SU(3) breaking and the pseudoscalar spectrum in multi-taste QCD

Michael Creutz
BNL

Are rooted staggered fermions QCD?

- NO despite 10 years of controversy

Issue: symmetries of rooted staggered fermions

- incompatable with the chiral anomaly

Concentrate on two ideosyncracies in the pseudoscalar spectrum

- 1. Different octet states have different taste degeneracies

- 2. Spurious states appear at unphysical masses

Review: Pseudoscalar octet masses in 3 flavor QCD

$$
\begin{array}{ccccc}
 & K_0 & & K_+ & \\
\pi_- & & \pi_0, \eta & & \pi_+ \\
 & K_- & & \overline{K}_0 &
\end{array}
$$

Lowest order meson mass spectrum in the sigma model

Ignore the 9th pseudoscalar η'

acquires large mass from the anomaly

Effective theory for fluctuations about $\langle\overline{\psi}\psi\rangle = v$

- model fluctuations $\overline{\psi}_L^j \psi_R^k \sim v\ \Sigma^{jk}$ (j and k label flavors)
- pion fields encoded in $\Sigma = \exp(i\pi_\alpha\lambda_\alpha/f_\pi) \in SU(3)$
- Gell-Mann matrices λ_α generate $SU(3)$ $(\mathrm{Tr}\lambda_\alpha\lambda_\beta = 2\delta_{\alpha\beta})$
- f_π phenomenological constant, about 93 MeV.

Effective theory continued

- Kinetic term

$$L_0 = \frac{f_\pi^2}{4}\mathrm{Tr}(\partial_\mu \Sigma^\dagger \partial_\mu \Sigma)$$

- Expanding to second order in the pion fields

$$L_0 = \mathrm{const} + \frac{1}{2}\partial_\mu \pi_\alpha \partial_\mu \pi_\alpha + \ldots$$

Chiral symmetry: without mass, two independent $SU(3)$ symmetries

$$\begin{array}{l}\psi_L \to \psi_L \; g_L \\ \psi_R \to \psi_R \; g_R\end{array} \qquad\qquad g_L, g_R \in SU(3)$$

in the effective theory

$$\Sigma \to g_L^\dagger \; \Sigma \; g_R.$$

Spontaneous breaking gives usual octet of 8 pseudoscalars

Masses break SU(3) and chiral symmetries

- for the effective theory take

$$L = L_0 - \frac{v f_\pi^2}{4} \mathrm{Re}\ \mathrm{Tr}(m\ \Sigma).$$

- diagonalize the mass matrix (use $m \to g_R^\dagger\ m\ g_L$)

$$m = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}$$

- expand L to quadratic order in pion fields

$$L = \text{const} + \frac{1}{2}\partial_\mu \pi_\alpha \partial_\mu \pi_\alpha + \frac{1}{2}\pi_\alpha M_{\alpha\beta}\pi_\beta$$

