mu_p - \mu_n. \quad (9)$$

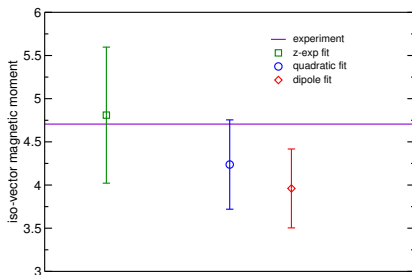
Numerical Results (Electric & Magnetic FFs)



we obtain

$$\sqrt{\langle r_E^2 \rangle} \nu = 0.917 \pm 0.099 [\text{fm}] \quad (8)$$

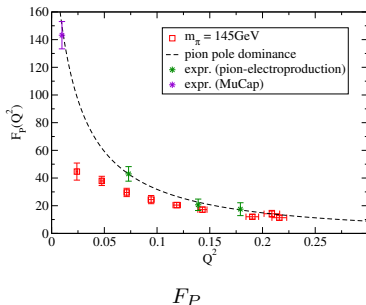
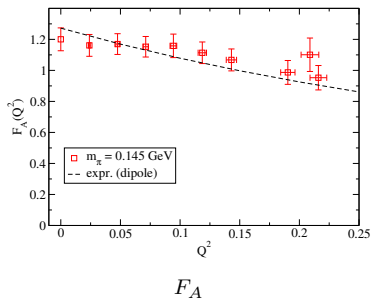
$$\mu \nu = 4.81 \pm 0.79 \quad (9)$$



Numerical Results (Axialvector & Pseudoscalar FFs)

We also show a preliminary analysis on FFs in axial channel.

$$\langle N | A_\mu^a(x) | N \rangle = \bar{u} \left(\gamma_\mu \gamma_5 F_A(Q^2) + i q_\mu \gamma_5 \frac{F_P(Q^2)}{2M_N} \right) \tau^a u e^{iq \cdot x} \quad (10)$$



Although F_A is barely consistent with experiments, F_P is underestimated in comparison with both experiments and the pion-pole dominance model.

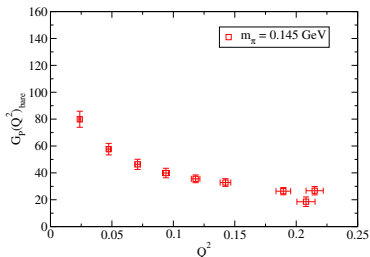
$$F_P(Q^2) = \frac{4M_N^2}{Q^2 + m_\pi^2} F_A(Q^2) \quad \text{PPD ansatz} \quad (11)$$

Numerical Results (Axialvector & Pseudoscalar FFs)

Although axial-vector current is not conserved when quark mass is non-zero, the current still satisfies the following Ward-Takahashi identity (AWTI)

$$\partial^\mu A_\mu^a(x) = 2mP^a(x), \quad (12)$$

where m is quark mass.



G_P

Goldberger-Treiman (GT) relation is derived from the nucleon matrix element of the currents on both sides of Eq.(12)

$$\begin{aligned} 2M_N F_A(Q^2) - \frac{Q^2}{2M_N} F_P(Q^2) \\ = 2m G_P(Q^2), \end{aligned} \quad (13)$$

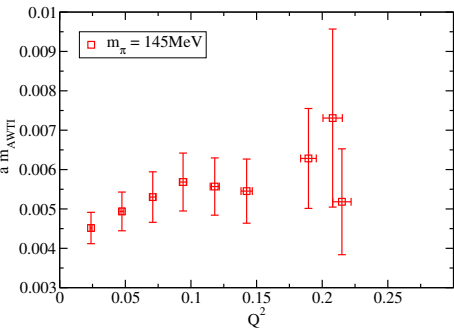
where $G_P(Q^2)$ is pseudoscalar form factor.

$$\langle N | P^a(x) | N \rangle = \bar{u} (\gamma_5 G_P(Q^2)) \tau^a u e^{iq \cdot x} \quad (14)$$

Numerical Results (Axialvector & Pseudoscalar FFs)

GT relation gives us a chance to determine a bare quark mass.

$$m = \frac{2M_N F_A(Q^2) - \frac{Q^2}{2M_N} F_P(Q^2)}{2G_P(Q^2)}. \quad (15)$$



- Ratio m has no apparent Q^2 dependence.
 - am is about 0.005 that corresponds to about 10 MeV.
- Recall that $am=0.001577(10)$ is obtained from the PCAC relation^a.

^aPACS Collaboration, arXiv:1511.08549 and arXiv:1511.09222

quark mass from Goldberger Treiman (GT) relation

We have studied the various nucleon form factors calculated in 2+1 flavor QCD near the physical point ($m_\pi = 145\text{MeV}$) on a large spacial volume $(8.1\text{fm})^3$.

- We examine both electric and magnetic form factor shapes with a model-independent analysis based on the z -expansion method.

As a result, we obtain

- RMS radius ($\sqrt{\langle r_E^2 \rangle_V} = 0.911(99)[\text{fm}]$) from isovector G_E
- magnetic moment ($\mu_V = 4.81(79)$) from isovector G_M

both which are consistent with experimental values.

- And we also analyzed axial and pseudoscalar form factors.
 - F_A is barely consistent with experiments ($g_A = 1.16(8)$)
 - Three form factors, F_A , F_P and G_P satisfy the GT relation, although F_P form factor is underestimated in comparison with experiments in low Q^2 region.