Higgs compositeness in $Sp(2N)$ gauge theories - Part II : The pure gauge model

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Outline

General methods for $Sp(2N)$

- HB algorithm for $Sp(2N)$.
- Resymplectization.
- Variational calculus and smoothing routines.

$Sp(4)$

- String tension.
- Glueball Spectrum.

The $Sp(2N)$ LGT

Definition

On a $\mathbb{Z}^4$ lattice with lattice spacing a

$$S_g[U] = \beta \sum_{x,\mu<\nu} \left(1 - \frac{1}{N} \mathrm{Re}\ \mathrm{Tr}\ \mathcal{P}_{\mu\nu}(x)\right) ,$$

with

$$\mathcal{P}_{\mu\nu}(x) = U_\mu(x) U_\nu(x+\hat{\mu}) U^\dagger_\mu(x+\hat{\nu}) U^\dagger_\nu(x) \ \in \ Sp(2N)$$

and $\beta = 2N/g^2$.

Algorithms

- Heathbath (this talk)
- Hybrid MonteCarlo (Talk by E. Bennett)

Algorithms

HB for Sp(2N)

The $Sp(2N)$ group

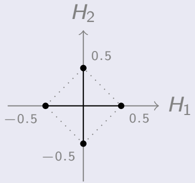
$$U \in Sp(2N) \Longleftrightarrow \left\{ U \in SU(2N) \,/\, \Omega^\dagger U \Omega = U^* \right\}$$

$$U = \begin{bmatrix} A & B \\ -B^* & A^* \end{bmatrix}, \text{ with } \begin{cases} A^\dagger A + B^\dagger B = \mathbb{I}, \\ A^T B = B^T A, \end{cases} \quad A, B \in \mathbb{C}^{N\times N}$$

The HB - Update on $SU(2)$ subgroups (Cabibbo-Marinari)

To choose the $SU(2)$ subgroups:

- $Sp(2N) \subset SU(2N)$
- Block structure of $Sp(2N)$ matrices.
- Weight diagram of the fundamental representation (Sp(4) in figure).



Resymplecticization

Modified Gram-Schmidt algorithm

GramSchmidt for $SU(N)$

normalize col(1).
  ortogonalize col(2) w.r.t col(1).
normalize col(2).
  ortogonalize col(3) w.r.t col(1).
  ortogonalize col(3) w.r.t col(2).
normalize col(3).
...

GramSchmidt for $Sp(2N)$

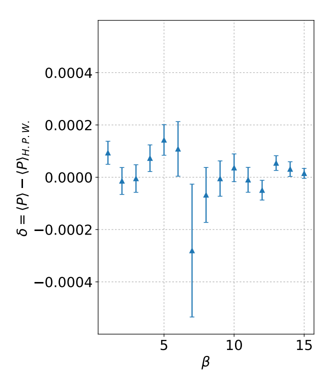
normalize col(1).
  column N is set to $\Omega\times$ col(1)*
  ortogonalize col(2) w.r.t col(1).
  ortogonalize col(2) w.r.t col(N).
normalize col(2)
...

- Fully generalizable to any N.
- Could reformulate as row-wise.
- Same numerical stability as Gram-Schmidt.
- Other possible approach in E. Bennett's Talk.

Simulation Algorithm

A comparison with existing results for $Sp(4)$.

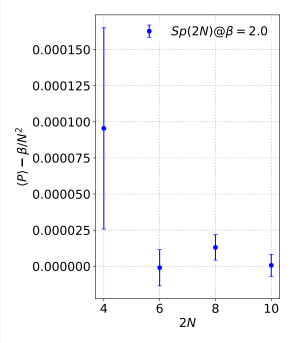
$Sp(4)$



[K. Holland, M. Pepe, U.-J. Wiese('03)]

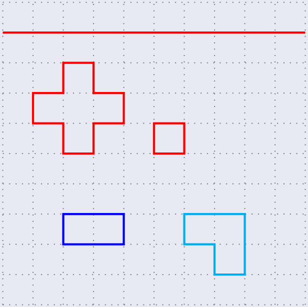
Str. Coupling for $Sp(2N)$

$$\langle P \rangle \sim \frac{\beta}{N^2} + \cdots$$



Gauge invariant operators

On a timeslice...



