Testing a non-perturbative mechanism for elementary fermion mass generation: numerical results

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(see also talk by M. Garofalo: *"Testing a non-perturbative mechanism for elementary fermion mass generation: lattice setup"*)

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Overview

- **Dynamical generation of fermion masses**
  - owing to a NP mechanism triggered by a Wilson-like (naively irrelevant) chiral breaking term.
- QCD extended to a theory with *enriched symmetry* offers possibility for tackling naturalness problem.
- Simplest **Toy-model** where the mechanism can be realised:
  - $SU(N_f = 2)$ doublet of strongly ($SU(3)_c$) interacting fermions coupled to scalars via Yukawa and Wilson-like terms
  - physics depends crucially related on the phase (Wigner or NG)
  - enhanced symmetry (naturalness à la t'Hooft) leads to $\langle \Phi \rangle$-independence of fermion masses
- The intrinsic NP character of the mechanism requires lattice numerical investigation of the toy model.

R. Frezzotti and G.C. Rossi, PRD 2015, [arXiv:1402.0389 [hep-lat]]

R. Frezzotti, M. Garofalo and G.C. Rossi, PRD 2016, [arXiv:1602.03684 [hep-ph]]

S. Capitani *et al.*, PoS LATTICE2016, [arXiv:1611.03997 [hep-lat]]

Theoretical setup

- Toy-model: QCD$_{N_f=2}$ + Scalar field + Wilson

  $L_{toy} = L_{kin}(Q, A, \Phi) + V(\Phi) + L_Y(Q, \Phi) + L_W(Q, A, \Phi)$, with:

$$
\begin{aligned}
L_{kin}(Q, A, \Phi) &= \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{Q}_L \gamma_\mu D_\mu Q_R + \bar{Q}_R \gamma_\mu D_\mu Q_L + \frac{1}{2} \mathrm{Tr}\left[\partial \Phi^\dagger \partial \Phi\right] \\
V(\Phi) &= \frac{1}{2} \mu^2 \mathrm{Tr}\left[\Phi^\dagger \Phi\right] + \frac{1}{4} \lambda \left(\mathrm{Tr}\left[\Phi^\dagger \Phi\right]\right)^2 \\
L_Y(Q, \Phi) &= \eta \left(\bar{Q}_L \Phi Q_R + \bar{Q}_R \Phi Q_L\right) \\
L_W(Q, A, \Phi) &= \rho \frac{b^2}{2} \left(\bar{Q}_L \overleftarrow{D}_\mu \Phi D_\mu Q_R + \bar{Q}_R \overleftarrow{D}_\mu \Phi^\dagger D_\mu Q_L\right)
\end{aligned}
$$

- Q: fermion $SU(2)$ doublet coupled to $SU(3)$ gauge field and to scalar field through Yukawa and Wilson terms.

- b^{-1}: UV cutoff.

Theoretical setup

- $\chi_L \times \chi_R$ transformations are symmetry of L_{toy}:

$$\chi_L : \tilde{\chi}_L \otimes (\Phi \to \Omega_L \Phi) \qquad \chi_R : \tilde{\chi}_R \otimes (\Phi \to \Omega_R \Phi)$$

$$\tilde{\chi}_L : Q_L \to \Omega_L Q_L, \qquad \tilde{\chi}_R : Q_R \to \Omega_R Q_R,$$

$$\bar{Q}_L \to \bar{Q}_L \Omega_L^\dagger \qquad \bar{Q}_R \to \bar{Q}_R \Omega_R^\dagger$$

$$\Omega_L \in SU(2)_L \qquad \Omega_R \in SU(2)_R$$

- **Exact symmetry** $\chi \equiv \chi_L \times \chi_R$ acting on fermions and scalars $\Rightarrow$ no power divergent mass terms.
- The (fermion) $\tilde{\chi} \equiv \tilde{\chi}_L \times \tilde{\chi}_R$ transformations are not a symmetry for generic (non-zero) η and ρ.
- P, C, T, gauge invariance . . . are symmetries.
- When the scalar potential $V(\Phi)$ has one minimum
  - $\chi_L \times \chi_R$ is realized à la Wigner.
  - *No* $\tilde{\chi}$−SSB phenomenon is expected to occur.