where

$$M_{\alpha\beta} = \text{Re Tr } \lambda_\alpha m \lambda_\beta$$

- diagonalize $M_{\alpha\beta}$ for the lowest order meson masses

And the answer is

$$M^2_{\pi_+} = M^2_{\pi_-} \propto \frac{m_u + m_d}{2}$$

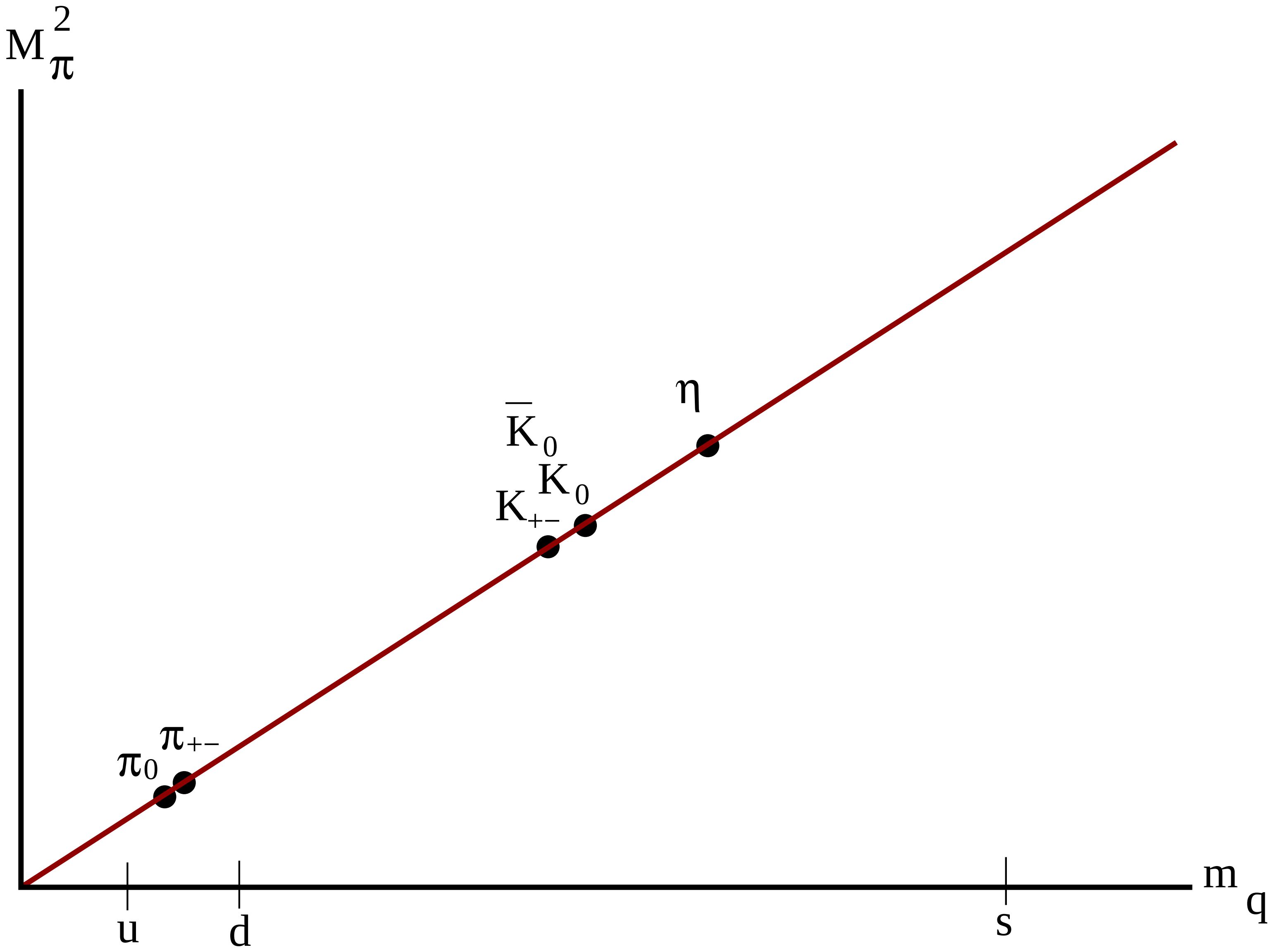
$$M^2_{K_+} = M^2_{K_-} \propto \frac{m_u + m_s}{2}$$

$$M^2_{K_0} = M^2_{\overline{K}_0} \propto \frac{m_d + m_s}{2}$$

$$M^2_{\pi_0} \propto \frac{1}{3}\left(m_u + m_d + m_s - \sqrt{m_u^2 + m_d^2 + m_s^2 - m_u m_d - m_u m_s - m_d m_s}\right)$$

$$M^2_{\eta} \propto \frac{1}{3}\left(m_u + m_d + m_s + \sqrt{m_u^2 + m_d^2 + m_s^2 - m_u m_d - m_u m_s - m_d m_s}\right)$$

