Coarse Dynamical Fermion Lattices

Lattice 2017
Granada, Spain
June 20, 2017

Robert Mawhinney
Columbia University
RBC and UKQCD Collaborations

Special thanks to Chulwoo Jung, David Murphy and Jiqun Tu

The RBC & UKQCD collaborations

BNL and RBRC

Mattia Bruno
Tomomi Ishikawa
Taku Izubuchi
Luchang Jin
Chulwoo Jung
Christoph Lehner
Meifeng Lin
Hiroshi Ohki
Shigemi Ohta (KEK)
Amarjit Soni
Sergey Syritsyn

Columbia University

Ziyuan Bai
Norman Christ
Duo Guo
Christopher Kelly
Bob Mawhinney
David Murphy
Masaaki Tomii
Jiqun Tu
Bigeng Wang
Tianle Wang

University of Connecticut

Tom Blum
Dan Hoying
Cheng Tu

Edinburgh University

Peter Boyle
Guido Cossu
Luigi Del Debbio
Richard Kenway
Julia Kettle
Ava Khamseh
Brian Pendleton
Antonin Portelli
Tobias Tsang
Oliver Witzel
Azusa Yamaguchi

KEK

Julien Frison

University of Liverpool

Nicolas Garron

Peking University

Xu Feng

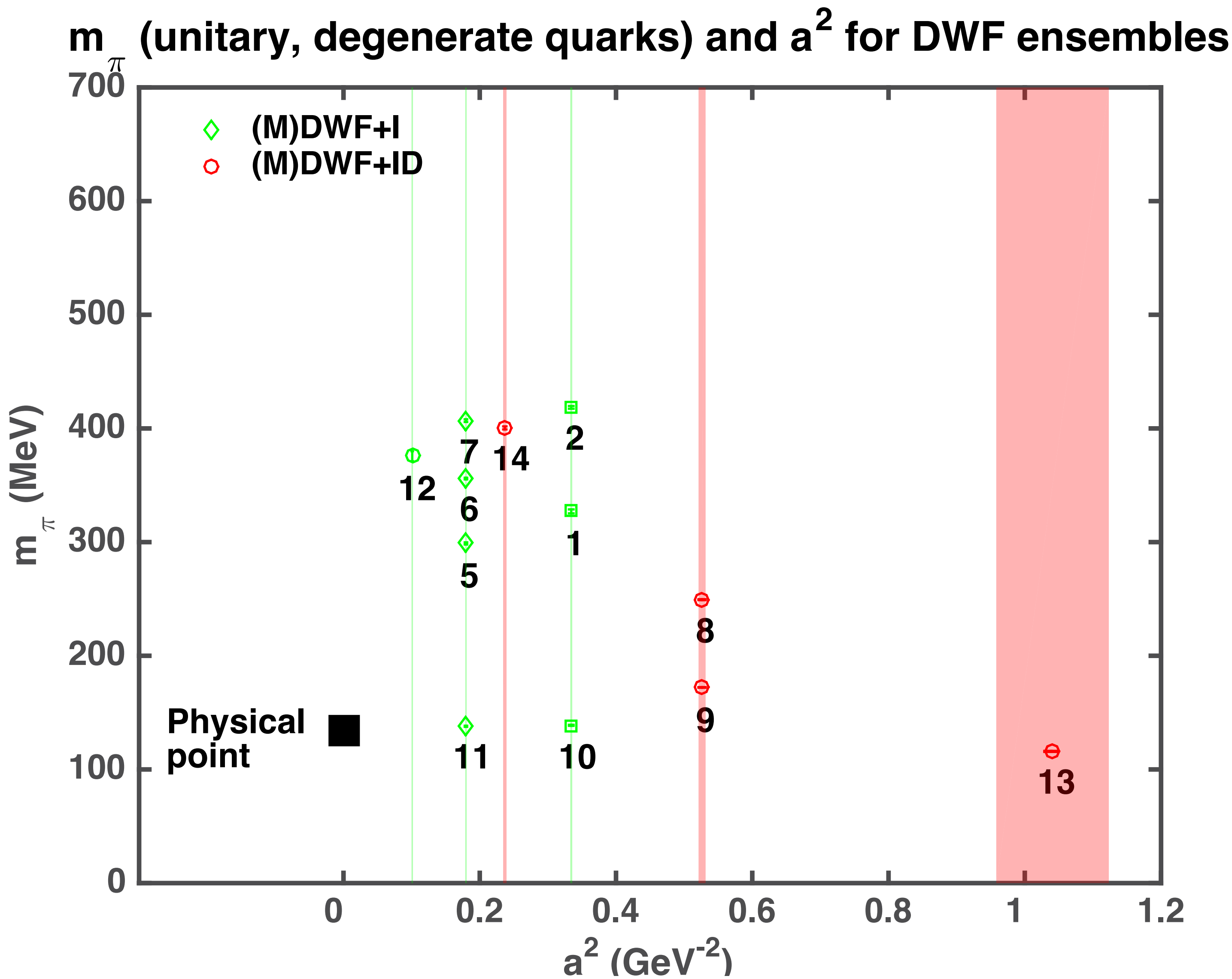
University of Southampton

Jonathan Flynn
Vera Guelpers
James Harrison
Andreas Juettner
Andrew Lawson
Edwin Lizarazo
Chris Sachrajda

York University (Toronto)

Renwick Hudspith

RBC/UKQCD 2+1 Flavor DWF Ensembles

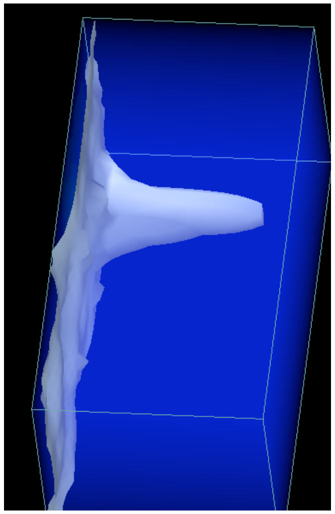


Balancing m_{res} and Topological Tunneling for DWF

- The propagation of light modes between the five-dimensional boundaries is controlled by the eigenvalues of the transfer matrix, H_T

$$H_T = \gamma_5 D_W(M) \frac{1}{2 + (b_i - c_i) D_W(M)}$$

- Zeros of $D_w(M)$ produce modes not bound to the five-dimensional boundaries
- These zeros occur when the gauge fields are changing topology (picture from PRD 77 (2008) 014509)



- Refer to this type of localized fluctuation in the gauge fields as a dislocation.
- For a given L_s, dislocations increase the size of the residual mass, m_{res}.

