Constrained Hybrid Monte Carlo on Multiscale Lattices

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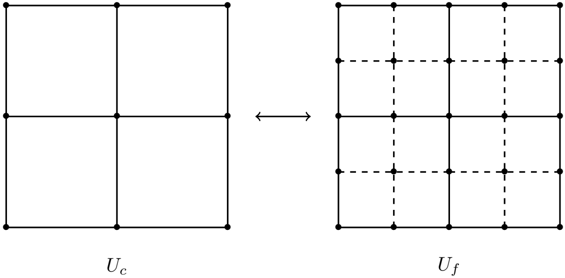
Renormalization Group (RG) Transformation

- Actions with different lattice spacings give identical low energy physics if they are related by an RG blocking,

$$e^{-S_c^b[U_c]} \equiv \int [dU_f] e^{-S_f[U_f]} G[U_c, U_f]$$

- Formally once given a *fine* action $S_f[U_f]$ and a *blocking kernel* $G[U_c, U_f]$, we can produce the *blocked coarse* action $S_c^b[U_c]$.
- The issue is this $S_c^b[U_c]$ has to be local and simple enough such that it can be handled numerically.



- Given an exact form for $S_c^b[U_c]$, observables can be calculated from constrained fine lattice Monte Carlos in an ensemble of background coarse lattice links.

$$\langle \mathcal{O} \rangle = \frac{\int [dU_f] e^{-S_f[U_f]} \mathcal{O}[U_f]}{\int [dU_f] e^{-S_f[U_f]}}$$

$$= \frac{\int [dU_c] G[U_c, U_f] [dU_f] e^{-S_f[U_f]} \mathcal{O}[U_f]}{\int [dU_c] G[U_c, U_f] [dU_f] e^{-S_f[U_f]}}$$

$$= \frac{\int [dU_c] e^{-S_c^b[U_c]} \frac{\int [dU_f] e^{-S_f[U_f]} G[U_c, U_f] \mathcal{O}[U_f]}{\int [dU_f] e^{-S_f[U_f]} G[U_c, U_f]}}{\int [dU_c] e^{-S_c^b[U_c]}}$$

- This is ambitious. We will only focus on the question of how to obtain $S_c^b[U_c]$, for now.

- Recent $2+1$ flavor ensembles generated with ID and MDWF action by RBC/UKQCD have $2-4\%$ scaling errors for currently measured observables with $a^{-1} = 1$ GeV. [cf. R. D. Mawhinney's talk today at 17:30]
- Iwasaki gauge action:

$$S_G = -\frac{\beta}{3}\left[(1-8c_1)\sum_{x,\mu>\nu} P_{\mu\nu}(x) + c_1 \sum_{x,\mu\neq\nu} R_{\mu\nu}(x)\right], \; c_1 = -0.331$$

- Möbius domain wall fermion(MDWF):

$$S_F(m) = \bar{\psi}\left[\frac{D_{\mathrm{MDWF}}(m)}{D_{\mathrm{MDWF}}(1)}\right]\psi, \;\; H = \frac{(b+c)\gamma_5 D_w}{2+(b-c)D_w}$$

- dislocation suppressed determinant ratio(DSDR):

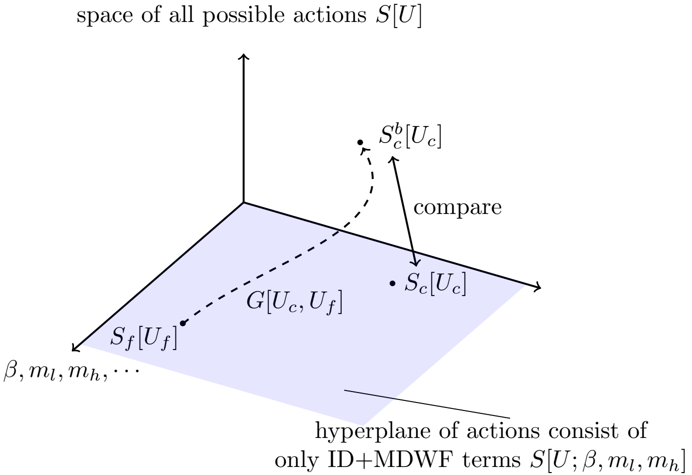
$$\det\left[\frac{H^2(-M_5)+\epsilon_f^2}{H^2(-M_5)+\epsilon_b^2}\right], \;\; H(-M_5) = \gamma_5 D_w(-M_5)$$

