Chiral transition of SU(4) gauge theory with fermions in multiple representations

Presented by Daniel Hackett
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Recap: "Multirep" Theories

General class of theories:

One gauge field

Multiple fermion fields coupled to the gauge field

Fermion fields charged under different irreps of the gauge group

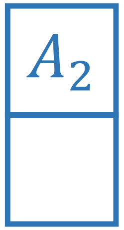
Motivation: Separated phases

This study ("Our theory"):

SU(4) gauge theory

$N_F = 2$ Dirac fermions in fundamental irrep of SU(4) [F q_i $\mathbf{4}$ ϕ]

$N_{A_2} = 2$ Dirac fermions in 2-index antisymmetric irrep of SU(4) [A_2 AS2 Q_i $\mathbf{6}$ θ]



The story so far (see previous two talks by W. Jay, V. Ayyar):

Zero-T spectroscopy looks QCD-like [William Jay's talk]

Confinement transitions for F, A_2 coincide [Venkitesh Ayyar's talk]

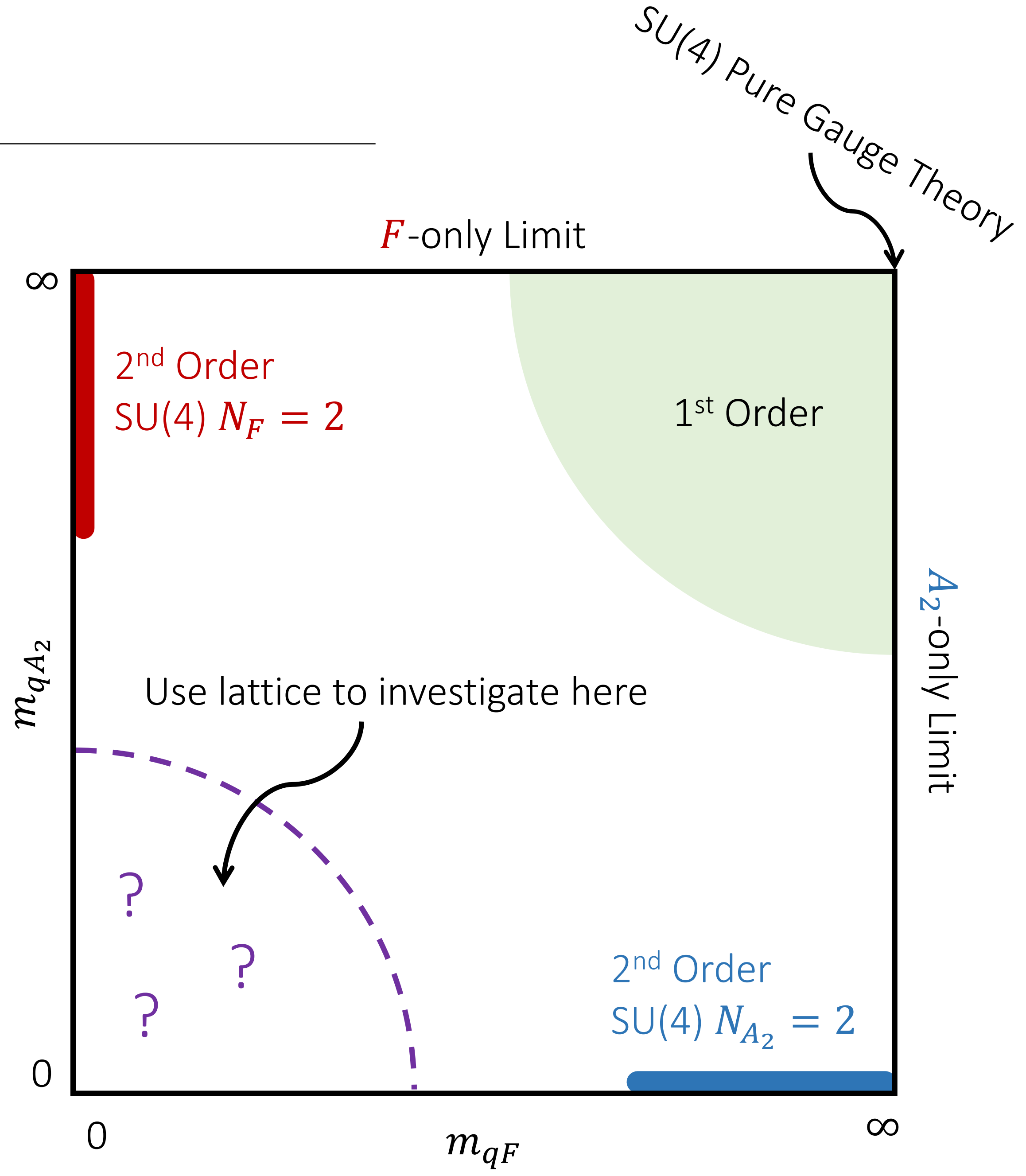
Outline

Chiral phase structure

- Are chiral transitions separated or simultaneous?
- Are chiral transitions separated from confinement transition?
- Lattice results