Choices of Action

- For 1/a in range 1.5 - 2.5 GeV, Iwasaki gauge action suppresses dislocations sufficiently with 2+1 flavors of fermions to allow physical light quark masses to be reached.

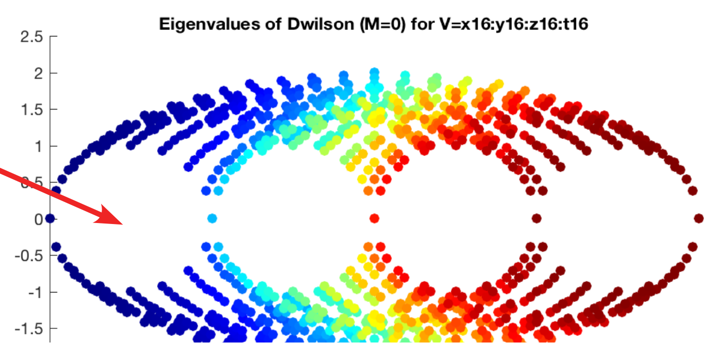
  * 1/a = 1.73 GeV: L_s = 24 for MDWF (b+c=2) gives $m_{res} = 0.45\ m_{ud}$

  * 1/a = 2.31 GeV: L_s = 12 for MDWF (b+c=2) gives $m_{res} = 0.32\ m_{ud}$

- For stronger couplings, add the Dislocation Suppressing Determinant Ratio (DSDR) to suppress topological tunneling

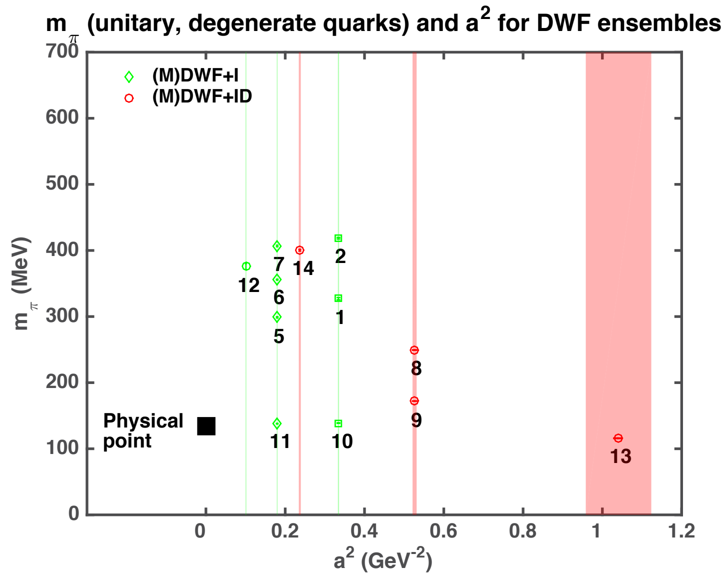
$$\det\left(\frac{D_W^\dagger(M)\,D_W(M)+\varepsilon_f^2}{D_W^\dagger(M)\,D_W(M)+\varepsilon_b^2}\right)=\prod_\lambda \frac{\lambda^2+\varepsilon_f^2}{\lambda^2+\varepsilon_b^2} \qquad \varepsilon_f<\varepsilon_b$$

choose $M = -M_5$



  * 1/a = 1.35 GeV: L_s = 12 for MDWF (b+c=32/12) gives $m_{res} = 0.95\ m_{ud}$

2+1 Flavor Iwasaki + DSDR (M)DWF ensembles



- Original DSDR ensemble had 1/a = 1.37(1) GeV, m_π = 170 MeV and V = (4.7 fm)^3

  * Another ensemble, with G-parity boundary conditions, generated for K –› ππ matrix elements calculations with m_π = 170 MeV

- For HotQCD thermodynamics study of the QCD phase transition with MDWF quarks, two T=0 DSDR ensembles were generated at 1/a = 0.98(4) and 2.02(1) GeV (PRL 113 (2014) no.8 082001).

- Global fits show small O(a^2) errors for MDWF ensembles, even at 1/a = 1 GeV.

SU(2) ChPT Fits to m_{PS} and f_{PS}

- We can simultaneously fit lattice data for different lattice spacings, actions and volumes using expansions of the form (SU(2) NLO example):

$$
\begin{aligned}
(m_{ll}^{\mathbf{e}})^2 &= \chi_l^{\mathbf{e}} + \chi_l^{\mathbf{e}} \cdot \left\{ \frac{16}{f^2}\Big((2L_8^{(2)} - L_5^{(2)}) + 2(2L_6^{(2)} - L_4^{(2)})\Big)\chi_l^{\mathbf{e}} + \frac{1}{16\pi^2 f^2}\chi_l^{\mathbf{e}} \log \frac{\chi_l^{\mathbf{e}}}{\Lambda_\chi^2} \right\} \\
f_{ll}^{\mathbf{e}} &= f\left[1 + c_f (a^{\mathbf{e}})^2\right] + f \cdot \left\{ \frac{8}{f^2}(2L_4^{(2)} + L_5^{(2)})\chi_l^{\mathbf{e}} - \frac{\chi_l^{\mathbf{e}}}{8\pi^2 f^2} \log \frac{\chi_l^{\mathbf{e}}}{\Lambda_\chi^2} \right\}
\end{aligned}
$$

with

$$
\chi_l^{\mathbf{e}} = \frac{Z_l^{\mathbf{e}}}{R_a^{\mathbf{e}}} \frac{B^{\mathbf{1}} \widetilde{m}_l^{\mathbf{e}}}{(a^{\mathbf{e}})^2}
$$

