Dirac-mode analysis for quark number density and its application for deconfinement transition

Takahiro Doi (RIKEN)

in collaboration with
Kouji Kashiwa (Yukawa Institute for Theoretical Physics, Kyoto University)

reference

"Dirac-mode expansion of quark number density
and its implications of the confinement-deconfinement transition,"
TMD, K. Kashiwa, arXiv:1706.00614 [hep-lat].

Lattice 2017, Granada Conference and Exhibition Centre, Jun. 20, 2017, Granada, Spain

Contents

- Introduction
  - Quark confinement at imaginary chemical potential and quark number holonomy
  - Chiral symmetry breaking and Dirac eigenmode
- Dirac mode analysis for the quark number holonomy
  - Large quark mass regime
  - Small quark mass regime

Introduction: Quark-confinement at imaginary chemical potential

- Roberge Weiss (RW) periodicity at imaginary chemical potential

$$Z_{\rm QCD}(T,\theta) = Z_{\rm QCD}(T,\theta + 2\pi k/3)$$

$\mu = i\mu_{\rm I}$: Imaginary chemical potential

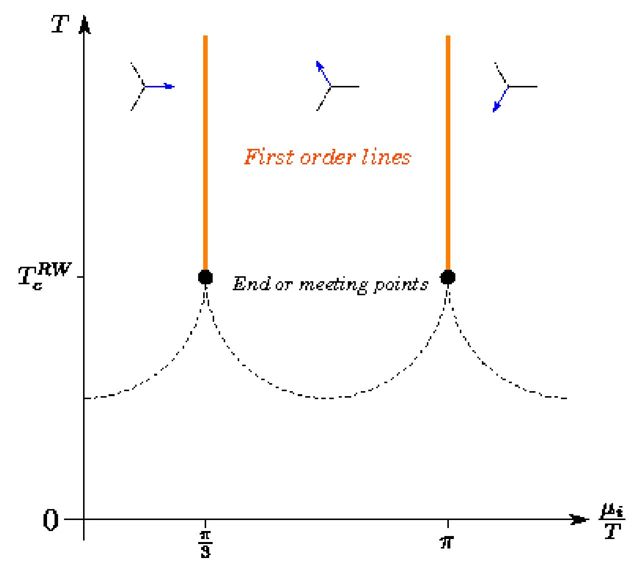
$\theta = \mu_{\rm I}/T$: dimensionless imaginary chemical potential

- The RW periodicity has deep relations with the free-energy degeneracy.

Free energy is

$T \gg T_{\rm RW}$: degenerated

$T \ll T_{\rm RW}$: not degenerated



A. Roberge and N. Weiss, Nucl. Phys. B275, 734 (1986).

C. Czaban, et al., Phys.Rev. D93 (2016) 054507.

Introduction: Quark number holonomy

K. Kashiwa and A. Ohnishi, Phys. Lett. B750 (2015).
K. Kashiwa and A. Ohnishi, Phys. Rev. D93 (2016) 116002.

- Quark number holnomy is defined as a integral of the dimensionless quark number $\tilde{n}_q$ on a loop for the direction of the dimensionless imaginary chemical potential θ.

$$\Psi(T) = \left[\oint_0^{2\pi} \left\{ \mathrm{Im} \left(\frac{d\tilde{n}_q}{d\theta} \Big|_T \right) \right\} d\theta \right]$$