Theoretical setup

- The (fermion) $\tilde{\chi} \equiv \tilde{\chi}_L \times \tilde{\chi}_R$ transformations generate Schwinger-Dyson Eqs (unrenormalised).
- They get renormalised after considering the operator mixing procedure.
- *Critical Model*: $\tilde{\chi}$-symmetry restoration occurs when the Yukawa term is compensated by the Wilson (6-d) term. This takes place (in the Wigner phase) at a certain value of the Yukawa coupling.
- In fact, for $\tilde{J}_\mu^{L,i}$ (or $\tilde{J}_\mu^{R,i}$) get
  $\partial_\mu \langle \tilde{Z}_{\tilde{J}} J_\mu^{L,i}(x) O(0)\rangle = (\eta - \overline{\eta}(\eta; g_0^2, \rho, \lambda))\langle [\bar{Q}_L \tau^i \Phi Q_R - h.c.](x) O(0)\rangle + O(b^2)$
  (SDE renrm/tion here analogous to chiral SDE renrm/tion in Bochicchio *et al. NPB 1985*)
  - then *enforce* the current $\tilde{J}_\mu^{L,i}$ (or $\tilde{J}_\mu^{R,i}$) conservation $\Longrightarrow$
    $\eta - \overline{\eta}(\eta; g_0^2, \rho, \lambda) = 0 \quad \rightarrow \quad \eta_{cr}(g_0^2, \rho, \lambda)$.
- In the Wigner phase the low-energy effective action reads

$$\Gamma^{Wig}_{\mu_\Phi^2 > 0} = \frac{1}{4}(F \cdot F) + \bar{Q} \not{D} Q + (\eta - \eta_{cr})(\bar{Q}_L \Phi Q_R + \text{h.c.}) + \frac{1}{2}\mathrm{Tr}\left[\partial_\mu \Phi^\dagger \partial_\mu \Phi\right] + \mathrm{V}_{\mu_\Phi^2 > 0}(\Phi)$$

- in the critical theory ($\tilde{\chi}$ is a symmetry, up to $O(b^2)$)
  - Φ decoupled (up to cutoff effects) from quarks and gluons.
  - no fermionic mass ($m_Q = 0$ up to $O(b^2)$).

Numerical investigation

▶ Determination of η_{cr} in the Wigner phase

- **Lattice simulation details**

  - Lattice discretization, $L_{latt.}$, with exact χ-symmetry.
  - Use naive fermions with symmetric covariant derivative, $\tilde{\nabla}_\mu$, throughout.
  - To avoid "exceptional configurations" introduce twisted mass regulator $L_{latt.} + i\mu_Q \bar{Q}\gamma_5\tau^3 Q$. (Frezzotti, Grassi, Sint and Weisz, JHEP 2001)
  - Locally smeared Φ in $\bar{Q}D_{lat}[U,\Phi]Q$ for noise reduction.
  - We limit our first study to the quenched approximation ($\rightarrow$ it is quite certain that the mechanism under investigation survives quenching).
  - Quenching: independent generation of gauge (U) and scalar (Φ) configurations.

(see also discussion and details on the lattice action in M. Garofalo's talk, this session.)

Numerical investigation

▶ Determination of η_{cr} in the Wigner phase

- **Lattice simulation parameters**

$\beta = 5.85$, $L/b = 16 \,\&\, T/b = 40$

$b = 0.12$ fm, from $r_0 = 0.5$ fm (motivated from QCD, for illustration)

$\lambda = 0.59$, $\rho = 1.96$ (in the Wigner & in the NG phases)

$\mu_\Phi^2 \equiv \mu^2 - \mu_{cr}^2 = 0.074(1)\, b^{-2} > 0$ (in the Wigner phase)

statistics: #configs (gauge $\times$ scalar) $\sim 750 - 1400$

@ several values of the Yukawa coupling η (and μ_Q).

▶ Determination of η_{cr} in the Wigner phase

- Compute correlation function

$$C_{\tilde{J}\tilde{D}}(x-y) \equiv \langle \tilde{J}_0^{V3}(x)\ \tilde{D}^{S3}(y)\rangle$$

where

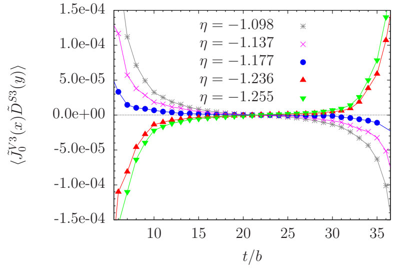
$$\begin{aligned}
\tilde{J}_0^{V3}(x) &= \tilde{J}_0^{L\,3}(x) + \tilde{J}_0^{R\,3}(x) \\
\tilde{J}_0^{L/R\,3}(x) &= \frac{1}{2}\Big[\bar{Q}_{L/R}(x-\hat{0})\gamma_0\frac{\tau_3}{2}U_0(x-\hat{0})Q_{L/R}(x) + \bar{Q}_{L/R}(x)\gamma_0\frac{\tau_3}{2}U_0^\dagger(x-\hat{0})Q_{L/R}(x-\hat{0})\Big] \\
\tilde{D}^{S3}(y) &= \bar{Q}_L(y)\left[\Phi,\frac{\tau^3}{2}\right]Q_R(y) - \bar{Q}_R(y)\left[\frac{\tau^3}{2},\Phi^\dagger\right]Q_L(y)
\end{aligned}$$