Order of Transition

- Motivation: fill in “multirep Columbia Plot”
- Analytic result: “Multirep Pisarski-Wilczek” calculation to predict order of transition(s)
- Lattice results



Chiral Phase Diagnostic: Parity Doubling

Diagnose condensation by comparing masses of parity partners:

Chirally restored:

$m_{\mathrm{s}} - m_{\mathrm{ps}} = 0$ [Scalar and pseudoscalar are degenerate]

$m_{\mathrm{pv}} - m_{\mathrm{v}} = 0$ [Pseudovector and vector are degenerate]

Chirally broken:

$m_{\mathrm{s}} - m_{\mathrm{ps}} > 0$ [Scalar and pseudoscalar are non-degenerate]

$m_{\mathrm{pv}} - m_{\mathrm{v}} > 0$ [Pseudovector and vector are non-degenerate]

Do observables for each irrep respond simultaneously or separately?

(**Note:** Wilson fermions explicitly break chiral symmetry, so degeneracy in chirally restored phase is only approximate – but, small effect in this study.)

Dataset

3D bare parameter space $(\beta, \kappa_F, \kappa_{A_2})$

Look at data along densely interpolated slice across the transition (boxed data in plot)

Lattices are $12^3 \times 6$

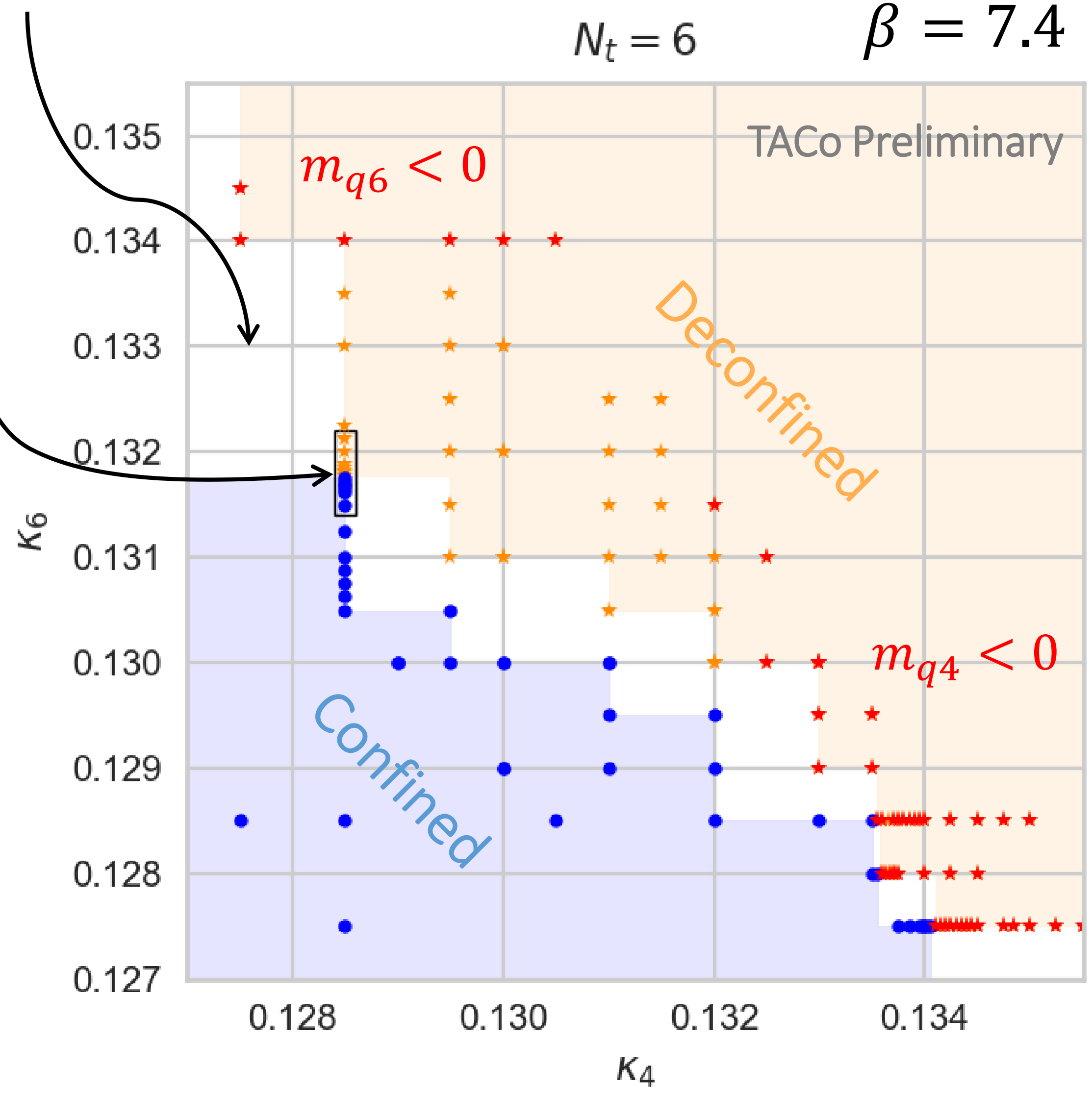
Have also looked at larger volumes, but have best interpolation on $12^3 \times 6$

