Flux tubes in $N_f = 2 + 1$ QCD with external magnetic fields

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Outline

- Phenomenological Motivation
- Anistropic static $Q\overline{Q}$ potential in the presence of (eB)
- Lattice observables and numerical setup.
- Flux Tube at $(eB) = 0$. Possible systematics.
- Flux Tube at $(eB) \neq 0$. Discussion of anisotropies.
- Summary and Perspectives

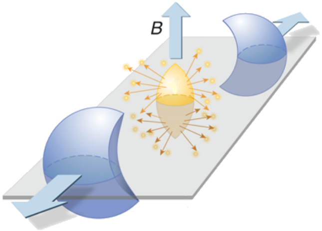
Phenomenological Motivation

Everyday magnetic fields are much smaller than the scale of strong interactions. Anyhow, although rare, particular phenomenological contexts in which magnetic fields, eB, are comparable with the scale Λ^2_{QCD} exist:

- Astrophysics - in a class of neutron stars, called **magnetars**: $eB \sim 10^{10}$ T
  [Duncan and Thompson, '92]
- Cosmology - during the **ElectroWeak phase transition**: $eB \sim 10^{16}$ T
  [Vachaspati, '91]
- Heavy ion collisions - at LHC in **non-central HIC**: $eB \sim 10^{15}$ T $\sim 15 m_\pi^2$
  [Skokov, Illarionov and Toneev, '09]

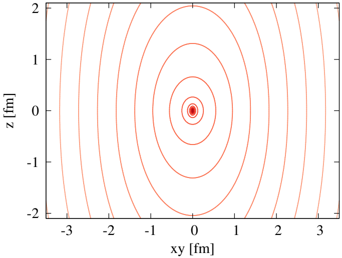
$1\ \text{GeV}^2 \sim 5 \cdot 10^{15}$ T

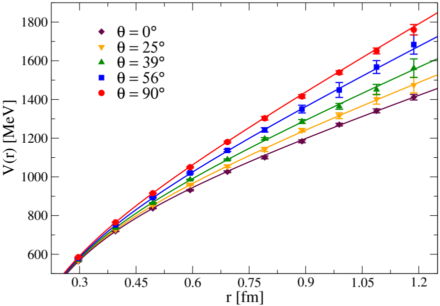
¿what happens to QCD physics and nonperturbative properties?



The effect of B on the static $Q\overline{Q}$-Potential

At nonzero magnetic field the static $Q\overline{Q}$-potential becomes anisotropic. With respect to the case at zero field, it grows steeply along the X and Y directions and lowly along the Z direction.





Results for $N_f = 2 + 1$ QCD at the physical point at $a = 0.0989$ fm on a $48^3 \times 96$ lattice.

Magnetic field value: $(eB) \simeq 1\text{GeV}^2$.

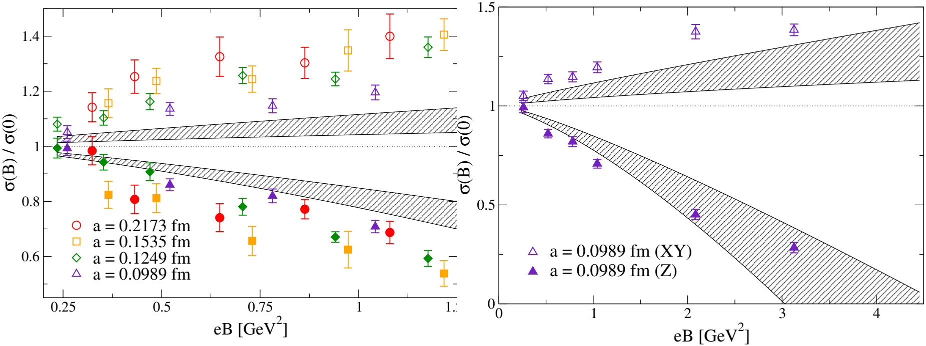
Figures from [Bonati, D'Elia, Mariti, Mesiti, N., Rucci and Sanfilippo, PRD 2016].

The effect of B on the static $Q\overline{Q}$-Potential

If we consider the standard Cornell parametrization

$V_{Q\overline{Q}}(\vec{r}) = C + \sigma|\vec{r}| + \frac{\alpha}{|\vec{r}|}$

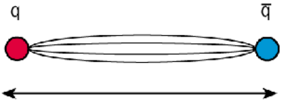
we find that both α and σ gets different values according to the angle between the field and the charge separation directions.