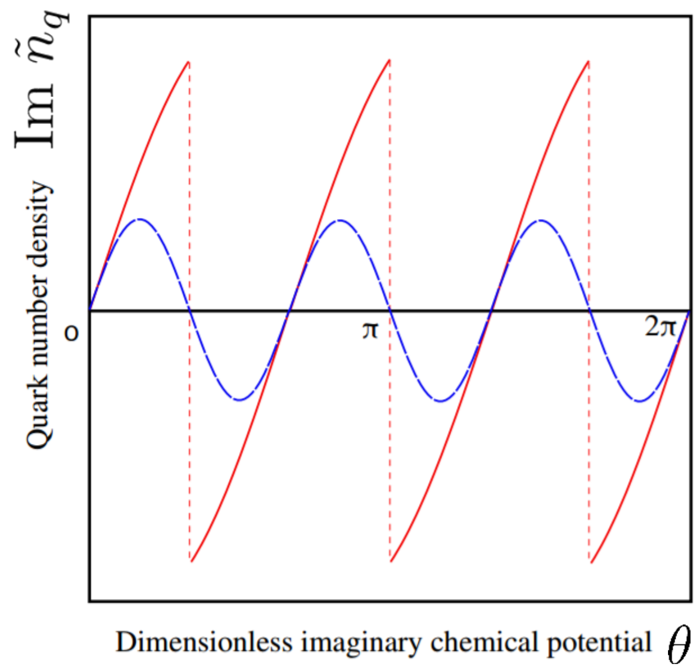
$\tilde{n}_q = C n_q$: dimensionless quark number

- it counts gapped points of the quark number density along θ direction.

- Quark number holonomy is a topological order parameter for the confinement-deconfinement transition.

Red solid line: high temperature

Blue dotted line: low temperature



Introduction – Chiral Symmetry Breaking

- Chiral symmetry breaking : chiral symmetry is spontaneously broken

$$\mathrm{SU(N)_L} \times \mathrm{SU(N)_R} \underset{\text{CSB}}{\rightarrow} \mathrm{SU(N)_V}$$

for example $\mathrm{SU(2)}$

- u, d quarks get dynamical mass(constituent mass)
- Pions appear as NG bosons

- Chiral condensate : order parameter for chiral phase transition

$$\langle \bar{q}q \rangle \begin{cases} \neq 0 & \text{(chiral broken phase)} \\ = 0 & \text{(chiral restored phase)} \end{cases}$$

- Banks-Casher relation

$$\langle \bar{q}q \rangle = - \lim_{m \to 0} \lim_{V \to \infty} \pi \langle \rho(0) \rangle$$

$\hat{\not{D}}$:Dirac operator

$\hat{\not{D}}|n\rangle = i\lambda_n |n\rangle$:Dirac eigenvalue equation

$\rho(\lambda) = \frac{1}{V} \sum_n \delta(\lambda - \lambda_n)$:Dirac eigenvalue density

Contents

- Introduction
  - Quark confinement at imaginary chemical potential and quark number holonomy
  - Chiral symmetry breaking and Dirac eigenmode
- Dirac mode analysis for the quark number holonomy
  - Large quark mass regime
  - Small quark mass regime

Dirac mode analysis for the quark number holonomy

- Quark number holonomy: a topological order parameter for deconfinement transition.

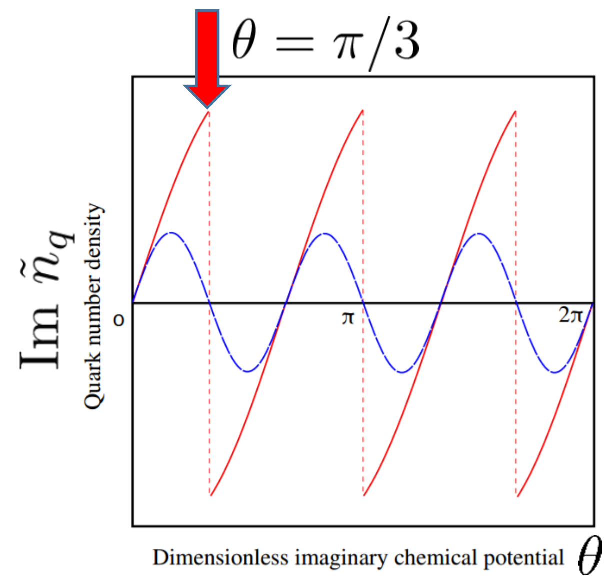
$$\Psi(T) = \left[\oint_0^{2\pi} \left\{ \mathrm{Im} \left(\frac{d\tilde{n}_q}{d\theta} \bigg|_T \right) \right\} d\theta \right]$$

- Low-lying Dirac eigenmodes: the important modes for chiral symmetry breaking.

$$D|n\rangle = \lambda_n |n\rangle$$

D : Wilson-Dirac operator

We investigate the relation between confinement and chiral symmetry breaking by calculating the contribution from the low-lying Dirac modes to the quark number holonomy using lattice QCD.