Typically 30-200 decorrelated configs/ensemble

Fit spatial-direction correlators to get screening masses

- ● Confined ensemble, $m_q > 0$ (both reps)
- ● Confined ensemble, $m_q < 0$ (one or both reps)
- ★ Deconfined ensemble, $m_q > 0$ (both reps)
- ★ Deconfined ensemble, $m_q > 0$ (one or both reps)

White band: phase is ambiguous
(Transition lives in here)



Chiral Transitions Coincide

Chiral transitions occur simultaneously with each other

Chiral transition occurs simultaneously with confinement transition

This slice is typical for the region of parameter space that we have explored

We have not observed phase separation.

Bottom: Parity doubling (chiral transition) observables

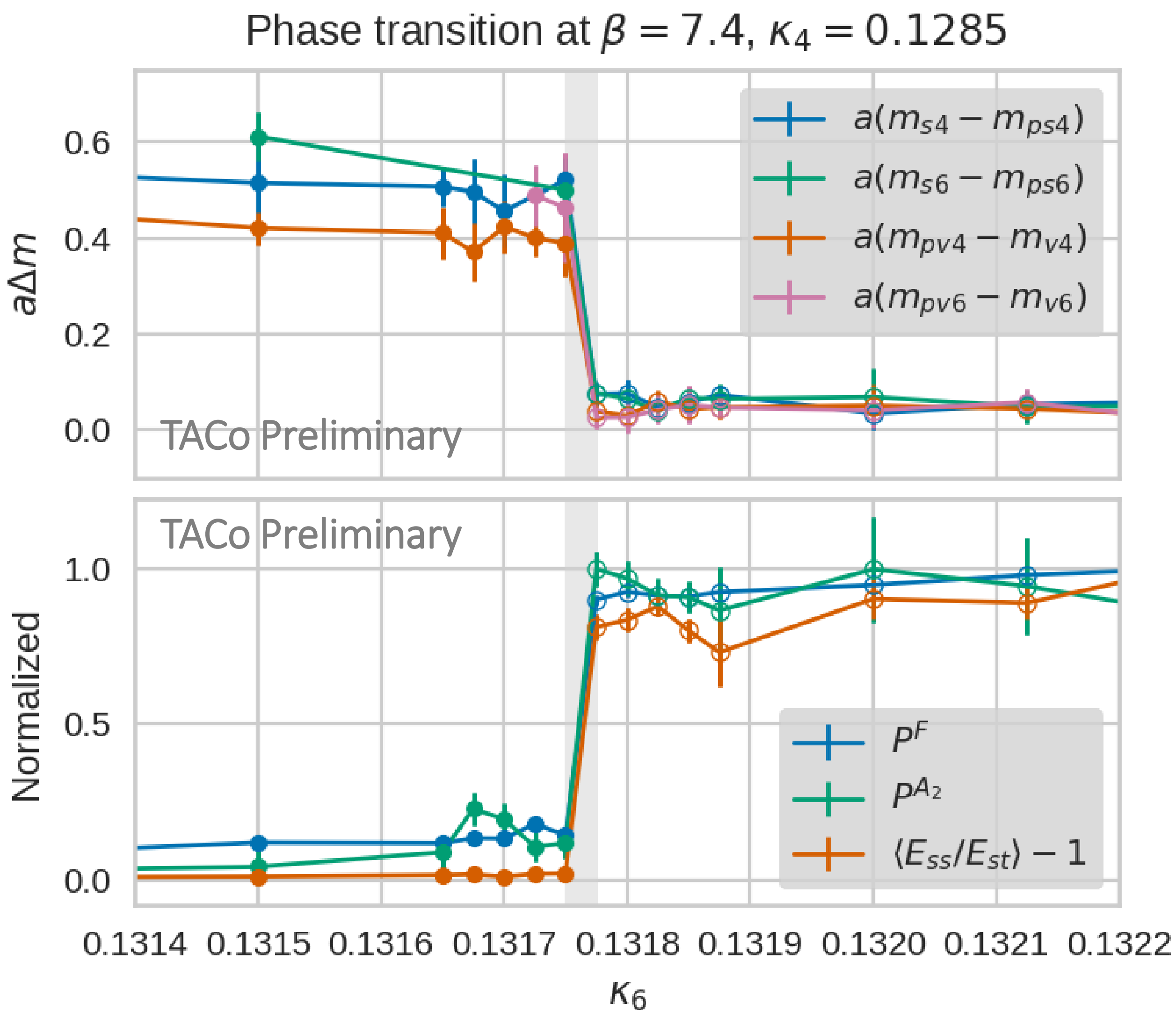
Top: (Normalized) confinement observables