ID+MDWF ensembles with $m_\pi = 300$ MeV

- We have produced ensembles with $a^{-1} = 1$ and 2 GeV with closely matched masses.

	coarse, S_c	fine, S_f	% diff.	
size	$12^3 \times 32 \times 12$	$24^3 \times 64 \times 12$	–	
β	1.633	1.943	–	
am_l	0.008521	0.000787	–	
am_h	0.065073	0.019896	–	
a^{-1}[GeV]	1.015(16)	2.001(18)	–	
$am_{\rm res}(m_l)$	0.007439(86)	0.004522(12)	–	
m_π[MeV]	307(5)	300(3)	2.3	tunning error
m_K[MeV]	506(8)	491(5)	3.0	
m_Ω[MeV]	1652(27)	1557(71)	5.9	
f_π[MeV]	147(2)	138(2)	6.3	tunning and scaling error
f_K[MeV]	166(3)	155(2)	6.8	

- Good $O(a^2)$ scaling indicates an approximate RG trajectory in the hyperplane of actions consisting of only ID+MDWF terms.

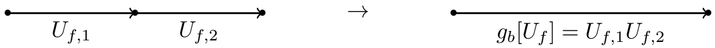


Blocking Kernel Forms Studied

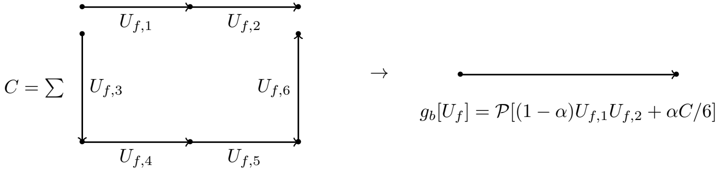
- We consider blocking kernels with the form:

$$G[U_c, U_f] = \prod_{x,\mu} \delta(U_c(x,\mu) - g_b[U_f; x, \mu])$$

- A *naive* blocking kernel:



- A single step APE-like blocking kernel:



- Given a configuration generated by gauge action we can introduce a series of *demon* variables to probe the underlining β's in the action. S_i can be plaquette(P), rectangular(R), chair loop(C), twist loop(T), $\cdots$

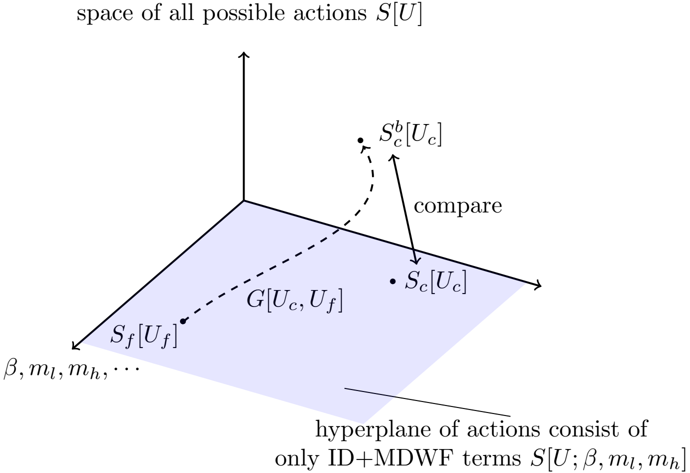
$$\int [DU] \int \prod_i [dE_i] \exp\left[- \sum_i \left(\beta_i S_i[U] + \beta_i E_i\right)\right].$$

- The update scheme consists of two parts,
  1. update U only.
  2. update U and E_i's at the same time while keeping $S_i + E_i$ constant. In this case the accept/reject part does not require knowledge of β_i's.
- Since the E_i's integration factorizes we can measure the average value of them and probe the underlying β_i's.

$$\langle E_i \rangle = \frac{1}{\beta_i} - \frac{E_{max}}{\tanh(\beta_i E_{max})}.$$

*See [T. Takaishi, 1995].