- Renormalised Schwinger-Dyson eqs of $\tilde{V}^3$-type (in the form of a would be $\tilde{\chi}$-WTI):

$\partial_\mu \tilde{J}_\mu^{V3} = (\eta-\eta_{cr})\tilde{D}^{S3} + O(b^2)$ and $\langle 0|\,\partial_0\tilde{J}_0^{V3}\,|M_S\rangle \equiv f_{M_S}M_{M_S}^2$

▶ Determination of η_{cr} in the Wigner phase

In the Wigner phase at $\eta = \eta_{cr}$ the restored $\tilde{\chi}$-symmetry is realised à la Wigner (*first example in a local setting*) and leads to vanishing correlation function $C_{\tilde{J}\tilde{D}}(x-y)$



(at $a\mu_Q = 0.0224$)

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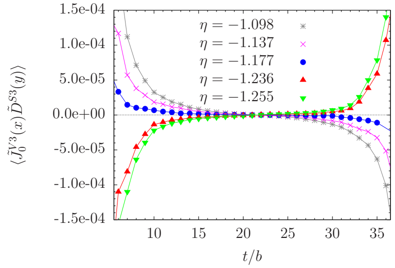
vanishing matrix element $\quad \mathrm{ME} = \langle 0|\tilde{J}_0^{V3}|M_S\rangle\langle M_S|\tilde{D}^{S3}|0\rangle \xrightarrow{\eta\to\eta_{cr}} 0$



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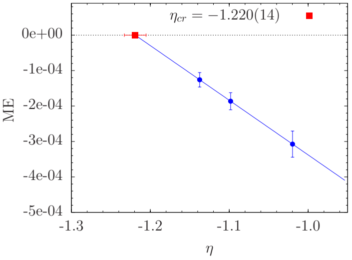

(after extrapolating to $a\mu_Q = 0$)

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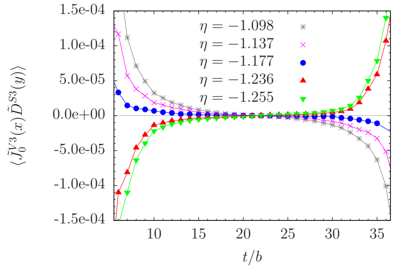

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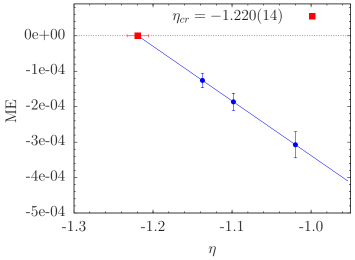

(after extrapolating to $a\mu_Q = 0$)

$\eta_{cr} = -1.220(14)$ (@ $\beta = 5.85$) PRELIMINARY!

Features and properties of the toy-model in NG-phase

- $V(\Phi)$ of mexican hat shape $\to \chi_L \times \chi_R$ realised à la NG.
- $\chi_L \times \chi_R$ spontaneously broken: $\Phi = v + \sigma + i\vec{\tau}\vec{\pi}$, $\quad \langle\Phi\rangle = v \neq 0$
- $L_W(Q, A, \Phi) = \frac{\rho b^2}{2}\left(\bar{Q}_L \overleftarrow{D}_\mu \Phi D_\mu Q_R + \text{h.c.}\right) \underset{a\leftrightarrow b}{\overset{r\leftrightarrow bv\rho}{\sim}} L_W^{QCD}(Q, A) = -\frac{ar}{2}\left(\bar{Q}_L D^2 Q_R + \text{h.c.}\right)$
- In the *critical* theory $\eta = \eta_{cr}$:
  - the (Yukawa) mass term, $v\bar{Q}Q$, gets cancelled.
  - $\tilde{\chi}-$ breaking due to residual $O(b^2 v)$ effects is expected to trigger dynamical χSB.