Closed circles: Confined ensembles

Open circles: Deconfined ensembles

Gray band: Location of transition

(Some points missing in bottom plot because scalar & pseudovector are noisy and hard to fit in confined phase)



Multirep Pisarski-Wilczek: Setup

Examine critical behavior of EFT of bilinear quark condensates

- $\Leftrightarrow$ Analyze fixed points of a linear sigma model

Procedure [Pisarski & Wilczek 1984]:

- Identify symmetries and SSB pattern
- Write down most general Lagrangian consistent with symmetries
  - Only include relevant and marginal terms
- Compute β functions (to one loop)
  - We only observe one transition, so drive both irreps to criticality simultaneously
  - ϵ expansion: At finite T, theory becomes effectively 3D, so set $\epsilon = 4 - d = 1$
- Perform stability analysis
  - No IR-stable fixed points exist $\Rightarrow$ transition **must** be first order
  - Any IR-stable fixed points exist $\Rightarrow$ transition **can** be second-order

Anomalies in Multirep Theories

Theories with two fermion irreps have two independent axial rotations

This study: $\mathrm{U}(1)_A^{(F)}$, $\mathrm{U}(1)_A^{(A_2)}$

There is only one axial anomaly

$\Rightarrow$ There is a non-anomalous $\mathrm{U}(1)_A$ symmetry!

In our theory, the non-anomalous $\mathrm{U}(1)_A$ obeys the condition:

$\alpha_F = 2\alpha_{A_2}$

[Clark, Leung, Love, Rosner 1986]

[DeGrand, Golterman, Neil, Shamir 2016]

where

$$q \to e^{i\alpha_F \gamma_5} q$$
$$Q \to e^{i\alpha_{A_2} \gamma_5} Q$$

[Behavior under axial rotations]

Multirep P-W: Symmetry Breaking Pattern

F is a complex irrep, so get the "usual" SSB pattern

$$SU(N_F)_L \times SU(N_F)_R \to \mathrm{SU}(N_F)_V$$

A_2 is a real irrep ⇒ Different SSB pattern

$$\mathrm{SU}(2N_{A_2}) \to \mathrm{SO}(2N_{A_2})$$

Non-anomalous $U(1)_A$ is also spontaneously broken (Axial pNGB!)

Full SSB pattern:

$$SU(N_F)_L \times SU(N_F)_R \times SU(2N_{A_2}) \times U(1)_A \to SU(N_F)_V \times SO(2N_{A_2})$$

Induce SSB by tuning potential

⇒ LHS must be a good symmetry of the Lagrangian

Multirep P-W: F and A_2 Sectors

Fundamental Sector [PW 1984]

Complex irrep

⇒ Invariant under $\mathrm{SU}(N_F)_L \times \mathrm{SU}(N_F)_R$

$N_F \times N_F$ complex matrix field ϕ

Lagrangian must be invariant under

$$\phi \to \underbrace{e^{2i\alpha_F}}\underbrace{U_L}\phi\underbrace{U_R}$$

Phase from axial rotation $q \to e^{i\alpha_F\gamma_5}q$

Independent $\mathrm{SU}(N_F)$ transformations

Antisymmetric Sector [BPV 2005]

Real irrep

⇒ Invariant under $\mathrm{SU}(2N)$

$2N_{A_2} \times 2N_{A_2}$ symmetric complex matrix field θ

Lagrangian must be invariant under

$$\theta \to \underbrace{e^{2i\alpha_{A_2}}}\underbrace{V}\theta\underbrace{V^T}$$