- Generating a sequence of fine lattices and then blocking them, gives coarse, blocked lattices that appear with a probability proportional to $e^{-S_c^b[U_c]} \propto \int [dU_f] e^{-S_f[U_f]} G[U_c, U_f] \Big/ \int [dU_f] e^{-S_f[U_f]}$,



- Apply demon algorithm on the blocked coarse lattice with different α.

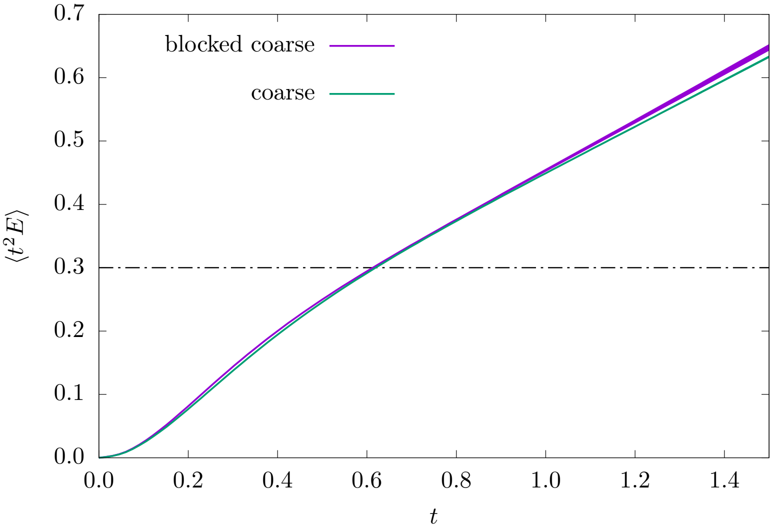
ensemble	β_P	β_R	β_C	β_T
ID+MDWF	$2.035(36)$	$-0.1018(33)$	$-0.0026(30)$	$-0.0006(30)$
$\alpha = 0.0$	$0.617(11)$	$0.0491(33)$	$0.0032(32)$	$0.0010(32)$
$\alpha = 0.5$	$1.478(35)$	$-0.0020(44)$	$0.0043(42)$	$-0.0016(43)$
$\alpha = 0.688$	$2.030(28)$	$-0.1522(30)$	$-0.0021(24)$	$0.0038(24)$
$\alpha = 0.7$	$2.069(33)$	$-0.1589(33)$	$0.0009(27)$	$-0.0003(27)$

- We will use $\alpha = 0.688$.