(π_0 η mixing since $M_{38} \neq 0$)


M_π^2
η
$\overline{K}_0$
K_0
K_{+-}
π_{+-}
π_0
u
d
s
m_q

Turn on a $N_t = 4$ factor for each flavor

Motivated by 4 “tastes” of staggered fermions

12 distinct quark species

Assume exact taste symmetry

Before flavor breaking: $SU(12)$ symmetry

- expect $143 = 12^2 - 1$ pseudo-goldstone bosons
- 8 by 8 meson mass matrix $\longrightarrow$ 143 by 143

Actually three distinct $SU(4)$ taste groups

- one for each flavor $u,\ d,\ s$
- classify states by representations in each
- relevant $SU(4)$ representations: $1, 4, \overline{4}, 15$

analog of $SU(3)$: $1, 3, \overline{3}, 8$

Kaons and charged pions

- each involves two distinct flavors
- appear in a $(4_q, \overline{4}_{\overline{q}'})$ representation

  16 equivalent taste combinations
- meson masses average constituents $M^2 \propto \frac{1}{2}(m_q + m_{q'})$
- this accounts for $16 \times 6 = 96$ of 143 expected pseudo-Goldstones

Neutral mesons: $q = q'$ $\longrightarrow$ $4 \otimes \overline{4} \rightarrow 1 \oplus 15$

- for each flavor $\overline{q}q$ $\rightarrow$ a taste 15 plus a taste singlet

The taste 15 combinations cannot mix: (independent taste groups)

a taste 15 of $i\overline{u}\gamma_5 u$ states $M^2 \propto m_u$

a taste 15 of $i\overline{d}\gamma_5 d$ states $M^2 \propto m_d$

a taste 15 of $i\overline{s}\gamma_5 s$ states $M^2 \propto m_s$

Remaining: three taste singlet combinations $\overline{u}u, \overline{d}d, \overline{s}s$