Phase from axial rotation $Q \to e^{i\alpha_{A_2}\gamma_5}Q$

$\mathrm{SU}(2N_{A_2})$ transformation

$$\mathcal{L}_F = \mathrm{Tr}[\partial_\mu\phi^\dagger\partial^\mu\phi] + r_F\mathrm{Tr}[\phi^\dagger\phi] + v_F(\mathrm{Tr}[\phi^\dagger\phi])^2 + u_F\mathrm{Tr}\left[(\phi^\dagger\phi)^2\right]$$

$$\mathcal{L}_{A_2} = \mathrm{Tr}[\partial_\mu\theta^\dagger\partial^\mu\theta] + r_{A_2}\mathrm{Tr}[\theta^\dagger\theta] + v_{A_2}(\mathrm{Tr}[\theta^\dagger\theta])^2 + u_{A_2}\mathrm{Tr}\left[(\theta^\dagger\theta)^2\right]$$

Multirep P-W: Anomaly Sector

Implement anomaly by adding explicit $\mathrm{U}(1)_A$-breaking terms like

$$\det\phi \to e^{2iN_F\alpha_F}\det\phi$$

$$\det\theta \to e^{4iN_{A_2}\alpha_{A_2}}\det\theta$$

[Behavior under chiral rotations]

For our theory, require $\mathcal{L}\to\mathcal{L}$ when $\alpha_F = 2\alpha_{A_2}$

Lowest order term that obeys symmetry:

$$\det\phi\,\det\theta^\dagger \to e^{4i(\alpha_F-2\alpha_{A_2})}\det\phi\,\det\theta^\dagger$$

[Plugged in $N_F=2$, $N_{A_2}=2$]

But:

$$\left[\det\phi\,\det\theta^\dagger\right] = N_F + 2N_{A_2} = 2+4=6$$

[Operator dimension]

Lowest-order anomaly term is irrelevant!

Anomaly does not play a role in physics of this theory's phase transition.

Multirep P-W: Full Lagrangian

No anomaly terms, both $\mathrm{U}(1)_A^{(F)}$ and $\mathrm{U}(1)_A^{(A_2)}$ are good symmetries separately.

Multirep Sector

Only one non-irrelevant term with ϕ and θ that respects symmetries:

$$\mathcal{L}_{\text{multirep}} = w\mathrm{Tr}\left[\phi^\dagger\phi\right]\mathrm{Tr}\left[\theta^\dagger\theta\right]$$ [Irrep coupling term]

Putting all the sectors together, the full Lagrangian is:

$$\mathcal{L} = \mathrm{Tr}\left[\partial_\mu\phi^\dagger\partial^\mu\phi\right] + r_F\mathrm{Tr}\left[\phi^\dagger\phi\right] + v_F\left(\mathrm{Tr}\left[\phi^\dagger\phi\right]\right)^2 + u_F\mathrm{Tr}\left[\left(\phi^\dagger\phi\right)^2\right]$$
$$+ \mathrm{Tr}\left[\partial_\mu\theta^\dagger\partial^\mu\theta\right] + r_{A_2}\mathrm{Tr}\left[\theta^\dagger\theta\right] + v_{A_2}\left(\mathrm{Tr}\left[\theta^\dagger\theta\right]\right)^2 + u_{A_2}\mathrm{Tr}\left[\left(\theta^\dagger\theta\right)^2\right]$$
$$+ w\mathrm{Tr}\left[\phi^\dagger\phi\right]\mathrm{Tr}\left[\theta^\dagger\theta\right]$$

Multirep P-W: Results

Now, turn the crank:

- Compute β functions to one loop (Complex ϕ^4 + group theory)
- Find fixed points (with mass parameters r_F, r_{A_2} tuned to zero)
- Stability analysis: determine IR-stable/IR-unstable

Find 6 fixed points

- (Of which only two fixed points have $w \neq 0$)

At one loop in the ϵ expansion, none of these fixed points are stable.

$\Rightarrow$ Prediction: transition must be first order!