- At NNLO order, using codes from Bijnens and collaborators, we fit to

$$
X(\tilde{m}_q, L, a^2) \simeq X_0 \Big(1 + \underbrace{X^{\rm NLO}(\tilde{m}_q) + X^{\rm NNLO}(\tilde{m}_q)}_{\text{NNLO Continuum PQChPT}} + \underbrace{\Delta_X^{\rm NLO}(\tilde{m}_q, L)}_{\text{NLO FV corrections}} + \underbrace{c_X a^2}_{\text{Lattice spacing}} \Big)
$$

- For SU(2), we use m_π, m_K and m_Ω to set the scale.
- There are different a^2 corrections to the decay constants for I and ID actions.
- Heavy quark ChPT used for light quark extrapolation of kaon.
- $t_0^{1/2}$ and w_0 are also fit using a linear chiral ansatz.

Scaling Errors for f_π and f_K

- Fits use different $O(a^2)$ coefficients for Iwasaki and Iwasaki+DSDR actions
- Results for these coefficients from PRD 93 054502 (2016):

	NLO (370 MeV cut)	NNLO (450 MeV cut)
Iwasaki f_π a^2 coeff.	0.059(47) GeV^2	0.065(45) GeV^2
DSDR f_π a^2 coeff.	-0.013(17) GeV^2	0.012(16) GeV^2
Iwasaki f_K a^2 coeff.	0.049(39) GeV^2	0.069(36) GeV^2
DSDR f_K a^2 coeff.	-0.005(15) GeV^2	0.019(15) GeV^2

- For 1/a = 1 GeV, percent scaling error:

	NLO (370 MeV cut)	NNLO (450 MeV cut)
Iwasaki f_π	6 ± 5%	7 ± 5%
DSDR f_π	-1 ± 2%	1 ± 2%
Iwasaki f_K	5 ± 4%	7 ± 4%
DSDR f_K	-1 ± 2%	2 ± 2%

- Canonical scaling errors should be $(a\Lambda_{QCD}^{(3)})^2 \sim (330\,\mathrm{MeV}/980\,\mathrm{MeV})^2 \sim 0.11$.
- 2+1 flavor physical quark mass simulations at strong coupling well behaved.

Scaling Errors For More Observables

- We have preliminary fits with more observables, including the $\pi\pi$ I=2 scattering length (David Murphy)
- Show results for SU(2) NNLO fits with pseudoscalar masses below 450 MeV

	Iwasaki a^2 coefficient	DSDR a^2 coefficient
f_π	0.070±0.041	0.022±0.017
f_K	0.079±0.034	0.030±0.014
$t_0^{1/2}$	-0.017±0.041	-0.021±0.020
w_0	-0.117±0.360	-0.039±0.018
a_0^2 (I=2 pi-pi scattering)	-0.15±0.33	-0.04±0.45

1 GeV Ensembles

- Evidence presented shows that we have an action that allows strong coupling simulations with 2+1 flavors at physical quark masses

  * Small a^2 corrections
  * Rapid topological tunneling
  * No execeptional configurations
  * Good chiral symmery properties from MDWF

- We expect these ensembles will be very useful for

  * Studying finite volume effects for QCD and QCD+QED physics, like g-2
  * Developing and testing methods at physical quark masses
  * Measurements requiring large statistics and/or good topological sampling.

- We are generating 3 ensembles with 1/a = 1 GeV

  * 24^3: physical volume is $(4.8\ \mathrm{fm})^3$, $m_\pi L = 3.4$, currently ~1900 MD time units
  * 32^3: physical volume is $(6.4\ \mathrm{fm})^3$, $m_\pi L = 4.5$, currently ~500 MD time units
  * 48^3: physical volume is $(9.6\ \mathrm{fm})^3$, $m_\pi L = 6.7$, currently ~500 MD time units

Preliminary Measurements on 24^3 1 GeV Ensemble

- Results from 378 configurations.
- Input quark masses for this ensemble from our global chiral fits.
- Global fits give 1/a = 0.981 GeV and small $O(a^2)$ scaling errors.
- Statistical errors probably underestimated since autocorrelations not included (yet).