ARE OTHER CONFINEMENT-RELATED OBSERVABLES AFFECTED BY B? WHAT ABOUT THE FLUX TUBE?

Flux tube: lattice observable

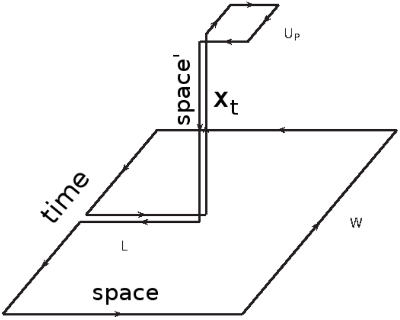
We consider a $Q\overline{Q}$ pair at relative distance $\vec{d} = d\hat{u}$ (with $\hat{u} = \hat{x},\ \hat{y},\ \hat{z}$).



We consider the chromoelectric field E_l longitudinal with respect to the $Q\overline{Q}$ separation. We measure it in between the pair, along a transverse direction.

$$E_l(x_t) = \lim_{a\to 0} \frac{1}{a^2 g} \left[\frac{\langle Tr(WLU_P L^\dagger)\rangle}{\langle Tr(W)\rangle} - \frac{1}{N_c} \frac{\langle Tr(W)\, Tr(U_P)\rangle}{\langle Tr(W)\rangle} \right]$$

[Di Giacomo, Maggiore, Olejnik, 1990]
[Cea, Cosmai, Cuteri, Papa, 2017]



W is a squared Wilson Loop
U_P is a coplanar plaquette

Numerical Setup

In order to discretize the Euclidean Feynman path integral

$$\mathcal{Z} = \int \mathcal{D}U \, \mathcal{D}\overline{\psi} \, \mathcal{D}\psi \;\; e^{-S_{YM}[U] - \overline{\psi}_f M_f^D \psi_f} = \int \mathcal{D}U \;\; e^{-S_{YM}[U]} \prod_f (\det M_f^D [U])^{1/4}$$

we adopt the following discretizations for QCD with $N_f = 2 + 1$:

- Gauge sector: tree level improved Symanzik action [Weisz, Nucl Phys B '83; Curci, Menotti and Paffuti, Phys Lett B '83]
- Fermionic sector: rooted staggered fermions $\oplus$ stout smearing improvement [Morningstar and Peardon, PRD '04]

The bare parameters we adopted in our simulations have been taken from [Borsanyi, Endrodi, Fodor et al., JHEP '10 and following].
They correspond to the "physical" line of constant physics ($m_\pi^{LAT} = m_\pi^{PHYS}$).

Simulations performed on BlueGene/Q-Fermi and on KNL-Marconi at CINECA, Italy.

Lattices

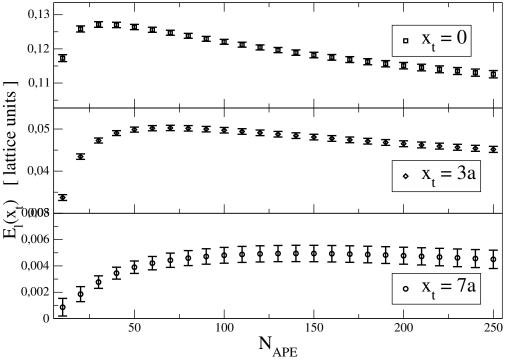
$a = 0.1249$ fm $\longrightarrow 40^4$	$\longrightarrow$	$V_4 = (5 \text{ fm})^4$	$(eB) = 0$
$a = 0.0989$ fm $\longrightarrow 48^3 \times 96$	$\longrightarrow$	$V_4 = (5 \text{ fm})^3 \times 10$ fm	$(eB) = 0$ and $(eB) \neq 0$
$a = 0.0898$ fm $\longrightarrow 48^4$	$\longrightarrow$	$V_4 = (4.3 \text{ fm})^4$	$(eB) = 0$
$a = 0.0796$ fm $\longrightarrow 48^4$	$\longrightarrow$	$V_4 = (3.8 \text{ fm})^4$	$(eB) = 0$