Transition is First Order: Discontinuities

Quark masses for each species and plaquette jump discontinuously at transition

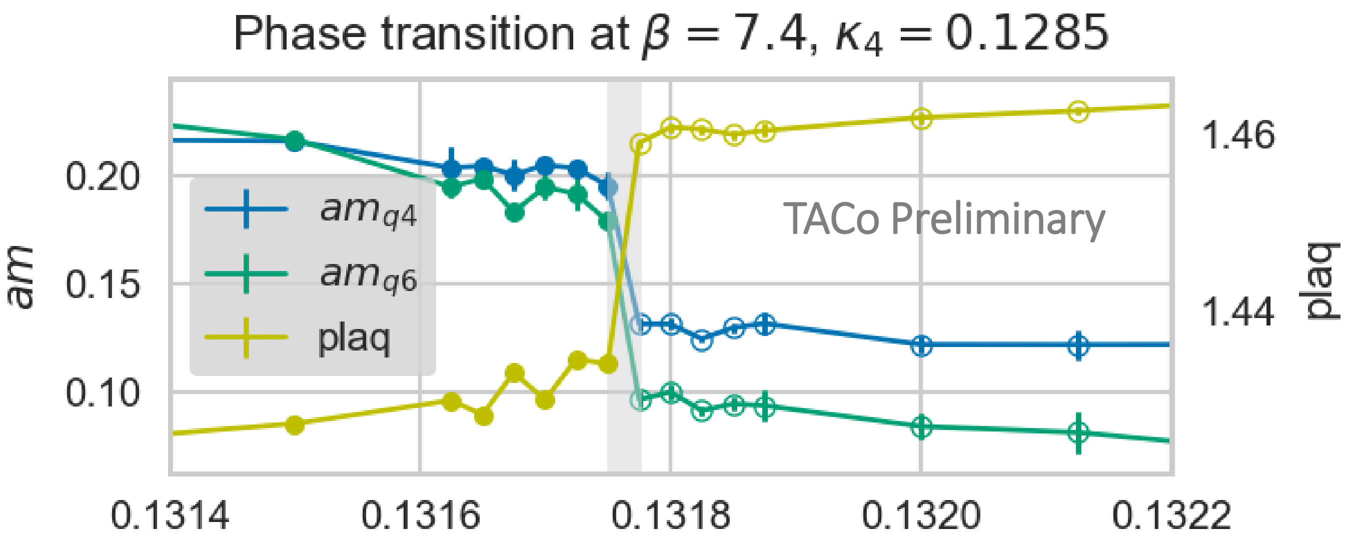
Every observable we've looked at jumps at the transition

Transition is sharp

- Interpolating more densely does not smooth out the transition
- Observables are either "confined-like" or "deconfined-like," with no interpolation

Discontinuity is present everywhere that we've looked in parameter space

⇒ (Violently) first-order transition!

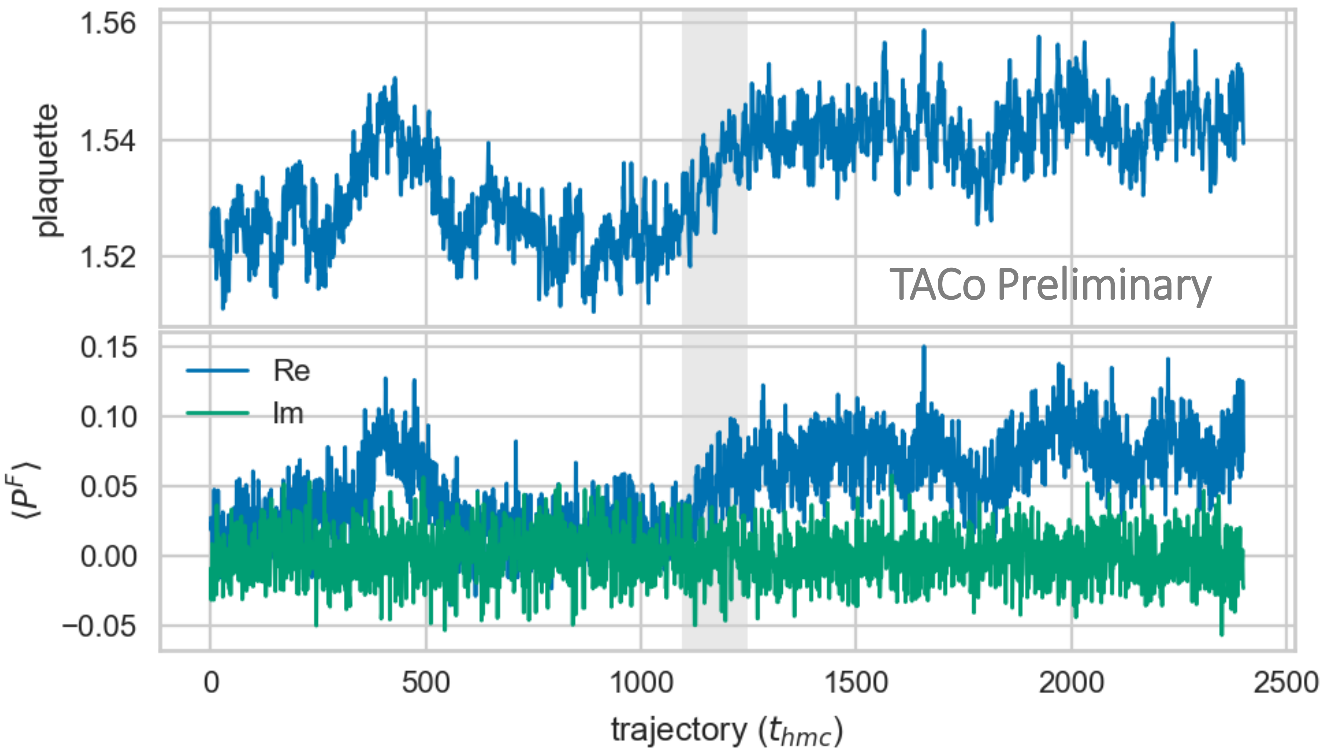


Same slice as previous slide
Left axis: axial Ward identity quark masses in lattice units
Right axis: Plaquette expectation value
Gray band: Location of transition (Same place as gray bands from previous slides)