The quark number holonomy
Is determined by the behavior of $\tilde{n}_q(\theta = \pi/3)$,
so we mainly investigate it in terms of the Dirac modes.

Dirac mode analysis for the quark number holonomy in the large quark number regime

- We consider large quark mass m.
  So, we can expand the quark number density by 1/M. (Hopping parameter expansion)

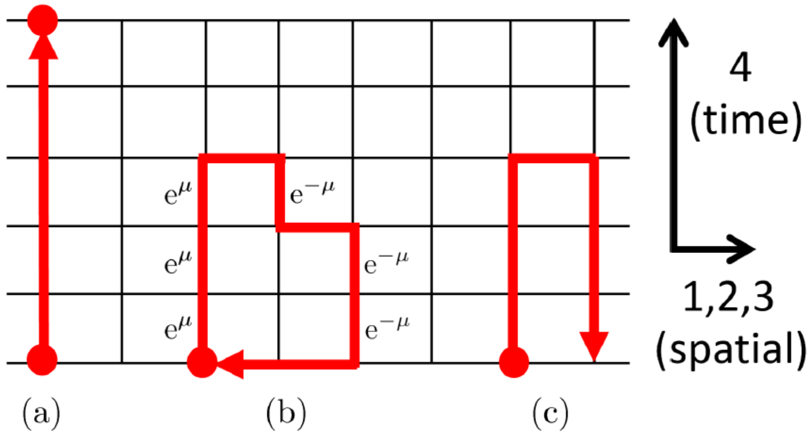
$$n_q = \frac{1}{V}\left\langle \mathrm{Tr}_{\gamma,c}\left[\frac{\partial D}{\partial \mu}\frac{1}{D+m}\right]\right\rangle$$

$$= \frac{1}{MV}\left\langle \mathrm{Tr}_{\gamma,c}\left[\frac{\partial D}{\partial \mu}\sum_{n=0}^{\infty}\left(-\frac{\hat{D}}{M}\right)^n\right]\right\rangle$$

$$= \sum (\text{temporally widing loops})$$

Wilson-Dirac operator with μ

$$D = -\frac{1}{2}\sum_{k=1}^{3}\left[P(+k)\hat{U}_k + P(-k)\hat{U}_{-k}\right] - \frac{1}{2}\left[e^{\mu}P(+4)\hat{U}_4 + e^{-\mu}P(-4)\hat{U}_{-4}\right] + M\cdot\hat{1} \qquad (M = 4+m)$$



Possible paths appearing in the expansion

(a): temporally winding loop

(b): spatially loop ⇒ μ-independent term
⇒ no contribution to n_q

(c): gauge-variant ⇒ no contribution to n_q

Dirac mode analysis for the quark number holonomy in the large quark number regime

- Any temporally winding loops can be expressed using the Dirac eigenmodes.

e.g.) Polyakov loop L TMD, H. Suganuma, T Iritani PRD90 (2014) 094505.

$$L = -\frac{2^{N_4-1}}{12V}\sum_n \lambda_n^{N_4-1}\langle n|\hat{U}_4|n\rangle \qquad D|n\rangle = \lambda_n|n\rangle$$

(Note) Low-lying Dirac-modes have little contribution to the Polyakov loop because of damping factor: $\lambda_n^{N_4-1}$

- Low-lying Dirac modes have little contribution to the other temporally winding loops

- The quark number density shows the similar behavior with the Polyakov loop.

$$n_q \sim L$$

c.f.) "dual chiral condensate～dressed Polyakov loop"

C. Gattringer, PRL97 (2006) 032003.
E. Bilgici, F. Bruckmann, C. Gattringer, C. Hagen,PRD77 (2008) 094007.

Dirac mode analysis for the quark number holonomy in the small quark number regime

- Next, we consider small quark mass m.
  In this case, it is difficult to analytically discuss the quark number density in terms of the Dirac eigenmodes.
  So, we numerically investigate it.