Zero momentum symmetrized operators

$$\phi_{\mathcal{C}}(t) = \sum_{\vec{x}} \phi_{\mathcal{C}}(x,t) = \sum_{\vec{x}} \mathrm{Tr}\left(\prod_{\mathcal{C}} U_l\right)$$

- $\mathcal{C}$ closed loop: contractible (glueballs) or non-contractible (fluxtubes).
- Symmetry of $|i\rangle$ dictated by $\mathcal{C}$: subduced reps of the rotation group:

$$R^P = A1^{\pm}, A2^{\pm}, T1^{\pm}, T2^{\pm}, E^{\pm}$$

- Sum over $\vec{x}$ for zero momentum operators,

To be measured...

$$\Gamma_{\mathcal{C}}(t) = \frac{\langle \phi^{\dagger}_{\mathcal{C}}(0)\phi_{\mathcal{C}}(t)\rangle}{\langle \phi^{\dagger}_{\mathcal{C}}(0)\phi_{\mathcal{C}}(0)\rangle} = \frac{\sum_i |\langle i|\phi_{\mathcal{C}}(0)|0\rangle|^2 \, e^{-m_i t}}{\sum_i |\langle i|\phi_{\mathcal{C}}|0\rangle|^2} \sim A^{(0)}_{C} e^{-mt}$$

Exponential decay of signal to noise ratio

- Problem 1: How to Maximize $A_{\mathcal{C}}^{(0)}$?
- Problem 2: $A_{\mathcal{C}}^{(0)} \rightarrow 0$ when $a \rightarrow 0$.

Solution to 1 - Variational calculus

$$C_{ij}(t) = \frac{\langle 0|\phi_i^\dagger(0)\phi_j(t)|0\rangle}{\langle 0|\phi_i^\dagger(0)\phi_j(0)|0\rangle}$$

Diagonalize C with

$$\Phi(t) = \sum_{i \in R^P} \alpha_i \phi_i(t),$$

obtain $\tilde{C}$,

$$m_{eff}(t) = -\log \frac{\tilde{C}_{ii}(t)}{\tilde{C}_{ii}(t-1)}.$$

[K. Ishikawa, M. Teper, G. Schierholz ('82)]

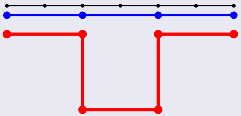
Solution to 2 - Improved Smearing

- Blocking: $U_l \rightarrow U_l^b$, (N_b, δ_b).
- Smearing: $U_l \rightarrow \tilde{U}_l$, (N_s, δ_s).

Combine the two:

$$U_l \rightarrow U_l^b \rightarrow \tilde{U}_l^b$$

and use Cooling to reproject on $Sp(2N)$.



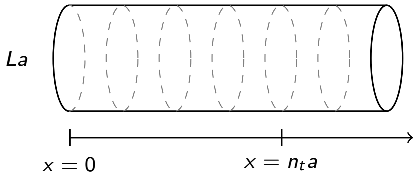
[M. Teper, B. Lucini, U. Wenger('04)]

General setting

Lattice parameters

- We implemented the tools to simulate $Sp(2N)$ into the *HiRep* code.
- Variational analysis fully automatized. [B. Lucini, A. Rago, E. Rinaldi ('10)]
- $L = 10 - 32$
- $\beta = 7.7 - 8.3$
- Update pattern: 1/4 sweeps of HB/OR,
- Thermalization and decorrelation: $1000 - 5000$ initial sweeps, 20 decorrelation sweeps.
- 10000 configurations at each L and each β.
- Improved smearing: 200 operators, $\delta_b = 0.4$, $\delta_s = 0.16$.
- Cooling: 15 steps.