Transition is First Order: Metastability

Plot: Metastability in HMC time

- Ensemble seeded from a confined lattice with parameters close to transition
- Ran ensemble at deconfined parameters close to transition
- Plaquette wanders until "locking in" around trajectory ~ 1200
- Appears confined until trajectory ~ 1200, when it tunnels to deconfined state
- Spectroscopy observables also changed from "confined-like" to "deconfined-like" after tunneling
- **Typical equilibration time for $12^3 \times 6$ lattices is less than 100 trajectories**



Metastability is another signal of a first-order transition (cf. hysteresis)!

We have observed several tunneling events in the course of running our data

Conclusion

Did not observe phase separation

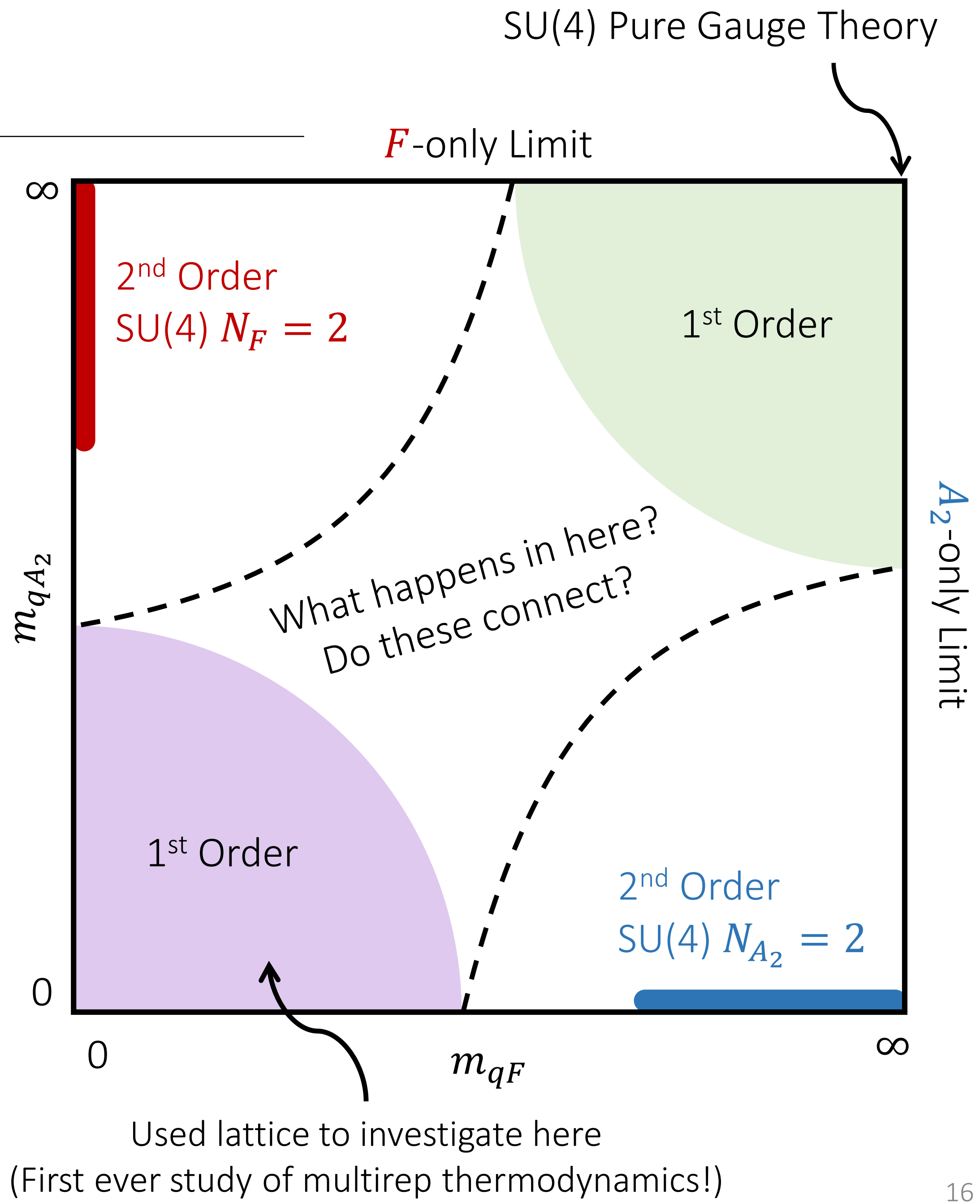
We have not explored parameter space exhaustively, but it's certainly not a typical or ubiquitous feature of the dynamics.