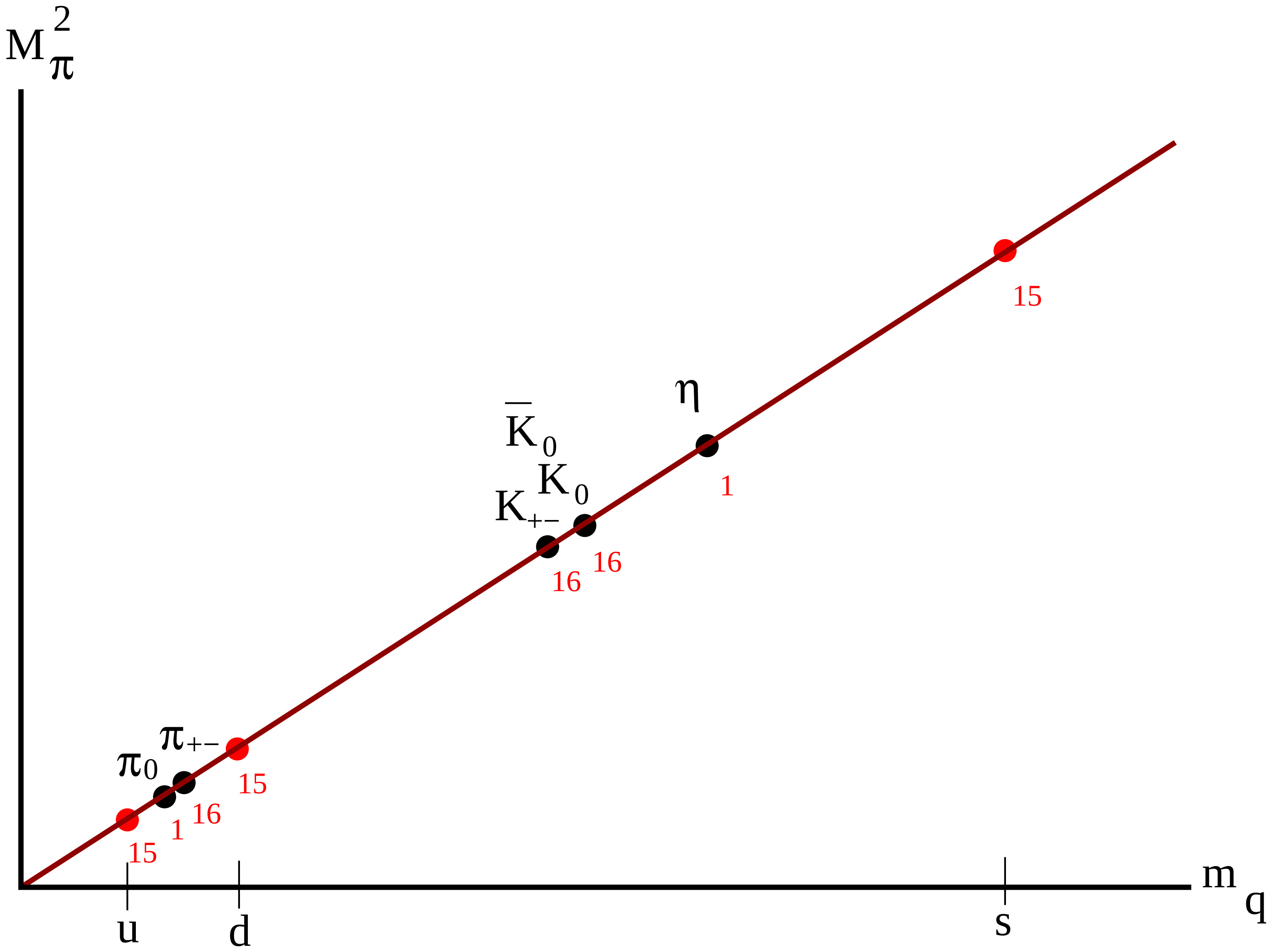
- η': taste and flavor singlet, heavy and ignored here
- flavor non-singlet combinations do mix

  same mixing matrix as the single taste theory

$$M_{\pi_0}^2 \propto \frac{1}{3}\left(m_u + m_d + m_s - \sqrt{m_u^2 + m_d^2 + m_s^2 - m_u m_d - m_u m_s - m_d m_s}\right)$$

$$M_{\eta}^2 \propto \frac{1}{3}\left(m_u + m_d + m_s + \sqrt{m_u^2 + m_d^2 + m_s^2 - m_u m_d - m_u m_s - m_d m_s}\right)$$

Aubin and Bernard



$$143 = 16 \times 6 + 15 \times 3 + 2$$

Rooting replicas

Replace $|D| \longrightarrow |D^4|^{1/4}$

With N_t copies of a valid fermion formulation; i.e. Wilson fermions

- rooting is a mathematical identity (if $|D| > 0$)
- propagator always has only one pole
- $16 \to N_t^2$ goes to unity as $N_t \to 1$
- $15 \to N_t^2 - 1$ goes to zero as $N_t \to 1$

Staggered fermions

NOT replicas: doublers are chiral partners

- propagator always has four poles, even with rooting
- the taste 15 multiplets remain