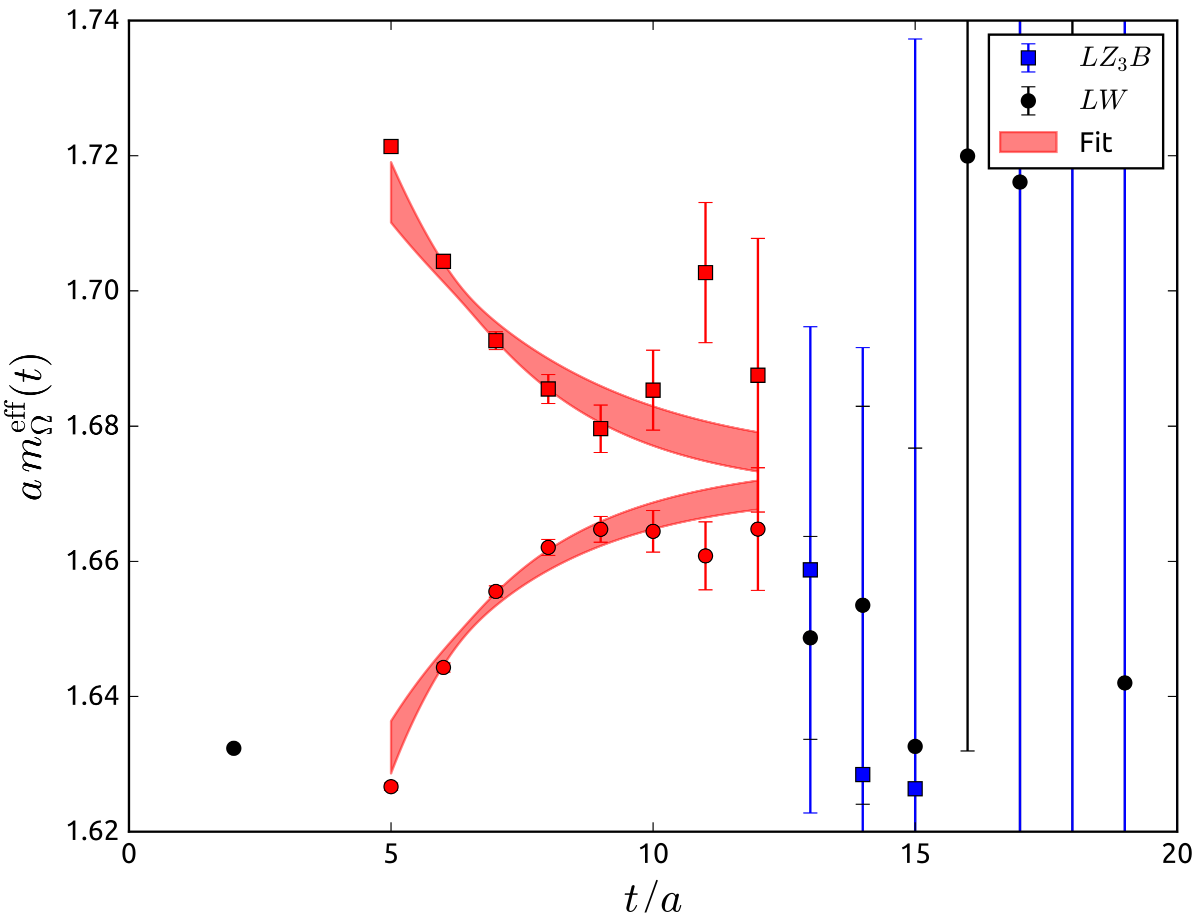
L	24
T	64
L_s	24
β	1.633
am_l	0.00107
am_h	0.0850

Table 1: Ensemble parameters

Observable	Fit	% err.
$am'_{\rm res}(m_l)$	0.0022824(70)	0.31
am_π	0.13975(10)	0.07
am_K	0.504154(89)	0.02
am_Ω	1.6726(25)	0.15
am'_Ω	2.040(63)	3.09
af_π	0.13055(11)	0.09
af_K	0.15815(13)	0.09

Omega Baryon Effective Mass on 24^3 1 GeV Ensemble

- Two sources: Coulomb gauge fixed wall source and 8 smaller Coulomb gauge fixed wall sources.
- Fit to common ground and excited states.



Comparing 24^3 and 32^3 1 GeV Ensembles

- Results from 378 configurations on 24^3 and 87 configurations on 32^3.

24^3

Observable	Fit	% err.
$am'_{\text{res}}(m_l)$	0.0022824(70)	0.31
am_π	0.13975(10)	0.07
am_K	0.504154(89)	0.02
am_Ω	1.6726(25)	0.15
am'_Ω	2.040(63)	3.09
af_π	0.13055(11)	0.09
af_K	0.15815(13)	0.09

32^3

Observable	Fit	% err.
$am'_{\text{res}}(m_l)$	0.0022764(76)	0.33
am_π	0.139474(96)	0.07
am_K	0.50457(20)	0.04
am_Ω	1.6704(28)	0.17
am'_Ω	2.20(11)	4.90
af_π	0.13122(18)	0.14
af_K	0.15887(35)	0.22

Results are preliminary

32^3 1 GeV Ensemble with $m_K \approx 300$

- To continue our studies of SU(3) ChPT fits, we are also generating an ensemble at 1 GeV with physical pion masses and a much lighter kaon, to test the accuracy of SU(3) ChPT.
- Ensembles have identical input parameters, except for a different m_s.

32^3 with physical m_K

Observable	Fit	% err.
$am'_{\rm res}(m_l)$	0.0022764(76)	0.33
am_π	0.139474(96)	0.07
am_K	0.50457(20)	0.04
am_Ω	1.6704(28)	0.17
am'_Ω	2.20(11)	4.90
af_π	0.13122(18)	0.14
af_K	0.15887(35)	0.22