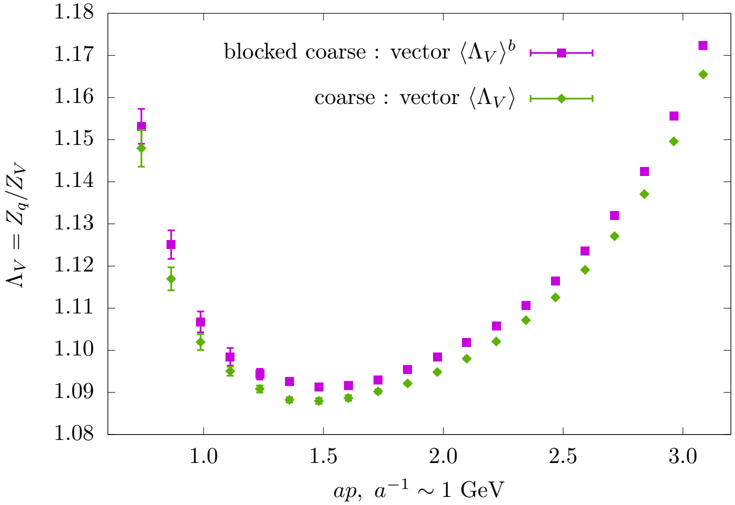
Spectrum

	coarse, $\langle \mathcal{O} \rangle$	blocked coarse[†], $\langle \mathcal{O} \rangle^b$	% diff.
size	$12^3 \times 32 \times 12$	$12^3 \times 32 \times 12$	–
β	1.633	–	–
am_l	0.008521	–	–
am_h	0.065073	–	–
$a^{-1}[\mathrm{GeV}]$	$1.015(16)$	$1.010(16)$	–
$am_{\mathrm{res}}(m_l)$	$0.007439(86)$	$0.00847(21)$	–
am_π	$0.3026(13)$	$0.3144(34)$	3.9
am_K	$0.4982(11)$	$0.5072(24)$	1.8
am_Ω	$1.628(10)$	$1.658(13)$	1.8
af_π	$0.14472(64)$	$0.14999(99)$	3.6
af_K	$0.16333(47)$	$0.16791(85)$	2.8

[†]Valence quark masses are taken to be the same as the coarse ensemble.

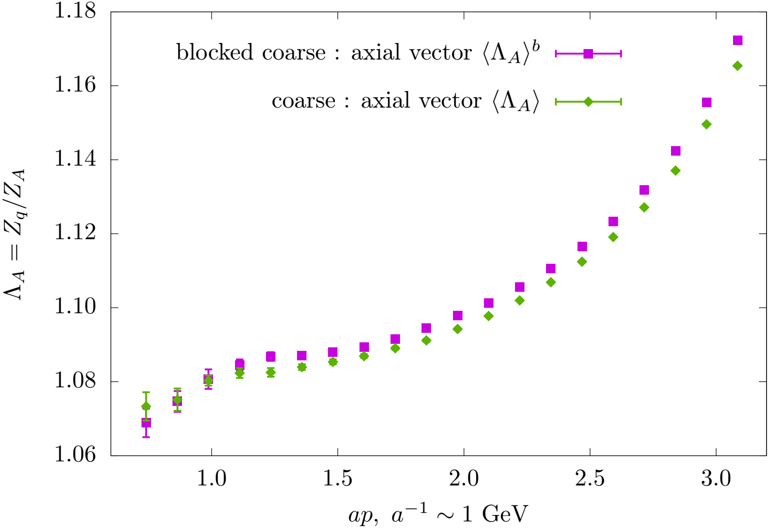


blocked coarse
coarse
$\langle t^2 E \rangle$
t
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4