Closed fluxtubes



Closed Fluxtubes

- Non-contractible loops create space-like and time-like propagating fluxtubes.
- With the algorithm above we can measure their masses $m_s(L)$, $m_t(L)$.

Effective String Theory

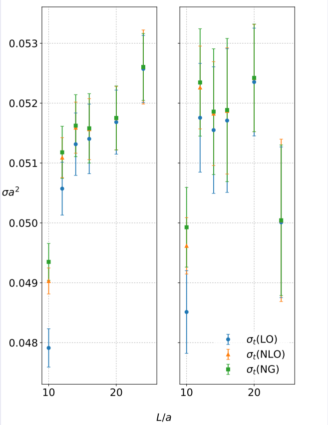
For a closed fluxtube of length L,

$$m(L) = \sigma_{\text{LO}} L - \frac{\pi}{3L} = \sigma_{\text{NLO}} L - \frac{\pi}{3L} - \left(\frac{\pi}{3}\right)^3 \frac{1}{2\sigma_{\text{NLO}} L^3} = \sigma_{\text{NG}} L \sqrt{1 - \frac{2\pi}{3\sigma_{\text{NG}} L}}$$

[M. Lüscher, P. Weisz, O. Aharony, M. Field. . .]

The string tension

Numerical results



Observations

- At this level of precision, we cannot detect the difference between NLO and NG.
- At $L \geq 14a$ we are in the infinite volume regime.
- σ_s and σ_t are compatible.
- Systematical errors at $L = 24a$.

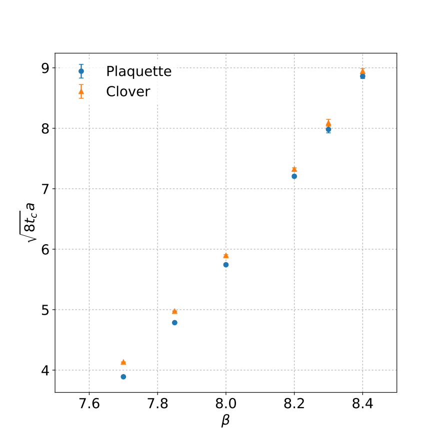
Estimate $\sqrt{\sigma}$ with

$$\sqrt{\sigma} = \frac{\frac{\sqrt{\sigma_s}}{(\Delta\sqrt{\sigma_s})^2} + \frac{\sqrt{\sigma_s}}{(\Delta\sqrt{\sigma_s})^2}}{\frac{1}{(\Delta\sqrt{\sigma_s})^2} + \frac{1}{(\Delta\sqrt{\sigma_s})^2}}$$

$@\beta = 7.7,\ \sigma a^2 = 0.0517(30)$

Scale setting

Using the Wilson flow. . .



Settings & Observations

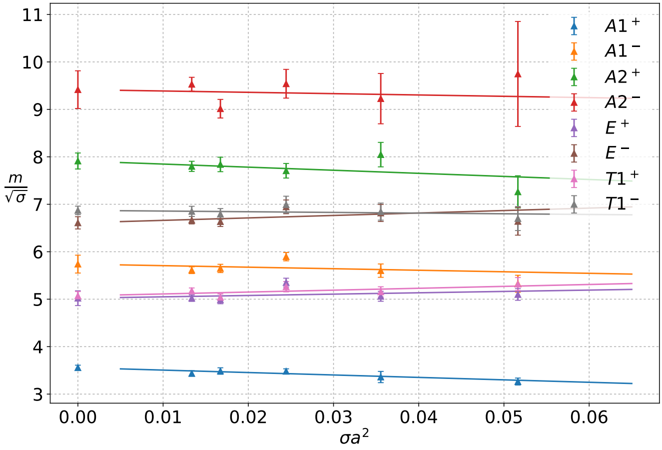
- 200 pure gauge configurations for each point
- L/a ranging from 16 to 32.
- $c_\tau = \frac{\sqrt{8t_c}}{L} \leq 4$ to avoid finite size effects.
- See E.J. Bennet's talk for more detailed account.