32^3 with $m_K \approx 300$ MeV

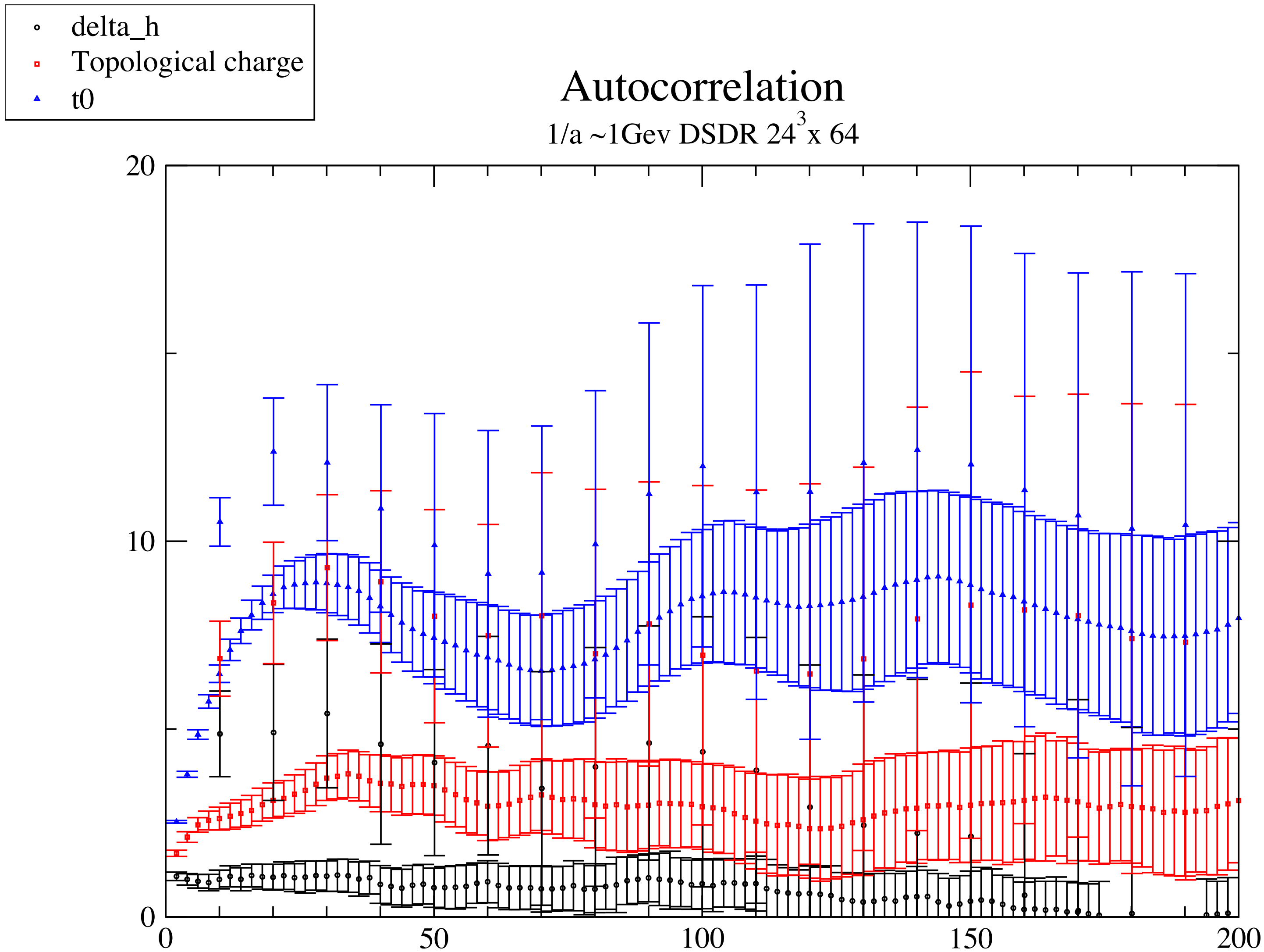
Observable	Fit	% err.
$am'_{\rm res}(m_l)$	0.0020539(96)	0.47
am_π	0.13412(27)	0.20
am_K	0.31498(20)	0.06
am_Ω	1.3316(43)	0.32
am'_Ω	1.832(56)	3.04
af_π	0.12435(21)	0.17
af_K	0.13734(22)	0.16

- One sees directly the dynamical strange quark mass dependence of f_π.