One exact chiral symmetry for each flavor

- a member of each 15 must survive as a Goldstone boson

The questions

1. How can the SU(3) octet be recovered?

   - Different flavors have different taste degeneracies

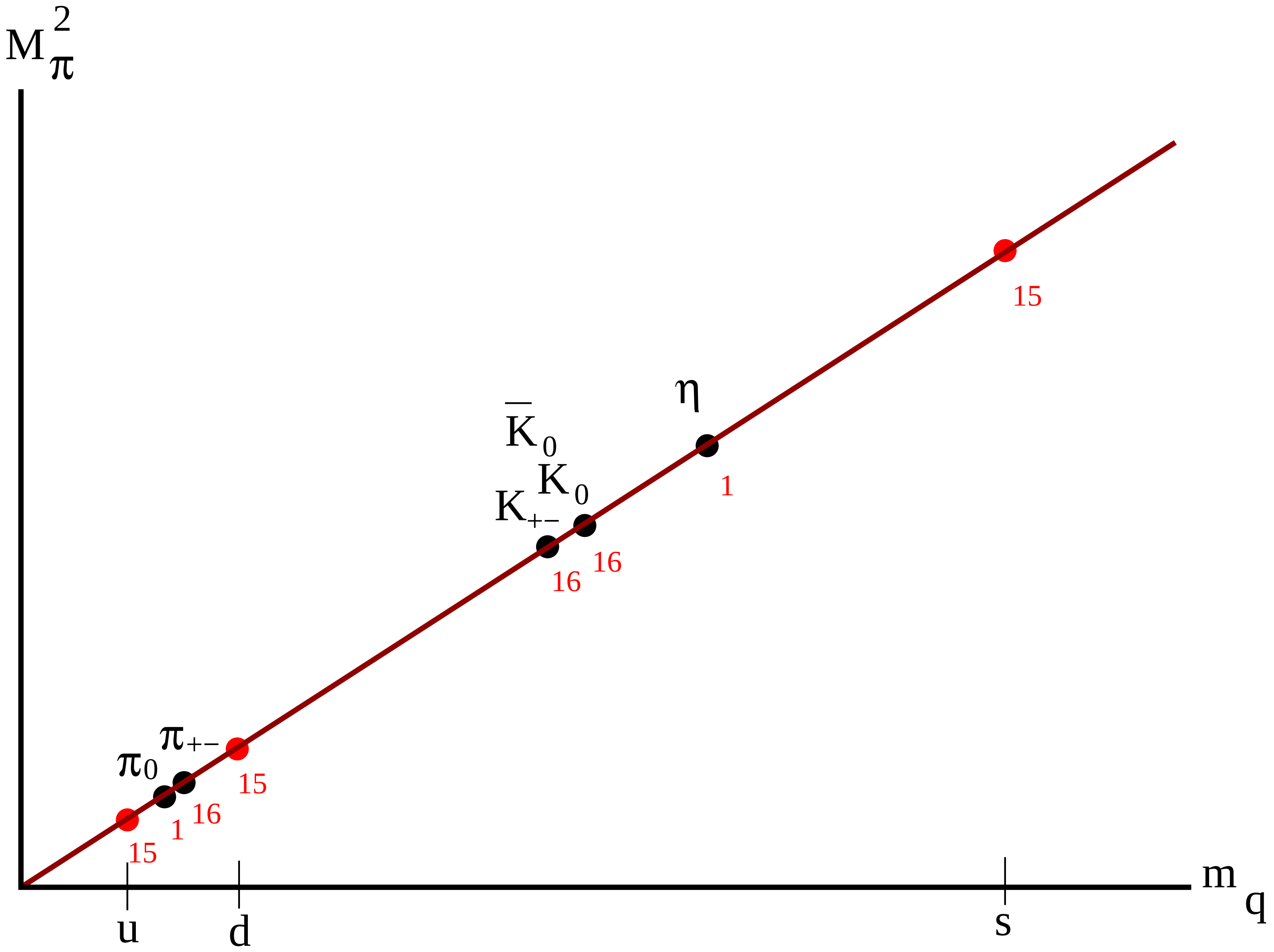
     π_0 and η: a single taste combination

     kaons and charged pions: 16 equivalent taste combinations

2. What happens to the 3 unphysical 15's?

   - masses incompatible with any physical particles

     $M^2 \propto m_u, \ m_d, \ m_s$



M_π^2
m_q
u
d
s
π_0
π_{+-}
K_{+-}
K_0
$\overline{K}_0$
η
15
1
16
15
16
16
1
15