Settings & Observations

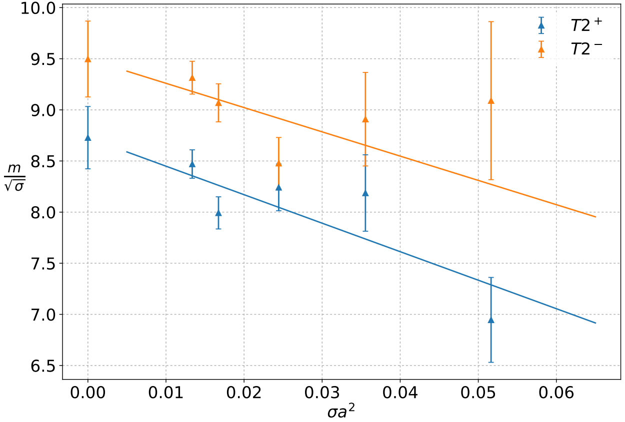
- We compute $m(R^P)/\sqrt{\sigma}$ in all the symmetry channels.
- Since $SP(2N)$ has only pseudoreal representations, no charge conjugation.
- ~ 200 operators in every symmetry channel were used.
- Physical volume $\sqrt{\sigma}L$ was kept approximately constant while $a \rightarrow 0$.
- Glueball masses have $\sim \sigma$ discretization effects.
- We expect that, in the continuum limit:

$$m(E^{\pm}) = m(T1^{\pm})$$

Continuum extrapolations - 1



Continuum extrapolations - 2



Numerical Results

The glueball spectrum in the continuum limit

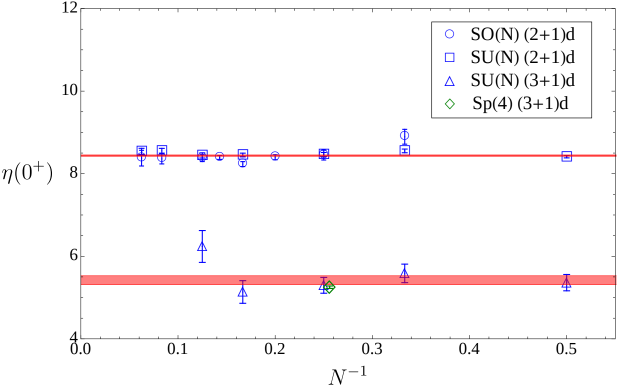
R^P	$m(R^P)/\sqrt{\sigma}$
$A1^+$	3.557(52)
$A1^-$	5.74(19)
$A2^+$	7.91(17)
$A2^-$	9.42(40)
E^+	5.02(16)
E^-	6.61(13)
$T1^+$	5.070(91)
$T1^-$	6.872(89)
$T2^+$	8.73(30)
$T2^-$	9.50(37)

Agreement with η conjecture

We conjecture that

$$\eta = \frac{m_{0^{++}}}{\sqrt{\sigma}} \frac{C_2(F)}{C_2(A)}$$

is a universal quantity that only depends on the dimension of spacetime and on the group: $SU(N)$, $SO(N)$, $Sp(2N)$. [B. Lucini, M. Piai, D.-K. Hong, J.-W. Lee, D.V.('17)]



Conclusions

- The tools to simulate pure gauge $Sp(2N)$ were developed: HeathBath, Resymplecticization, . . .
- The glueball spectrum for $Sp(4)$ was obtained by including these into the *HiRep* package for BSM on the lattice.
- The value of η for $Sp(4)$ is in agreement with its conjectured universality.
- Future directions: use the developped tools to study $Sp(2N)$ at large N.

Thank you for your attention!

(To be continued. . .)