Results are preliminary

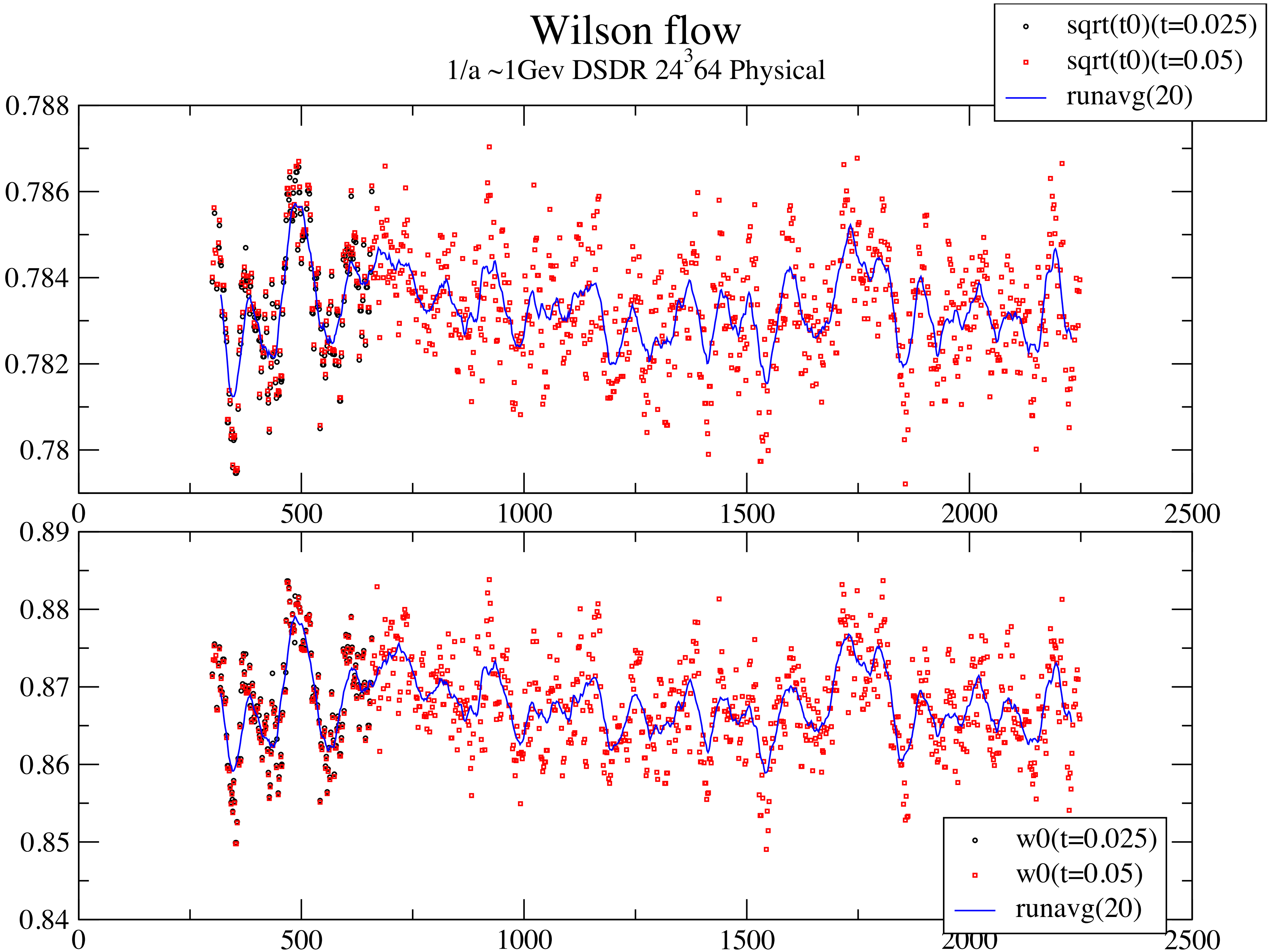
Algorithmic Questions

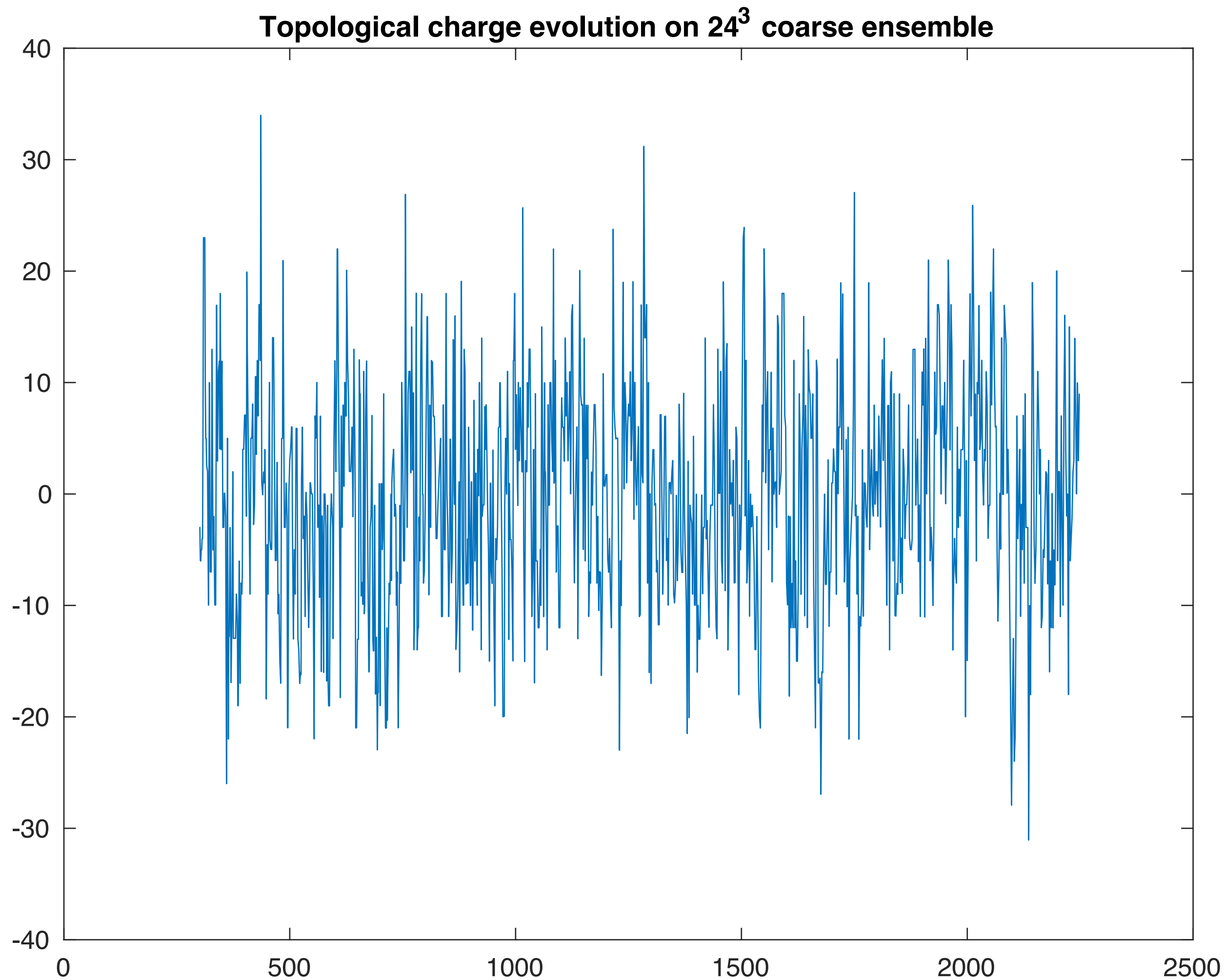
- Using HMC + RHMC for these evolutions.
- Given the strong coupling, should be a relatively easy area to study algorithms.
- First some integrated autocorrelation time estimates.