- We consider the IR-cutoff dependence of quark number density $n_q(\theta = \pi/3)$.

- Specifically, we calculated the IR-cut quark number density:

$$n_{\rm q}^{\rm cut}(\Lambda_{\rm IR}) = \frac{1}{n_{\rm q}} \sum_{|\Lambda_n|>\Lambda_{\rm IR}} n_{\rm q}^n$$

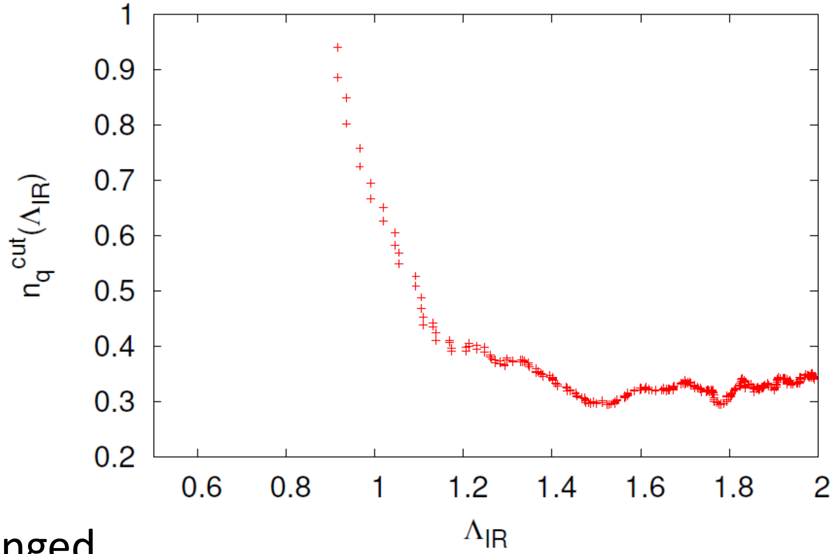
lattice setup:

- Configuration: quenched QCD
  - standard plaquette action
  - gauge coupling: $\beta = \frac{2N_{\rm c}}{g^2} = 6.0$
  - lattice size: $N_\sigma^3 \times N_\tau = 6^3 \times 5$
  - temperature: $T \simeq 400$ MeV
- Wilson-Dirac operator
  - periodic boundary condition for link-variables and Dirac operator
  - hopping parameter: $\kappa = 0.151515$
  - chemical potential: $\mu a = 0.2094i$
    $\Rightarrow \theta = \pi/3$

Dirac mode analysis for the quark number holonomy in the small quark number regime

- The absolute value of the quark number density is drastically changed by removal of the low-lying Dirac eigenmodes.

$$n_{\mathrm{q}}^{\mathrm{cut}}(\Lambda_{\mathrm{IR}}) = \frac{1}{n_{\mathrm{q}}} \sum_{|\Lambda_n|>\Lambda_{\mathrm{IR}}} n_{\mathrm{q}}^{n}$$



- However, the sign of the quark number density is not changed.

- Therefore, the quark number holonomy is not changed because it counts gapped points of the quark number density along θ direction.

- This result suggests that the important modes for chiral symmetry breaking are not important for confinement-deconfinement transition defined by the quark number holonomy.

Summary

1. We have analyzed the quark number density in terms of the Dirac eigenmodes.

2. In the large-quark mass regime, we analytically found that
the quark number density shows the same behavior as the Polyakov loop,
an usual order parameter for the confinement-deconfinemnet transition.

3. In the small-quark mass regime, we analytically found that
the quark number density is drastically changed by removal of the low-lying Dirac eigenmodes. However, its sign is not changed.
The results means that the quark number holonomy, the topological order parameter for the deconfinement transition, is insensitive to the low-lying Dirac eigenmodes, the important modes for chiral symmetry breaking.

4. Our results suggest that
low-lying Dirac modes do not play important roles in deconfinement transition as the topological phase transition defined by the quark number holonomy.