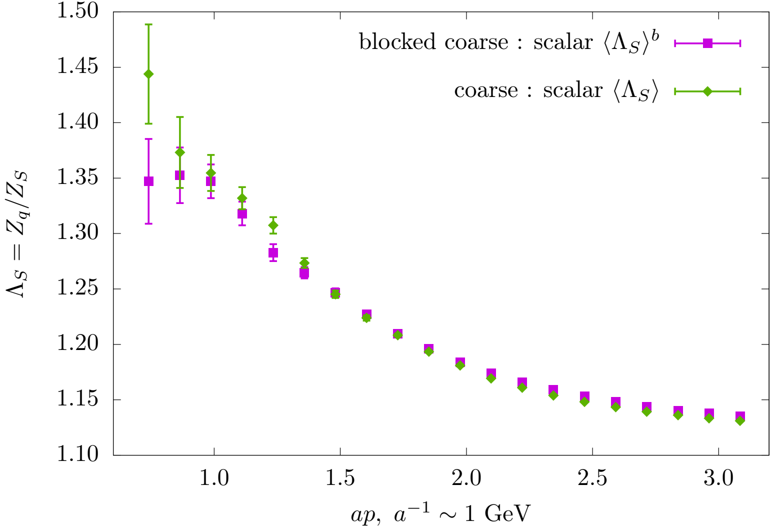


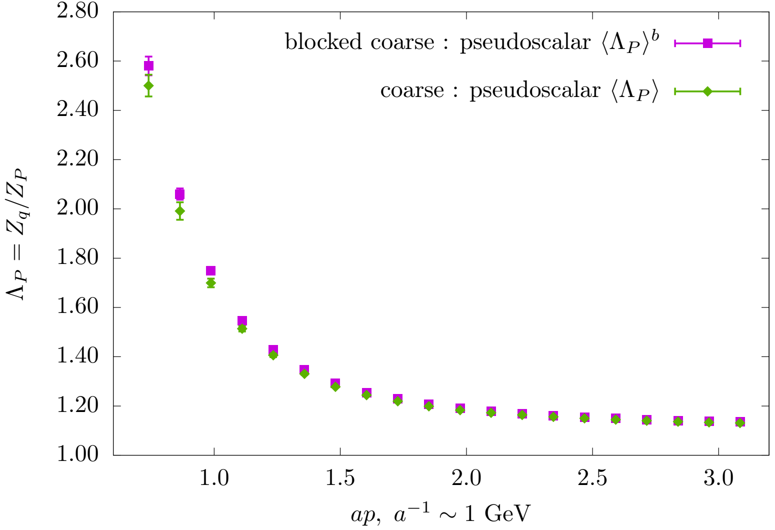
[‡] All valence quark masses are taken to be the sea quark masses of the coarse ensemble. Non-exceptional momenta are used. Refer to [R. Arthur, 2011, arXiv:1006.0422].

Nonperturbative Renomalization(cont'd)



Nonperturbative Renomalization(cont'd)





blocked coarse : pseudoscalar $\langle \Lambda_P \rangle^b$
coarse : pseudoscalar $\langle \Lambda_P \rangle$
$\Lambda_P = Z_q/Z_P$
$ap,\ a^{-1} \sim 1$ GeV
2.80
2.60
2.40
2.20
2.00
1.80
1.60
1.40
1.20
1.00
1.0
1.5
2.0
2.5
3.0

Conclusion

- We have produced 12^3 and 24^3 ensembles with $300\ \mathrm{MeV}$ pions using the ID+MDWF action.
- Using the demon algorithm and a simple APE-like blocking kernel, we have tuned the single blocking parameter to match the blocked coarse lattice and original ID+MDWF coarse lattice.
- It turns out the blocked coarse action produced by this simple APE-like blocking kernel is only a few percent different from the ID+MDWF coarse action.

So what?

- It might be possible to use this close matching to generate fine lattices from coarse lattices.
- An exact correction factor is likely needed to go from few percent discrepancies to an exact algorithm.

Parameter	NLO(370 MeV)	NLO(450 MeV)
$c^{\mathrm{I}}_{f_\pi}$ [GeV^2]	$0.059(47)$	$-0.028(51)$
$c^{\mathrm{ID}}_{f_\pi}$ [GeV^2]	$-0.013(17)$	$-0.058(19)$
$c^{\mathrm{I}}_{f_K}$ [GeV^2]	$0.049(39)$	$-0.035(38)$
$c^{\mathrm{ID}}_{f_K}$ [GeV^2]	$-0.005(15)$	$-0.044(14)$

Parameter	NNLO(370 MeV)	NNLO(450 MeV)
$c^{\mathrm{I}}_{f_\pi}$ [GeV^2]	$0.081(48)$	$0.065(45)$
$c^{\mathrm{ID}}_{f_\pi}$ [GeV^2]	$0.013(15)$	$0.012(16)$
$c^{\mathrm{I}}_{f_K}$ [GeV^2]	$0.070(41)$	$0.069(36)$
$c^{\mathrm{ID}}_{f_K}$ [GeV^2]	$0.011(15)$	$0.019(15)$

[§] See [P. A. Boyle, 2016, arXiv:1511.01950].

Observables

$$
\begin{aligned}
\langle \mathcal{O} \rangle \leftrightarrow \langle \mathcal{O} \rangle^b &= \frac{1}{Z} \int [dU_c] e^{-S_c^b[U_c]} \mathcal{O}[U_c] \\
&= \frac{1}{Z} \int [dU_c] \int [dU_f] G[U_c, U_f] e^{-S_f[U_f]} \mathcal{O}[U_c] \\
&= \frac{1}{Z} \int [dU_f] e^{-S_f[U_f]} \int [dU_c] G[U_c, U_f] \mathcal{O}[U_c] \\
&= \frac{1}{Z} \int [dU_f] e^{-S_f[U_f]} \tilde{\mathcal{O}}[U_f]
\end{aligned}
$$