$\Longrightarrow$ Look for dynamically generated fermion mass:

- At generic η, two $\tilde{\chi}$ breaking operators:

  Yukawa induced + dynamically generated ($\leftarrow$ conjecture)

- $\Gamma^{NG} = \ldots + (\eta - \eta_{cr})(\bar{Q}_L\langle\Phi\rangle Q_R + \text{h.c.}) + c_1\Lambda_s(\bar{Q}_L\mathcal{U}Q_R + \text{h.c.})$

  where $\mathcal{U} = \dfrac{\Phi}{\sqrt{\Phi^\dagger\Phi}} = \dfrac{(v+\sigma)\mathbb{1} + i\vec{\tau}\vec{\varphi}}{\sqrt{v^2 + 2v\sigma + \sigma^2 + \vec{\varphi}\vec{\varphi}}} \simeq \mathbb{1} + i\dfrac{\vec{\tau}\vec{\varphi}}{v} + \ldots$

  and $\Lambda_s \equiv$ RGI NP mass scale.

  Note that (χ-inv. term): $c_1\Lambda_s(\bar{Q}_L\mathcal{U}Q_R + \text{h.c.}) \simeq c_1\Lambda_s\bar{Q}Q + \ldots$

▶ Features and properties of the toy-model in NG-phase

- Work at the same lattice parameters (β, λ, ρ and volume) as in the Wigner phase
- Determine M_{PS} (from e.g. $\langle PP \rangle$ correlation function) and $m^{WTI} = \dfrac{\partial_0 \langle 0 | \widetilde{J}_0^{A\pm} | M_{PS^\pm} \rangle}{\langle 0 | P^\pm | M_{PS^\pm} \rangle}$ in the NG-phase.

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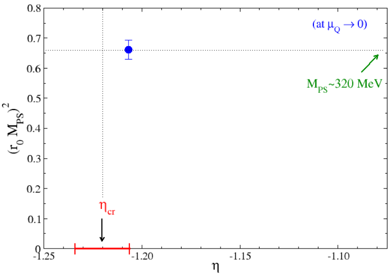

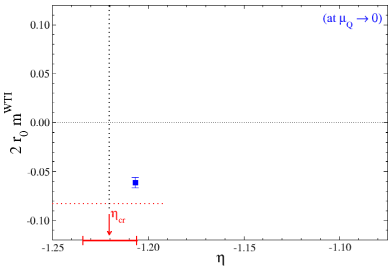

At $\eta = \eta_{cr}$: $\begin{cases} M_{\rm PS} \neq 0 \\ m^{WTI} \neq 0, \quad \text{Note } m^{WTI} = (\eta - \eta_{cr})v + c_1\Lambda_s \xrightarrow{\eta = \eta_{cr}} m^{WTI} = c_1\Lambda_s \end{cases}$

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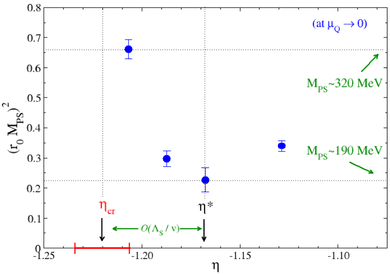

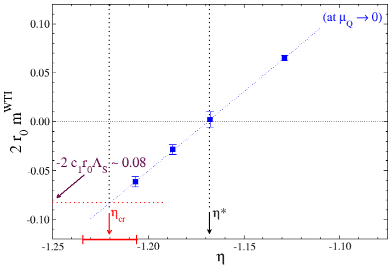

$$\text{At } \eta = \eta_{cr} : \begin{cases} M_{\rm PS} \neq 0 \\ m^{WTI} \neq 0, \quad \text{Note } m^{WTI} = (\eta - \eta_{cr})v + c_1\Lambda_s \xrightarrow{\eta=\eta_{cr}} m^{WTI} = c_1\Lambda_s \end{cases}$$

- m^{WTI} cancels at $\eta^* = \eta_{cr} - c_1\Lambda_s/v \Rightarrow \eta_{cr} \neq \eta^* \leftrightarrow c_1\Lambda_s \neq 0$
- At $\eta = \eta_{cr}$ for $\beta = 5.85$: $M_{PS} \sim 400$ MeV and $m^{WTI}_{\rm bare} \sim 16$ MeV.

(Preliminary results!)

Conclusions & Outlook

- We have presented a toy-model that exemplifies a *novel* NP mechanism for fermion mass generation.
  R. Frezzotti and G.C. Rossi, PRD 2015, [arXiv:1402.0389 [hep-lat]]

- The **toy model** is a non-Abelian gauge model with an SU($N_f = 2$)-doublet of strongly interacting fermions coupled to scalars through Yukawa and Wilson-like terms: at the *critical point*, where (fermion) $\tilde{\chi}$ invariance is recovered in Wigner phase (up to UV-effects) the model is *conjectured* to give rise in NG phase to dynamical $\tilde{\chi}$-SSB and hence to non-perturbative fermion mass generation.