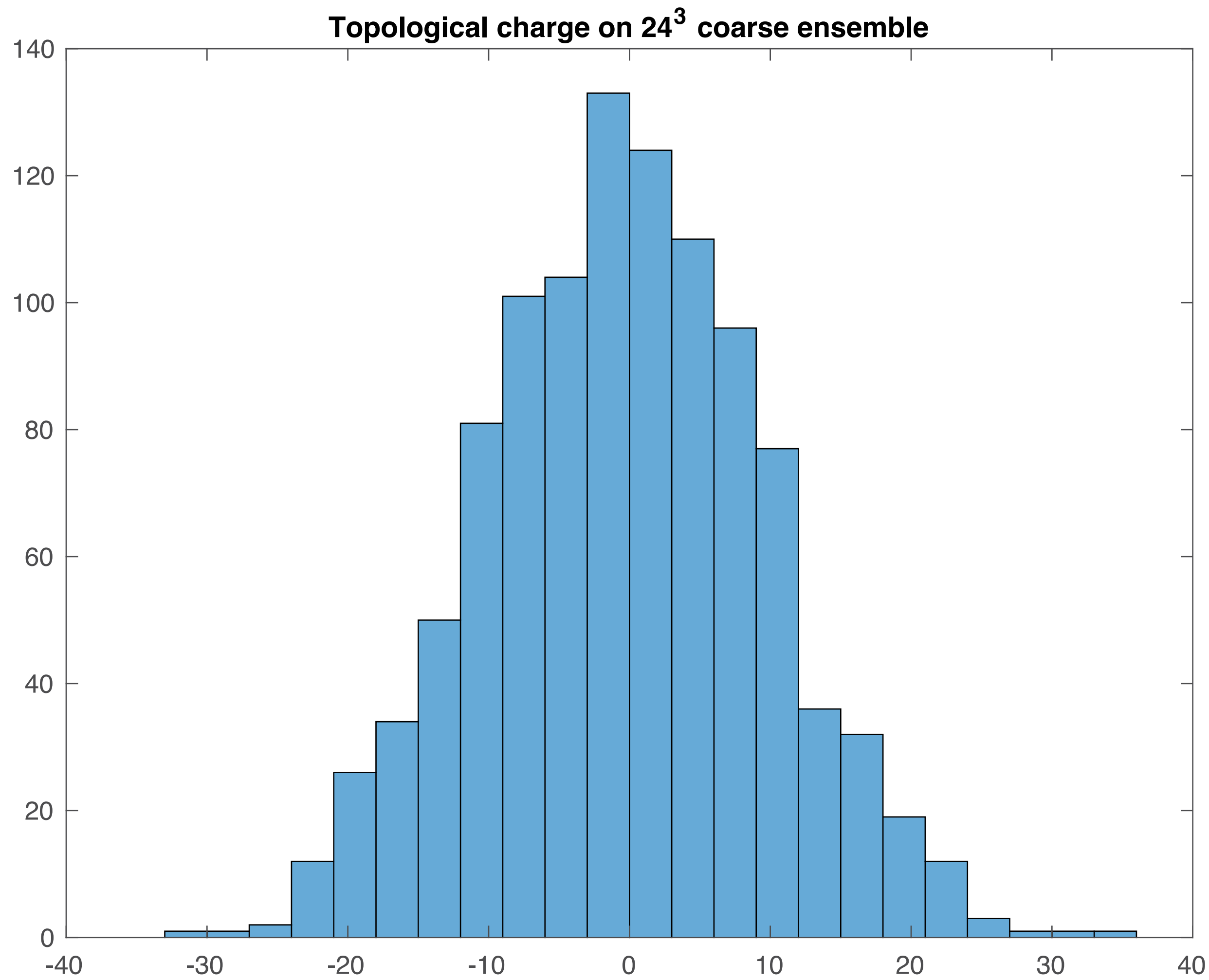


Wilson flow

1/a ~1Gev DSDR $24^3 64$ Physical




Topological charge evolution on 24^3 coarse ensemble
40
30
20
10
0
-10
-20
-30
-40
0
500
1000
1500
2000
2500



Topological charge on 24^3 coarse ensemble
140
120
100
80
60
40
20
0
-40
-30
-20
-10
0
10
20
30
40

Diffusion of Topological Charge - quenched theory

- In McGlynn and Mawhinney (PRD 90 (2014) 074502) the diffusion of topological charge was measured for lattices with both open and periodic temporal boundary conditions.