Flux tubes under smearing

Smearing

Wilson Loops-related observales are extremely noisy. We smear the confs:

1) 1 HYP smearing on the temporal links
2) N_{APE} spatial APE smearing steps $\alpha = 0.1666$ on spatial links



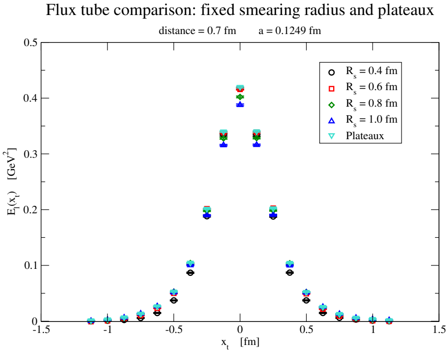
$48^3 \times 96$ lattice at $a = 0.0989$ fm and $(eB) = 0$

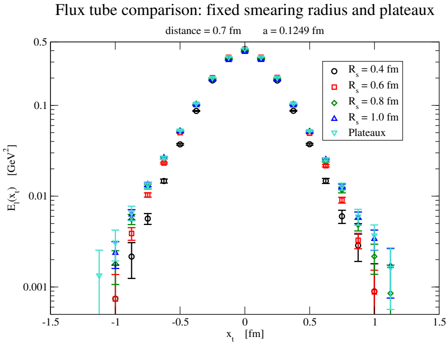
Measures every 10 steps

WHICH SMEARING LEVEL IS THE "CORRECT" ONE?

Peak/plateaux and fixed smearing comparison

Example: flux tube at $d = 0.7$ fm on the 40^4 lattice at $a = 0.1249$ fm.





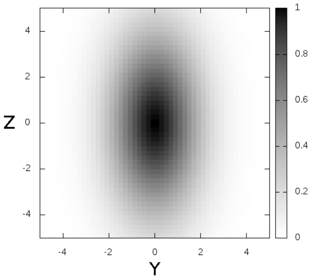
The hope is to avoid this ambiguity by taking the continuum limit. Although it is very interesting, we will not discuss here this issue.

Effect of B on the flux tube

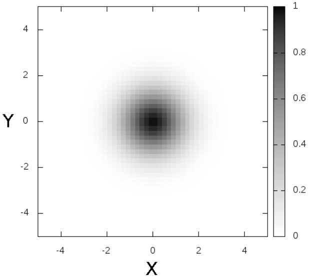
We consider a magnetic field oriented along the Z axis.

¿Does the B field modifies the flux tube shape? Does it induce any anisotropy? Is the cylindrical symmetry of the tube broken?

X (or Y) separation



Z sepration



We need to study individually the possible direction combinations!

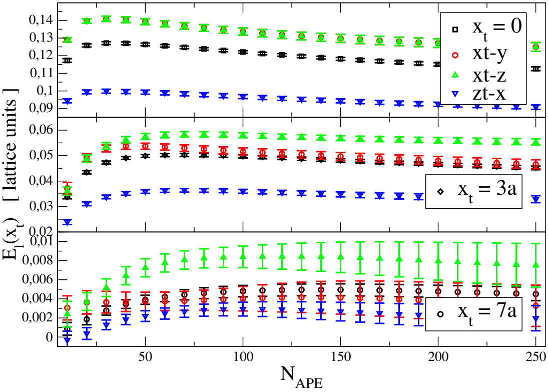
$[XT - Y \;\; \& \;\; YT - X]$

$[XT - Z \;\; \& \;\; YT - Z]$

$[ZT - X \;\; \& \;\; ZT - Y]$

Flux tube vs smearing steps at non zero field

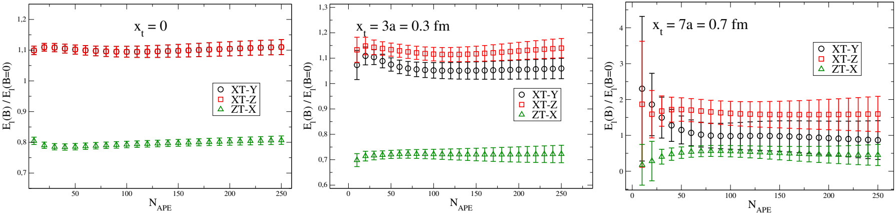
As an example, we plot the $(eB) \simeq 2$ GeV^2 case.
Physical distance between the quarks: $7a \simeq 0.7$ fm.