- The main physical implications of the conjecture above can be *verified/falsified* by numerical simulations of the toy-model (rather cheap in the quenched approximation).

Conclusions & Outlook

- A *preliminary* study at one value of the lattice spacing (~ 0.12 fm) in the quenched approximation has been presented.

- We have discussed one possible method for the accurate determination of the critical value of the Yukawa coupling in the Wigner phase at which $\tilde{\chi}$ is restored. Then we explored the effects of dynamical SSB of the (restored) $\tilde{\chi}$-symmetry in the NG phase which (barring cutoff effects) look very well compatible with the generation of a non-zero (effective) fermion mass and $M_{PS} \sim O(\Lambda_s)$.

- These findings have to be checked and verified at different values of the lattice spacing (*on-going work*) in order to confirm the persistence of the dynamical mass generation mechanism in the continuum limit.

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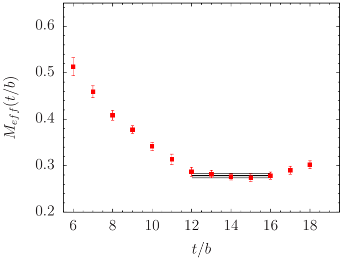
Thank you for your attention!

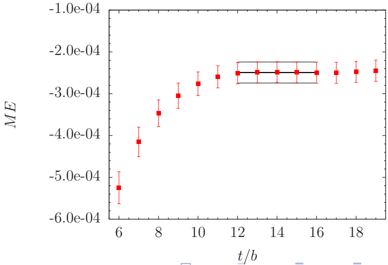
Extra slides

- Correlation function (example case $\eta = -1.020$ @ $a\mu_Q = 0.0224$in Wigner phase):

$$C_{\tilde{J}\tilde{D}}(x-y) \equiv \langle \tilde{J}_0^{V3}(x)\ \tilde{D}^{S3}(y) \rangle$$

$$\mathrm{ME} = \langle 0|\tilde{J}_0^{V3}|M_S\rangle\langle M_S|\tilde{D}^{S3}|0\rangle$$

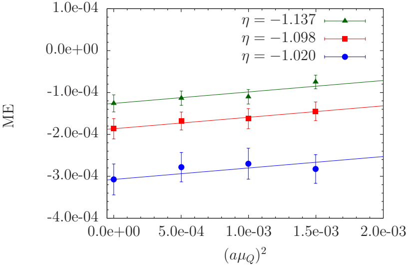


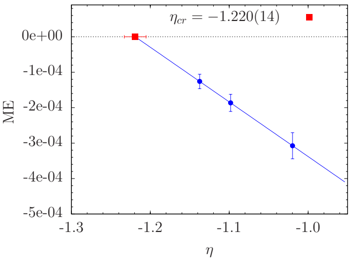


Extrapolation in $\mu_Q = 0$ and in η of

$$\mathrm{ME} = \langle 0|\tilde{J}_0^{V3}|M_S\rangle\langle M_S|\tilde{D}^{S3}|0\rangle$$

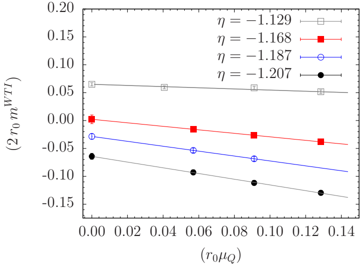
in order to determine η_{cr} where ME vanishes:

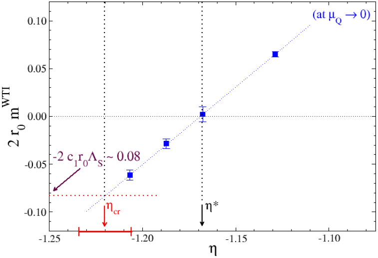




Extrapolation in $\mu_Q = 0$ (left panel) and η-dependence (right panel) for

$$2r_0 m^{WTI} = \frac{2r_0\partial_0\langle 0|\widetilde{J}_0^{A\pm}|M_{PS^\pm}\rangle}{\langle 0|P^\pm|M_{PS^\pm}\rangle}$$ in the NG-phase.





Extrapolation in $\mu_Q = 0$ (left panel) and η-dependence (right panel) for $(r_0 M_{\rm PS})^2$