- The correlation function

$$C(t, t_0, \tau) \equiv \langle Q(t, \tau_0 + \tau) Q(t_0, \tau_0) \rangle$$

was measured and fit to a diffusion model, given by

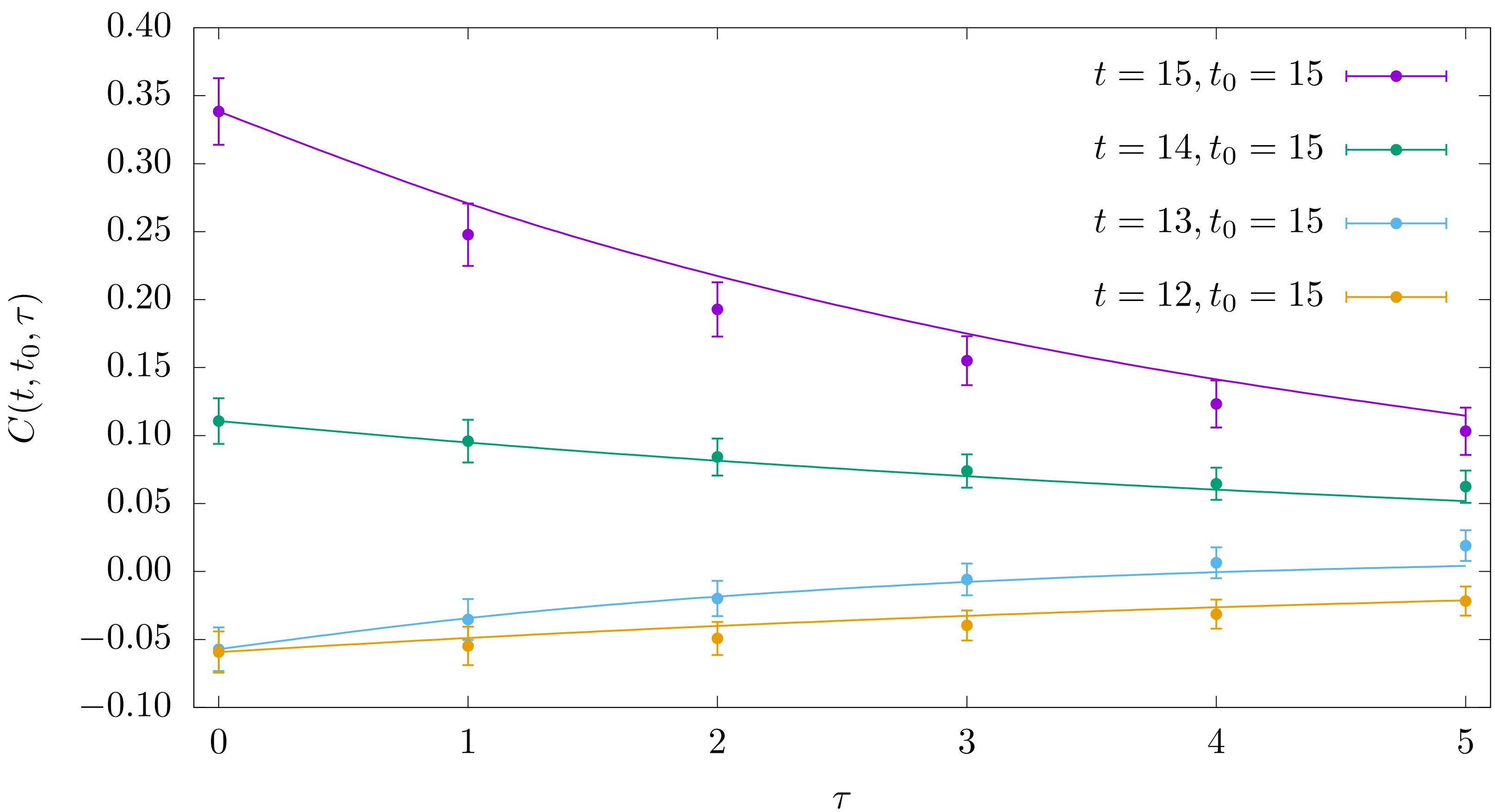
$$\frac{\partial}{\partial \tau} C(t, t_0, \tau) = \frac{\partial}{\partial t} \left(D(t) \frac{\partial}{\partial t} C(t, t_0, \tau) \right) - \frac{1}{\tau_{\rm tunn}} C(t, t_0, \tau)$$

- For pure gauge simulations with the DBW2 gauge action, we found

a (fm)	$\tau_{\rm tunn}$ Periodic	$\tau_{\rm tunn}$ Open	$D(\frac{T-a}{2})/a^2$ (MDU^{-1}) Periodic	$D(\frac{T-a}{2})/a^2$ (MDU^{-1}) Open
0.2000	20(1)	20(2)	0.090(12)	0.099(30)
0.1600	56(3)	51(3)	0.1018(73)	0.113(18)
0.1326	185(20)	162(19)	0.1085(97)	0.088(15)
0.1143	561(59)	737(143)	0.1080(56)	0.120(14)
0.1000	2350(389)	1973(621)	0.1155(29)	0.116(12)

Diffusion of Topological Charge - 2+1 flavors

- $12^3 \times 32 \times 12$, 1/a = 1 GeV, ID+MDWF ensemble with $m_\pi \approx 300$ MeV generated.
- $24^3 \times 64 \times 12$, 1/a = 2 GeV, ID+MDWF ensemble with $m_\pi \approx 300$ MeV generated.
- Producing these with both open and periodic boundary conditions and fitting to diffusion model.
- Example of fit for open boundary conditions case:



Diffusion of Topological Charge - 2+1 flavors

	D/a^2	τ	#
periodic	0.094(15)	6.94(76)	800/1
open	0.050(33)	7.20(96)	700/1

0.2 fm 2+1 flavor results

Quenched DBW2 Results

	τ_{tunn}		$D(\frac{T-a}{2})/a^2$ (MDU^{-1})	
a (fm)	Periodic	Open	Periodic	Open
0.2000	20(1)	20(2)	0.090(12)	0.099(30)
0.1600	56(3)	51(3)	0.1018(73)	0.113(18)
0.1326	185(20)	162(19)	0.1085(97)	0.088(15)
0.1143	561(59)	737(143)	0.1080(56)	0.120(14)
0.1000	2350(389)	1973(621)	0.1155(29)	0.116(12)

0.1 fm 2+1 flavor results

	D/a^2	τ	#
periodic	0.092(43)	79(22)	800/10
open	–	–	–

2+1 flavor results are preliminary

Conclusions

- The DSDR term has allowed us to simulate domain wall fermions on coarse lattices.
- Global chiral fits have revealed that our Iwasaki+DSDR ensembles have small a^2 scaling violations.
- We are currently generating $1/a = 1$GeV ensembles with physical volumes of $(4.8\ \text{fm})^3$, $(6.4\ \text{fm})^3$, and $(9.6\ \text{fm})^3$.
- These coarse lattices will allow studies of finite volume effects for a variety of observables. g-2 HVP is an important target
- Physical 2+1 flavor ensembles with size $24^3 \times 64 \times 24$ useful for high statistics and exploring techniques.
- We are measuring the production of and diffusion of topological charge on these ensembles.