Flux tube profile scales with the smearing similarly as in the $(eB) = 0$.
Likely, the ratios $E_l(x_t, B \neq 0)/E_l(x_t, B = 0)$ are smearing independent (or less dependent).

Ratios of flux tubes vs smearing steps

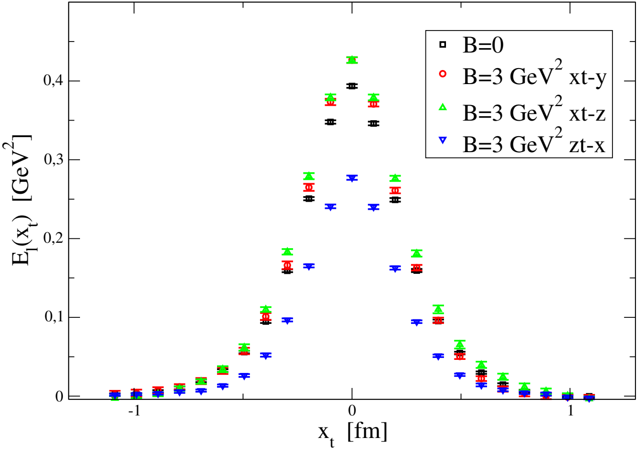
Actually, taking the ratios between observables at nonzero and zero field almost removes the smearing ambiguity.



For all the values of (eB) and for all the distances the class $ZT - X$ is significantly different from the other two.
At large fields and short $Q\overline{Q}$ separations, also the classes $XT - Y$ and $XT - Z$ becomes non-degenerate.

Flux tube profile at $(eB) \neq 0$

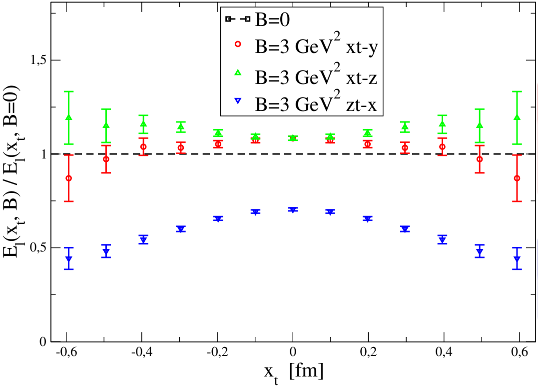
Profile of the flux tube at $(eB) = 3\text{GeV}^2$ compared to that at $(eB) = 0$.
We consider $R_s \simeq 0.7$ fm ($N_{APE} = 80$) and $Q\overline{Q}$ separation of $\simeq 0.7$ fm.



It is smearing dependent...

Ratio of flux tube profiles

The same data as before (symmetrized), but plotting now the ratio of the tube at $(eB) \neq 0$ and that at $(eB) = 0$.



$Q\overline{Q}$ separation X-Y

Enhancement and anisotropic behaviour.

$Q\overline{Q}$ separation Z

Reduction and shrinking.

Fitting the flux tube profile

We assume the QCD flux tube (inside the QCD vacuum) to be the dual counterpart of the Abrikosov tube inside an ordinary superconductor.
It has been observed [Cea, Cosmai, Cuteri and Papa, 2017] that the flux tube profile is well described by the function [Clem, 1975]:

$$E_l(x_t) = \frac{\phi\mu^2}{2\pi\alpha}\frac{K_0(\sqrt{\mu^2 x_t^2+\alpha^2})}{K_1(\alpha)} \qquad \rightarrow \text{fitting params: } \phi,\ \mu,\ \alpha$$

ϕ [total flux]
$\lambda = 1/\mu$ [London penetration lenght]
$\xi = \frac{\alpha}{\sqrt{2}\mu}(1 - K_0^2(\alpha)/K_1^2(\alpha))^{-1/2}$ [coherence lenght]
$\kappa = \lambda/\xi$ [Ginzburg-Landau parameter]

Other quantities can be computed from the fitting params, in particular:

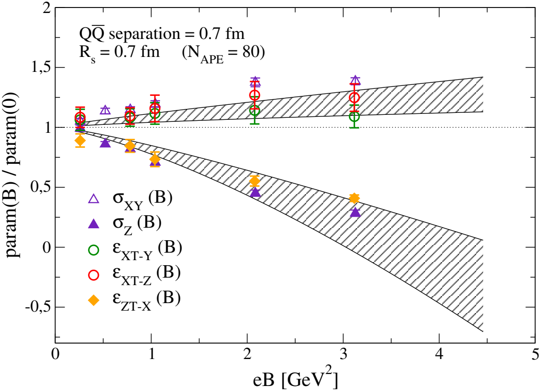
- energy per unit lenght
- width of the tube

Energy per unit lenght - string tension

We consider the energy per unit lenght of the flux tube

$$\epsilon = \int d^2 x_t E_l^2(x_t)/2 = \frac{\mu^2\phi^2}{8\pi}\left(1 - \left(\frac{K_0(\alpha)}{K_1(\alpha)}\right)^2\right)$$

It should be related to the string tension σ.

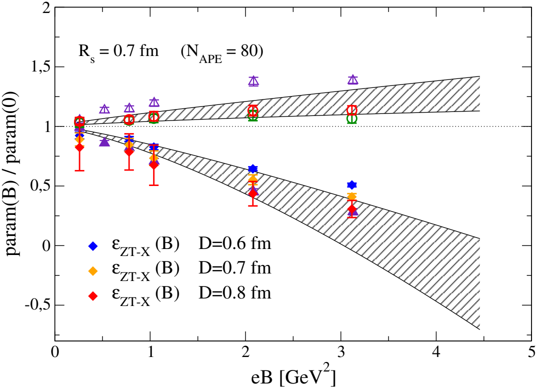


The bands are the continuum limit of $\sigma(B)/\sigma(0)$ based on 4 lattice spacings considering fields $(eB) < 1.25\text{GeV}^2$.

The behaviour of the ratios is rather similar.

Energy per unit lenght - string tension

The value of ϵ depends on the distance between the sources, and also the anisotropy does.
In the case of quarks separated along Z, we approach the value of the σ ratio by separating more the quarks.



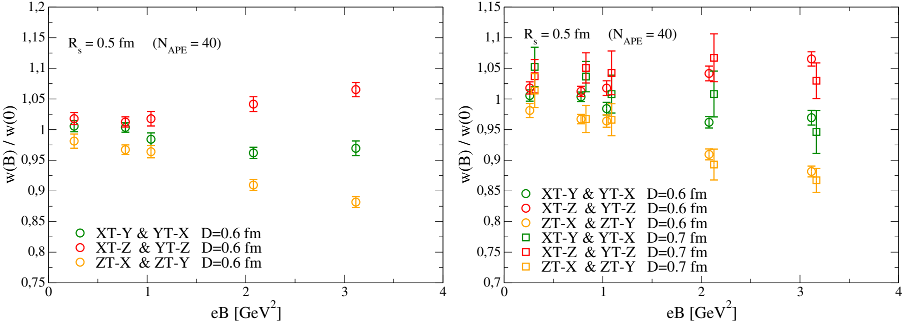
This is reasonable, since $\sqrt{\epsilon}$ takes contribution both from σ and α at short distances. On the other hand, at larger distances only σ should be relevant.

Width of the flux tube

We consider the mean square root width

$$\sqrt{w^2} = \sqrt{< x_t^2 >} = \frac{2\alpha K_2(\alpha)}{\mu^2 K_1(\alpha)}$$

And compute, again, the ratios $\sqrt{w^2(B)}/\sqrt{w^2(B=0)}$:



This ratio is less stable against smearing.

Summary

- All of the presented results are still preliminary.
- Gauge fields gets modified by the magnetic field
- Discussion of systematics for $E_l(x_t)$ at $(eB) = 0$.
- Determination of $E_l(x_t)$ at $(eB) \neq 0 \quad \longrightarrow$ anisotropy
- Study of the energy per unit lenght ratio and of the width ratio.
- Open issues and perspectives:
  - Systematic study of the flux tube at $(eB) = 0$. Continuum limit.
  - Continuum limit of the results at $(eB) \neq 0$.
  - ¿What happens at even larger fields?

Thank you for the attention!

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