Phase structure looks like QCD (as did the zero-T spectroscopy)

Transition is first order

Multirep Pisarski-Wilczek calculation suggests that it must be

Open question: First order all the way from pure gauge limit to multirep chiral limit?



Motivation: Separated Phases

Can gauge theories exhibit intermediate phases between the usual "confined & chirally broken" and "deconfined & chirally restored"?

Multirep theories have many (conceptually) distinct transitions

- Confinement and chiral transitions for each irrep
- Our theory has 4 transitions: chiral F, chiral A_2, confinement F, conf. A_2
- All occur simultaneously, or are they separated?

MAC hypothesis predicts separated phases [RSD 1980]

- (Specifically: predicts A_2s chirally condense at a higher scale than Fs)

Numerical Methods

Multirep MILC [Shamir]

- SU(4) gauge theory in 3+1 dimensions
- nHYP smearing
- Clover-improved Wilson fermions (This study: $c_{sw} = 1$)
  - (Can run arbitrary number of simultaneous species in F, A_2, S_2, G!)
- nHYP Dislocation Suppressing (NDS) action [DeGrand, Shamir, Svetitsky 2014]

Spectroscopy

- Screening masses
- "Periodic+Antiperiodic BCs" trick enables spectroscopy on small lattices

MAC Hypothesis & Predictions

Potential between quarks scales like Casimirs of irreps [RSD 1980]

$$V \sim g^2(\mu)\left[\lambda_{r_1} \cdot \lambda_{r_2}\right]$$
$$\sim g^2(\mu)\left[C^2_{r_1+r_2} - C^2_{r_1} - C^2_{r_2}\right]$$

As scale runs down, most attractive channel chirally condenses first

For our theory, possible channels:

$$V_{F,\bar{F}\to 1} \sim -15/2$$
$$\boxed{V_{A_2,A_2\to 1} \sim -10} \leftarrow \text{MAC}$$
$$V_{F,A_2\to \bar{F}} \sim -5$$
$$V_{F,A_2\to 20} \sim +1$$

Naïve MAC expectation: $A_2, A_2 \to 1$ first, then $F, F \to 1$

Multirep P-W: Beta functions

$$\beta_{u_F} = -u_F + (N_F^2 + 4)u_F^2 + 4N_F v_F u_F + 3v_F^2 + 2N_{A_2}^w\left(N_{A_2}^w + 1\right)w^2$$

$$\beta_{v_F} = -v_F + 2N_F v_F^2 + 6v_F u_F$$

$$\beta_{u_{A_2}} = -u_{A_2} + \frac{1}{2}\left({N_{A_2}^w}^2 + N_{A_2}^w + 8\right)u_{A_2} + 2\left(N_{A_2}^w + 1\right)u_{A_2}v_{A_2} + \frac{3}{2}v_{A_2}^2 + 4N_F^2 w^2$$

$$\beta_{v_{A_2}} = -v_{A_2} + \left(N_{A_2}^w + \frac{5}{2}\right)v_{A_2}^2 + 6v_{A_2}u_{A_2}$$

$$\beta_w = -w + w\left(2N_F v_F + (N_F^2 + 1)u_F + \left(N_{A_2}^w + 1\right)v_{A_2} + \frac{1}{2}\left({N_{A_2}^w}^2 + N_{A_2}^w + 4\right)u_{A_2} + 2w\right)$$

Multirep P-W: Caveats and Complications

More sophisticated analysis reveals stable fixed point for $N_F = 2$ fundamental-only theory (without anomaly terms)

But: we know this fixed point will be destabilized by irrep coupling

- This stable fixed point in F sector will only show up in "decoupled product" fixed points where $w = 0$
- $N_{A_2} = 2$ antisymmetric-only theory has no stable fixed points [BPV 2005]
- If $w = 0$, then multirep corrections cannot stabilize the A_2 sector
- $\Rightarrow A_2$ sector runs away, dragging F sector along with it via w coupling
- $\Rightarrow$ All "product" fixed points are unstable because A_2 sector is unstable.

Hard to say anything higher-order about "multirep" fixed points ($w \neq 0$) without doing a much more involved calculation.